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Machine Design

265 Illustrations

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MACHINE DESIGN
(Five Parts)

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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed

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CONTENTS

NOTE.—This volume is made up of a number of separate Sections, the page numbers of which usually begin with 1 To enable the reader to distinguish between the different Sections, each one is designated by a number preceded by a Section mark (§), which appears at the top of each page, opposite the page number In this list of contents, the Section number is given following the title of the Section, and under each title appears a full synopsis of the subjects treated This table of contents will enable the reader to find readily any topic covered.

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MACHINE DESIGN—(*Continued*)

MACHINE DESIGN

(PART 1)

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DESIGN IN GENERAL

DESIGN OF DETAILS

1. In this Section, rules and formulas for designing many of the most common details of machines are given. These rules are in some cases based on considerations of strength, as treated in *Strength of Materials;* in other cases, the wear to which the parts are to be subjected has been the principal element in determining the given proportions. In all cases, however, the practice of successful designers has been followed in preference to mere theoretical principles.

2. The first work a young designer is called on to do is usually that of making drawings of details of machines, the general plans of which have been developed by his superiors. The principal dimensions of these details will be given him, and he will be required to furnish drawings from which patternmakers, blacksmiths, and machinists can make up the parts ready to be put in their proper place in the machine.

3. In most shops, such parts as bolts, nuts, screws, pipe fittings, etc., and often other simple parts of machines, are obtained from factories, where they are made in large quantities by special machinery. If the shop is a large one, there may be a separate department where these parts are made according to fixed standards. The designer should ascertain the practice of the shop in this regard, and in all

cases make the details conform with these standards. He should also know the kind of material available, the methods employed by the shop in working this material, and the capacity and principal dimensions of the tools and appliances for doing the work, in order that the details as designed may be built in the most economical manner. When designing a machine part, the processes that will be used in making it should be kept in mind; this will often prevent constructions that would be very difficult and expensive if built with the machinery in use in the shop for which the design is made.

4. In making designs of details, it is always advisable to draw them to as large a scale as can be used conveniently. The scale commonly used for small details is full size; for larger ones, 6, 3, or $1\frac{1}{2}$ inches to the foot may be used. A scale of 4 or 2 inches to the foot should never be used if it can be avoided.

5. It should be remembered that the object of a detail drawing of a machine part is to show the workmen in the clearest possible manner how the part is to be made and finished, so that it will take its proper place in the completed machine and do the work for which it is intended. The designer must therefore be very careful to make the drawing show the form and dimensions of each portion as clearly as possible; the drawing should also show plainly the kind and quality of material to be used, and the finish, if any, to be given the different surfaces. Sections should be used wherever the general views do not show the form with perfect clearness.

It is well for the designer to imagine himself in the position of a man in the shop who knows nothing of the machine, and to study his drawing carefully to see whether anything can possibly be lacking that will be required to make perfectly clear the ideas he wants carried out.

6. Ordinary dimensions are expressed in feet and inches, and the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc. of an inch. The fractions $\frac{1}{3}$, $\frac{1}{5}$, or $\frac{1}{6}$ should never be used in dimensions, as the scales

that mechanics use are not divided into these fractions. The most common scales in use by mechanics for ordinary work are 2-foot rules divided into inches, numbering from 1 to 24; for this reason many draftsmen give all dimensions less than 2 feet in inches and fractions of an inch, and dimensions greater than 2 feet in feet, inches, and fractions of an inch.

Unless great accuracy is required, decimals are never used in giving dimensions. If decimal values are obtained from the calculations, they are expressed in the nearest $\frac{1}{8}$, $\frac{1}{16}$, or $\frac{1}{32}$ inch, according to the degree of accuracy required. In particular cases, where extreme accuracy is needed, the dimensions may be expressed in decimals; as, for example, in denoting the pitch of gear-teeth.

GENERAL DESIGN OF A MACHINE

7. The methods to be employed in the general design of a machine will vary so much with different conditions that no fixed rules or methods of procedure can be given. The first thing necessary is a thorough knowledge of the work that the machine must do, together with its location and surroundings and the conditions under which it must perform its work. Keeping these facts in mind, the designer must apply his knowledge of the principles of mechanics, strength of materials, and the design and construction of details in such a way as to accomplish the desired end in the simplest and most direct manner consistent with the conditions imposed.

All machines consist of different combinations of simple principles, and in order to be successful, the designer must become thoroughly acquainted with these principles and the relation they bear to one another. A study of machines that have been built for similar work will greatly assist in suggesting ideas for the new machine.

In many cases, it will be necessary to make more or less complete drawings of a number of different plans before a satisfactory result can be obtained A combination that appears feasible at first may be found to be impracticable

when drawn out in detail and the parts proportioned so as to give the necessary strength. In other cases, the motion of some part may be found to be limited in such a way as to interfere with the proper working of the machine. The difficulty or expense of manufacture may also make some otherwise good design impracticable.

8. In all classes of work—whether it is the designing of details or of more complicated combinations—all calculations, notes, and sketches should be kept in such form that they may readily be preserved for future reference. These notes should be dated and given titles that will make their purpose perfectly clear. In this way, ideas that may be impracticable for the particular case for which they were originally developed can be kept for a possible future use; moreover, the results of many hours spent in calculation will be preserved and thus make repetition unnecessary. Some engineering establishments supply their draftsmen with books made of Manila paper bound in board covers, and all calculations, notes, and sketches are made in these books instead of on loose sheets of paper that are likely to become lost.

9. The following practical rules are often neglected by inexperienced designers:

Make all parts that are subject to wear or breakage accessible for the purpose of inspection, repairs, or renewal.

Provide means for adjusting all parts that are subject to wear.

Make careful provision for lubrication.

Use links and rotating pieces for guiding motion in preference to slides.

Use cranks, levers, belts, and gear-wheels for transmitting motion in preference to cams, screws, or worm-wheels.

Wherever possible, make the motion of all parts positive; that is, avoid the use of weights or springs for producing motion.

Use through bolts or T*-headed bolts instead of tap bolts or studs, wherever it can be done.*

10. Designers are often required to furnish an estimate of the weight and cost of a machine from the drawings.

This is done in the following manner: The volume of the different details is estimated by means of the principles of mensuration; the weight can then be obtained by multiplying the volume of each piece by the weight of a cubic unit of the material of which it is composed. When the weights are known, the cost of the material is easily found from the known market values. The time that will be required for fitting and finishing the different pieces is then estimated and charged for according to the rates paid for that work. In this way the cost of the machine may be estimated with a degree of accuracy that will depend on the knowledge the estimator has of the time required to do different kinds of work in the shops, and his skill in making approximate calculations of the volumes of irregular-shaped bodies.

PROPERTIES OF MATERIALS

11. Cast Iron.—The general properties of the various metals used in the construction of machine parts have been given in *Materials of Construction*, but some additional remarks about cast iron are necessary.

The great advantage of cast iron is the ease with which it may be given any desired form. Shapes that cannot possibly be forged from wrought iron may be cast with comparative ease. The operation of casting is as follows: A pattern having the exact shape of the required part is first made. This pattern is usually of pine, though metal is sometimes employed when the castings are small and a great number are required. The pattern is placed in a bed of sand or loam, in which it leaves, after being removed, an impression, or cavity, called the *mold*. Molten metal is then poured into the mold, and, after cooling, the casting is withdrawn and finished to the required dimensions.

In cooling, cast iron contracts about $\frac{1}{8}$ *inch per foot in each direction*. This contraction, however, may vary from $\frac{1}{16}$ to $\frac{3}{16}$ inch per foot, according to the condition of the iron used and its preliminary treatment. On account of this contraction, commonly called the *shrinkage*, the pattern must be

made larger than the required casting. Therefore, in practice, a *shrink rule*, which is about $\frac{1}{8}$ inch longer per foot than the standard rule, is always used in constructing the pattern.

12. A serious difficulty experienced in the use of cast iron is its liability to be thrown into a state of *internal stress*, on account of inequality of cooling after being poured into the mold. It is a matter of experience that the amount of contraction depends on the size and thickness of the casting. In general, thick and heavy parts contract more than thin ones; consequently, a casting composed of both thick and thin parts will sometimes differ from the form desired. Again, one part of a casting may cool and solidify while another part is still in a molten condition; therefore, the contraction of the latter must strain the part already solidified. Internal stress is thus produced, which must to some degree reduce the effective strength of the casting.

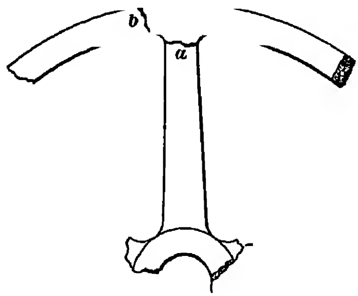


FIG 1

Take, for example, the case of a pulley. When the rim is thin, but rigid, it is likely to contract and solidify first, and the subsequent contraction of the arm may induce a fracture, as shown at *a* Fig. 1. If, however, the arms set first, the subsequent contraction of the rim may cause a fracture, as shown at *b*.

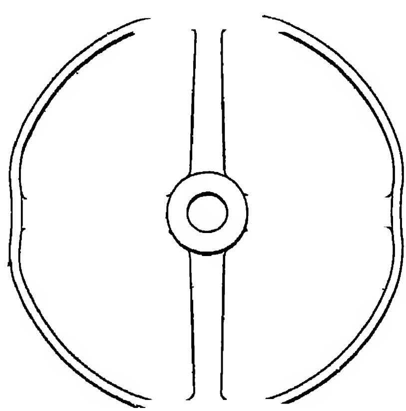
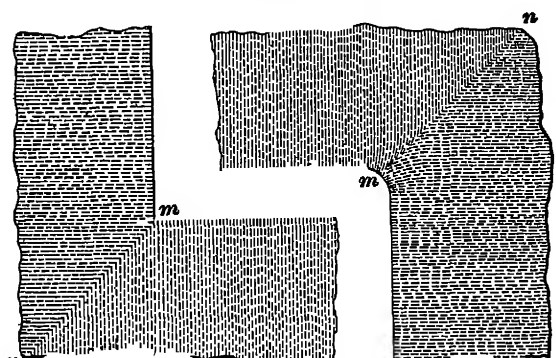


FIG 2

13. When pulleys are cast with thin rims that are not rigid, the casting sometimes takes the form shown in Fig. 2. The rim is drawn in at the points where it joins the arms, because the arms solidify and contract after the rim has set, and the latter, not being sufficiently rigid to withstand the pull of the arms, is distorted.

These internal stresses make cast iron an unreliable material for the construction of parts requiring strength; and it should be the aim of the designer to prevent these stresses, as far as may be, by making all parts that are to be cast as uniform in thickness as possible, and avoid having a large boss or hub appear in a comparatively thin part, or having a very thick part meet a very thin one.

14. It is found that, in cooling, the iron crystals arrange themselves perpendicularly to the surface of the casting. For this reason, an inside angle *m*, Fig. 3 (*a*), is a source of



(*a*) (*b*)

FIG 3

weakness, the casting having a tendency to break through the line *m n*. Such corners should be rounded, as shown at *m*, Fig 3 (*b*), in which case the crystalline arrangement renders the casting much stronger.

15. Variation of the Elastic Limit.—Experiments have indicated that if a machine part is subjected to repeated tensile stresses, the material shows an *increase* in its elastic limit while under a tensile stress, but that the elastic limit in

regard to *compressive* stresses has been *decreased*. For parts subjected to stresses of this nature, it would evidently be unsafe to apply working stresses that approach the elastic limit.

A machine part is generally supposed to be safe if so proportioned as to withstand the *maximum* stresses to which it may be subjected. This supposition leaves out of consideration whether these stresses are constant or whether they change in value, in direction, or in both. That this view is erroneous has been shown by a large number of experiments performed by various investigators of high repute. These experiments prove that the ultimate strength of a material is *lowered* when the load varies in amount alone or in both amount and direction.

16. Maximum and Minimum Stresses.—With a constant load, the usual custom is to make the working stress of the material $\frac{1}{4}$ or $\frac{1}{5}$ of its ultimate strength, in order to allow for possible defects and unforeseen stresses. The factors of safety are then 4 and 5, respectively. To increase this factor to 6 and 7 for variable stresses, as is customary, will not always bring the working stresses within safe limits. The reason for this will be seen from the following explanations. In a revolving shaft with a variable load, the stress at any one place will change between a maximum and a minimum if the shaft is subjected to torsion only. Denoting the values of the maximum and minimum stresses by S_{max} and S_{min}, respectively, the total variation v will be $S_{max} - S_{min}$.

In a revolving loaded axle, as, for instance, on a railroad car, the stresses at any one place are constantly alternating between tension and compression. Considering the tensile stresses as positive, and the other stresses as negative, and subtracting the latter from the former,

$$v = S_{max} - (- S_{min}) = S_{max} + S_{min}$$

To show the difference between these two cases, let it be supposed that in a revolving shaft the unit torsional stress, that is, the unit stress due to the action of a torsional force, varies between 10,000 and 3,000 pounds; v is therefore

10,000 − 3,000 = 7,000 pounds. In an axle revolving under the conditions just mentioned, the stress may change from 7,000 pounds tensile to 7,000 pounds compressive. The variation v is then 7,000 − (− 7,000) = 14,000 pounds.

17. Repeated, or Periodic, Stresses.—When a material is subjected to regularly repeated stresses, its ultimate strength suffers a further reduction. To ascertain the relative strengths of various materials working under such conditions, it is assumed that if two test pieces having identical dimensions will withstand the same amount of variation in stress, repeated a similar number of times, they are of the same strength. It was found by experiment that two bars of wrought iron were able to withstand the same number of repeated stresses before breaking, when the stresses were of the following nature: In one case the stresses were tensile, varying from 31 to 18,713 pounds per square inch; in the other they alternated from a tensile stress of 8,317 pounds to a compressive stress of 8,317 pounds. The average variation in both cases was about 17,000 pounds. Assuming the ultimate tensile strength of wrought iron to be about 50,000 pounds, it is seen at once that the effect of such stresses is to reduce the tensile strength in one case to 18,713 pounds and in another to 8,317 pounds.

18. Classification of Stresses.—The various stresses to which a machine part may be subjected can be arranged in three classes as follows:

1. Stresses that do not vary.
2. Stresses that vary in value, but not in direction.
3. Stresses that vary both in value and in direction.

It has been found that the ultimate stresses that machine parts coming under these three classes may resist, are, in round numbers, in the proportions of 3 : 2 : 1. In cases where the stresses vary between zero and the maximum or where the stresses change direction, it is supposed that the changes are gradual. If the load is applied suddenly, but without velocity, the values found from the proportion just given should be divided by 2.

SCREWS, BOLTS, AND NUTS

SCREWS AND SCREW THREADS

19. Screws are used in machine construction for three purposes: (1) As fastenings for clamping or joining parts together; (2) for the transmission of motion; and (3) for producing pressure.

A screw provided with a head and a nut for fastening parts together is generally called a *bolt.*

20. Sharp V Thread.—Screw threads are usually triangular or square in section, the triangular form being best for bolts, and the square form best for screws transmitting motion. The fundamental **V** thread is shown in Fig. 4, and is termed the **sharp V thread** to distinguish it from other triangular threads in which the top and bottom of the thread are flattened or rounded off.

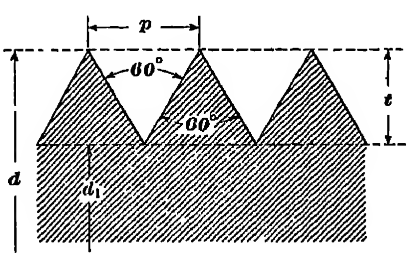


FIG 4

The angle between the sides of the thread is 60°. The distance p from a point on one thread to a corresponding point on the next is called the **pitch** of the screw. The **lead** of a screw is the distance that a nut will move, in an axial direction, during one complete turn of the screw. In the case of a single-threaded screw, like that shown in Fig. 5, the pitch and the lead are equal.

As shown in Fig. 4, a section of a single thread is an equilateral triangle, the altitude of which is t. Hence,

$$t = p \cos 30° = .866p \qquad (1)$$

Let d = outside diameter of bolt;

d_1 = diameter at root of thread.

Then, $d_1 = d - 2t = d - 1.732p$ (2)

The diameter d_1 must always be used in calculating the strength of a bolt.

Letting n represent the number of threads per inch in a screw, then

$$p = \frac{1}{n}, \text{ and } n = \frac{1}{p} \qquad (3)$$

The pitch of the sharp **V** thread corresponds in general with that of the United States standard thread.

21. United States Standard Thread.—In 1864 an improvement on the sharp **V** thread was proposed by William Sellers and subsequently adopted by the United States Navy and War Departments. From its originator, this thread is known as the **Seller's thread,** but is more commonly called the **American thread,** or the **United States standard thread.** In Fig. 5 is shown a general view, and in Fig. 6 an enlarged section of this thread.

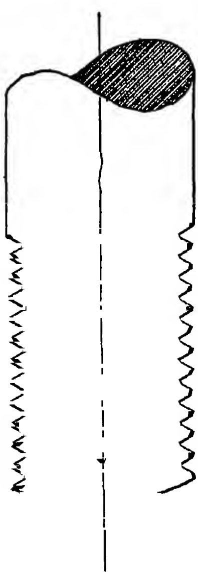


FIG 5

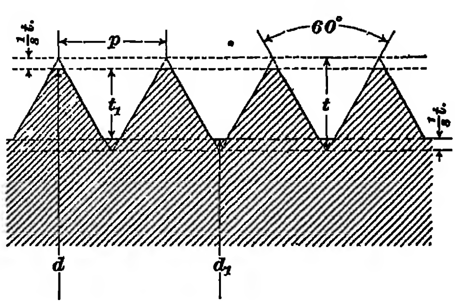


FIG 6

The difference between the sharp **V** thread and the United States standard thread will be seen by comparing Figs. 4 and 6. Thus, in the latter thread one-eighth the altitude of the triangle is cut off from the apex, and the angle at the

root is filled in to a like depth. Hence, the *real depth* of the thread t_1 is three-fourths the altitude of the triangle; that is, $t_1 = \frac{3}{4}t$.

But, $$t = p \cos 30° = .866p;$$
hence, $$t_1 = \frac{3}{4}t = .65p$$

22. The pitch of the United States standard thread depends on the diameter of the bolt, and may be ascertained approximately by the following empirical formula, in which d represents the outside diameter of the bolt:

$$p = .24\sqrt{d + .625} - .175 \text{ inch} \qquad (1)$$

The diameter d_1 at the root of the thread may be found by the formula

$$d_1 = d - 2t_1 = d - 1.3p \qquad (2)$$

Let n = number of threads per inch;
p = pitch.

Then, $$p = \frac{1}{n}, \text{ and } n = \frac{1}{p} \qquad (3)$$

Consequently, $$d_1 = d - \frac{1.3}{n} \qquad (4)$$

EXAMPLE.—The external diameter of a bolt is $1\frac{3}{8}$ inches. Find the pitch, the number of threads per inch, the depth of thread, and the diameter of bolt at root of a United States standard thread.

SOLUTION.—From formula **1**, just given,

$$p = .24\sqrt{1.375 + .625} - .175 \text{ in.} = .164 \text{ in. Ans.}$$

From formula **3**,

$$n = \frac{1}{.164} = 6, \text{ nearly}$$

Use six threads per inch, then, $p = \frac{1}{6} = .167$ in. Ans.
$t_1 = .65p = .65 \times .167 = \frac{7}{64}$ in., nearly. Ans.
$d_1 = d - 2t_1 = 1\frac{3}{8} - \frac{7}{32} = 1\frac{5}{32}$ in. Ans.

Table II, at the end of this Section, gives the number of threads per inch, diameter of bolt at root of thread, and effective area of bolt at root of thread, United States standard sizes.

23. Whitworth Thread.—In England, the **Whitworth system** of threads is used. This thread differs from the United States standard mainly in the size of the angle between the sides of the thread and in the rounding off of

its point and bottom. As shown in Fig. 7, the angle between the sides is equal to 55°, and the amount cut away from the top and bottom is $\frac{1}{6}t$. The remaining corners are not left sharp, as in the United States standard thread, but are

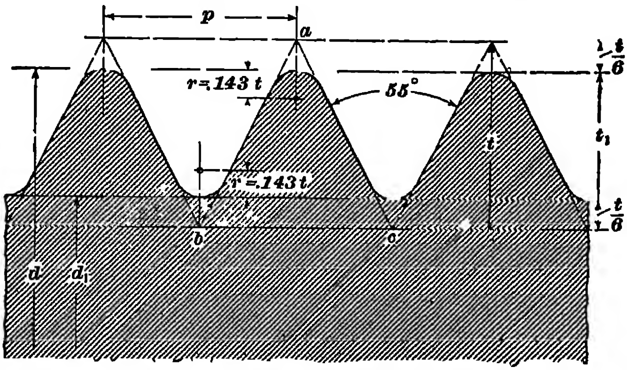


FIG 7

rounded by an arc with a radius equal to .143 t. The real depth of the thread t_1 is two-thirds the altitude of the triangle abc; that is, $t_1 = \frac{2}{3}t$.

But, $t = .5\,p\cot 27.5° = .96\,p$ (1)

and $t_1 = \frac{2}{3} \times .96\,p = .64\,p$ (2)

24. The pitch of the Whitworth thread depends on the diameter of the bolt, and may be obtained approximately by means of the following formula, in which d represents the diameter of the bolt:

$$p = \frac{1}{n} = .08\,d + .04 \qquad (1)$$

To find the diameter d_1 at the root of the thread, the following formula is used:

$$d_1 = d - 2\,t_1 = d - 1.28\,p \qquad (2)$$

The diameter d_1 must always be used in calculating the strength of the bolt.

Letting n represent the number of threads per inch in a screw,

$$n = \frac{1}{p} \qquad (3)$$

Consequently, $$d_1 = d - \frac{1.28}{n} \qquad (4)$$

EXAMPLE.—The external diameter of a bolt is $1\frac{1}{2}$ inches Assuming that the bolt is threaded according to the Whitworth system, find the pitch, the number of threads per inch, the depth of the thread, and the diameter of bolt at root of thread

SOLUTION.—By formula **1**,

$$p = .08 \times 1.5 + .04 = .16 \text{ in} \quad \text{Ans}$$

By formula **3**, $n = \dfrac{1}{.16} = 6$, nearly

Using six threads per inch, $p = .167$ in. Ans

By formula **2**, Art. **23**,

$$t_1 = .64 \times .167 = \tfrac{7}{64} \text{ in., nearly} \quad \text{Ans.}$$

By formula **2**, $d_1 = 1\frac{1}{2} - \frac{7}{32} = 1\frac{9}{32}$ in. Ans.

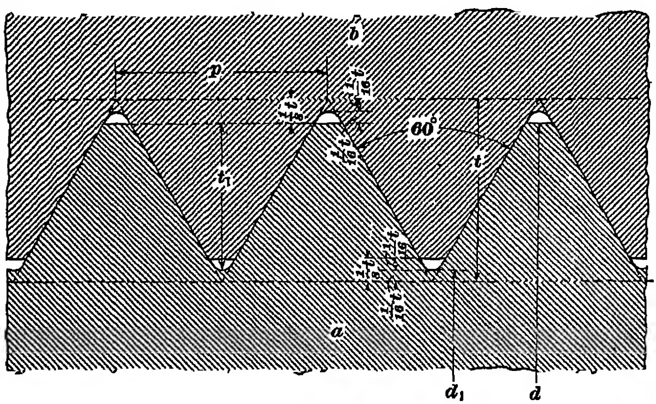


FIG 8

25. International Standard Screw Thread.—The Whitworth system has been used extensively on the continent of Europe, but on account of the inconvenience of combining the British with the metric units of length, attempts were made at various times to adopt a more convenient system as a standard. Finally, at a meeting held at Zurich, in 1898, an international committee agreed on proportions that in the main are the same as those of the French standard screw thread. The shape of the international thread is almost identical with the United States standard thread, but it has an advantage over the latter in that a clearance has been provided at the top of the thread, both in the nut and on the bolt. As shown in Fig. 8, where *a* represents the

bolt and b the nut, one-eighth of the altitude of the triangle has been cut off from the apex of the thread. The angle at the root is filled in to a depth of one-sixteenth of the total altitude, but the flat part thus produced is preferably rounded off as indicated The real depth of the thread t_1 is therefore thirteen-sixteenths of the altitude of the triangle; that is, $t_1 = \frac{13}{16} t$. As

$$t = p \cos 30° = .866 p$$
$$t_1 = \frac{13}{16} \times .866 p = .7036 p \qquad (1)$$

The diameter d_1 at the root of the thread may be found by the following formula, which may be used for values given in either millimeters or inches.

$$d_1 = d - 2 \times \frac{13}{16} t$$
$$= d - 1.625 t$$
$$= d - 1.407 p \qquad (2)$$

Table III, at the end of this Section, gives the proportions of the international standard screw thread in millimeters. To facilitate comparison with the United States standard thread, however, some of the values are expressed in inches; likewise, the number of threads per inch corresponding to the pitch are given in millimeters.

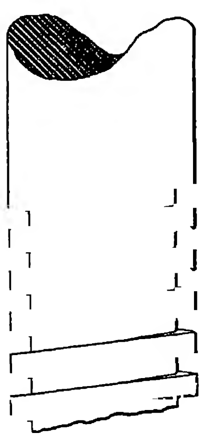


FIG 9

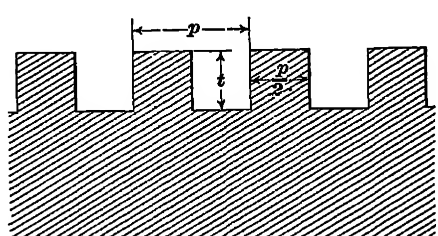


FIG 10

26. Square Thread.—In Fig. 9 is shown a screw with a **square thread**, and in Fig. 10 an enlarged section of the thread. As the name implies, and as seen from Fig. 10, the section of the thread is a square, each side of which is one-half the pitch.

The pitch of the square thread is usually taken as *double* that of the United States standard thread for the same diameter of bolt; for example, the pitch of a square thread on a $2\frac{1}{2}$-inch bolt or rod is $\frac{1}{2}$ inch. Hence, the pitch p may be found by inserting the values given for n in Table I in the following formulas:

$$p = \frac{2}{n} \qquad (1)$$

and

$$t = \frac{p}{2} \qquad (2)$$

EXAMPLE.—Find the pitch and the depth of the thread on a $1\frac{3}{8}$-inch bolt.

SOLUTION.—From Table I, $n = 6$. Inserting this value in formula 1, $p = \frac{2}{6} = \frac{1}{3}$ in. Ans

By formula 2, $t = \frac{p}{2} = \frac{1}{6}$. Ans.

The edges of the threads should be rounded off very slightly, so as to prevent them from being accidentally flattened, which would cause a nut to bind on the thread of the screw.

Several modifications of the square thread are in use. In some, the depth t is less than $\frac{p}{2}$, and in others the width of the thread is greater than $\frac{p}{2}$.

27. Acme Standard Thread.—As difficulties are encountered in making square-threaded screws and in giving the threads a smooth bearing surface, the tendency is to substitute in their place screws having threads that taper slightly from root to point, but retain the height of the corresponding square thread. By giving a taper to the thread, it does not have to be cut in the lathe, as is the case with the square thread, for it may also be cut with a die. Another advantage in favor of the tapering thread is that the nut—with which the screw engages—when composed of two parts, may be made to engage or disengage readily with the screw.

In the taper thread in general use the angle included between the sides of the thread is 29°. This variation of the square thread is known as the **Acme standard screw thread.** The angle of this thread is similar to that of the standard worm-thread, shown further on, but the threads differ in height.

In the Acme standard thread, the relation between the depth and the pitch is the same as that of the square thread; that is, the depth is one-half the pitch. To insure sufficient clearance, the bottom of the thread is lowered $\frac{1}{100}$ inch, making the true height of the thread

$$t = \frac{p}{2} + .01$$

Similarly, the diameter at the root of the thread in the nut is increased by $\frac{1}{100}$ inch, as shown in Fig. 11. The outside diameter of the screw and the diameter of the hole in the nut are not changed. This clearance of $\frac{1}{100}$ inch is con-

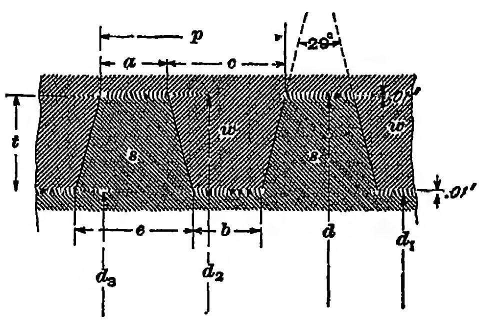


FIG 11

stant for all sizes of screws made with the Acme standard thread.

28. Fig 11 shows portions of a section through a screw s and nut w, d being the outside diameter of the bolt, and n the number of threads per inch of length. The dimensions of the various parts of the thread are as follows:

Diameter of screw at root of threads,

$$d_1 = d - \left(\frac{1}{n} + .02\right) \qquad \textbf{(1)}$$

Diameter at root of thread in nut, or the outside diameter of the tap,

$$d_s = d + .02 \qquad (2)$$

Diameter of drill for hole in nut,

$$d_s = d - \frac{1}{n} \qquad (3)$$

Depth of thread,

$$t = \frac{1}{2n} + .01 \qquad (4)$$

Thickness at top of thread,

$$a = \frac{.3707}{n} \qquad (5)$$

Width of space at bottom of thread,

$$b = \frac{.3707}{n} - .0052 \qquad (6)$$

Width of space at top of thread,

$$c = \frac{1}{n} - a \qquad (7)$$

Thickness at root of thread,

$$e = \frac{1}{n} - b \qquad (8)$$

Table IV, at the end of this Section, gives the dimensions of threads varying in number from one to ten per linear inch The diameters of the screws for the various numbers of threads per inch are not given, as there is no definite relation between pitch and diameter in the Acme standard thread. The pitch and diameter of the screw will depend on the purpose for which the screw is intended.

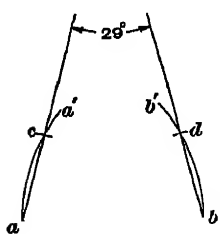


FIG. 12

To facilitate the laying off of an angle of 29°, the construction shown in Fig. 12 may be used. With the line $a\,b$ as a radius and the points a and b as centers, draw arcs $b\,b'$ and $a\,a'$. With a as a center and one-half of $a\,b$ as a radius, draw a short arc intersecting the arc $a\,a'$ at c; locate the point d in the same manner. Lines drawn through the points a and c and b and d will include an angle of 29°.

29. Knuckle Thread.—Another variation of the square thread is the **knuckle thread,** a section of which is shown in Fig. 13. In this type, the top and the root of the thread are rounded off, thus not only increasing its strength but also its friction surface. The knuckle thread is especially adapted to withstand rough usage.

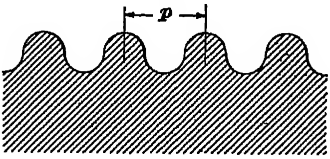


FIG 13

30. Trapezoidal, or Buttress, Screw Thread.—As shown in Fig. 14, one face of the trapezoidal thread is perpendicular to the axis of the screw, and the other is inclined at an angle of 45°. From the construction of the figure, it is evident that $t = p$, the pitch. To form the actual thread, an amount equal to $\frac{1}{8}\,t$ is cut off from the top and bottom of the triangle; hence, the real depth t_1 is $\frac{3}{4}\,t$.

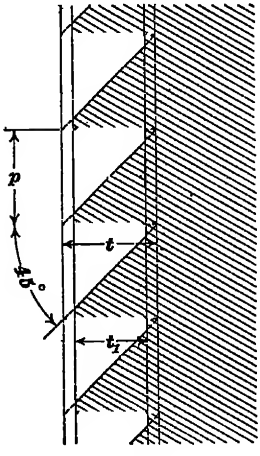


FIG 14

When this thread is used, it is generally for communicating motion, or where great resistance without any bursting tendency of the nut is required. The usual dimensions are given by the following formulas, in which the letters have the same meaning as before:

$$p = \frac{2d}{15} = \frac{d_1}{6} \qquad (1)$$

$$t_1 = \frac{d}{10} = \frac{d_1}{8} \qquad (2)$$

For example, suppose that the diameter of the screw is $1\frac{1}{2}$ inches; then, the pitch of a trapezoidal thread is

$$p = \frac{2d}{15} = \frac{2 \times 1\frac{1}{2}}{15} = \frac{1}{5} \text{ inch}$$

The number of threads per inch is $\frac{1}{p} = \frac{1}{\frac{1}{5}} = 5$, and the depth of the thread is

$$t_1 = \frac{d}{10} = \frac{1\frac{1}{2}}{10} = .15 \text{ inch}$$

A variation of the buttress thread is used for breech blocks in modern guns and for bolts that fasten the armor plates to the hull of a warship. The thread is rounded off at the top and root, and the apex angle, instead of having one side coincide with a normal to the axis of the screw, is increased to 60° by letting this side make an angle of 15° with the normal.

31. Comparison of Various Threads.—The relative advantages of the various forms of screw threads may be shown by considering the forces acting on the thread.

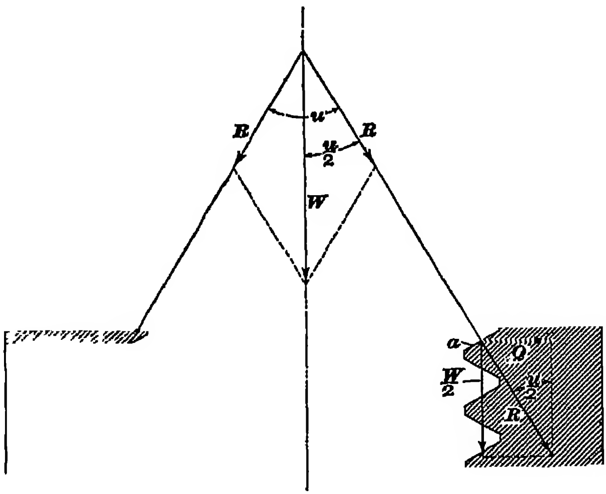


FIG. 15

Usually, the load on a bolt or screw acts in the direction of its axis; that is, a bolt used as a fastening is in tension, while a screw used to produce pressure is in compression. In either case, the load is carried by the reaction between the thread surfaces of the screw and the nut.

Let Fig. 15 represent part of a nut provided with a United States standard thread. The screw—not shown in the illustration—is supporting a load W, which is resolved into the two reactions R, one for each half of the nut. As the forces R are normal to any single element of the thread surface,

the angle $\frac{u}{2}$ between R and W will be $60° \div 2 = 30°$. The reaction R may again be resolved into two forces, one $\frac{W}{2}$ parallel with the axis of the bolt and the other Q, perpendicular to the axis. Then, $\frac{W}{2}$ represents the portion of the load carried by the surface a of the screw thread, while Q is the force that tends to burst the nut. For a given load W the force $\frac{W}{2}$ will remain constant whatever the angle u of the thread may be. On the other hand, the forces R and Q will increase as the angle u increases, and as the friction between two surfaces is proportional to the normal pressure between them, it follows that *the greater the angle of a screw thread, the greater is the friction between the threads of the bolt and the nut, and also the greater is the force tending to burst the nut.*

The relation between the total load W and the bursting pressure Q may be found as follows:

$$\frac{W}{2} = R \cos \frac{u}{2}; \; W = 2R \cos \frac{u}{2}; \; R = \frac{W}{2 \cos \frac{u}{2}}$$

$$Q = R \sin \frac{u}{2} = \frac{W \sin \frac{u}{2}}{2 \cos \frac{u}{2}} = \frac{W}{2} \tan \frac{u}{2} \qquad (1)$$

When $\frac{u}{2} = 30°$,

$$Q = \frac{W}{2} .577 = .289W$$

In this instance, the friction between the threads on the bolt and the nut has not been considered. This friction would tend to decrease the pressure Q, the reduction depending on the coefficient of friction between the contact surfaces. Assuming a coefficient of .132, the angle of friction e would be the angle of which .132 is the tangent, or 7° 30′. The angle $\frac{u}{2}$ would be reduced by this amount, making

$$Q = \frac{W}{2} \tan\left(\frac{u}{2} - e\right) = \frac{W}{2} \tan 22° \, 30' = .207\,W \qquad (2)$$

The pressure Q is shown in Fig. 15, as referring only to the one thread marked *a*, but it should be understood that the pressure is divided among all the threads and along the whole circumference of the nut. In calculating the resistance of the nut against the bursting pressure, the minimum cross-sectional area will be considered, which is that defined by a longitudinal section taken along the short diameter of the nut.

According to formula **2**, the pressure Q is equal to .207 of the total load supported by the bolt. The required sectional area of the nut should therefore be at least .207 of the bolt area for the coefficient of friction assumed. The equations that follow will show whether this condition is fulfilled by the nut.

Let d = diameter of bolt;
D = short diameter of nut;
h = height of nut = d.

Then, the cross-sectional area of the nut is $d(D - d)$. This area should be at least equal to 207 times the bolt area, or,

$$d(D - d) = .7854\,d^2 \times .207$$

Solving for D, $\quad D - d = .1626\,d$

and $\quad D = 1.1626\,d \qquad (3)$

It will be shown in a subsequent article that the diameter D for standard nuts is $D = 1\frac{1}{2}\,d + \frac{1}{16}$. This is in excess of that required by formula **3**, and the sectional area of the nut is therefore well within the safe limit In the foregoing considerations it was assumed that the full diameter of the bolt was its effective diameter and that the height of the nut was equal to *d*.

32. In the square thread, the angle between the sides is zero; hence, there is no force tending to burst the nut. The reaction R at one-half of the nut becomes equal to $\frac{W}{2}$; therefore, the friction of a square thread is less than that of a triangular thread. On the other hand, the triangular thread

is nearly twice as strong as a square thread. This will be seen more clearly by referring to Fig. 16, which illustrates a section of two bolts *a* and *b* of the same diameter, *a* having a square and *b* a United States standard thread. Since the number of threads per inch of the United States standard screw is twice as great as that of the square-threaded screw, the pitch p of the former is half that of the latter. The shearing surface of a single triangular thread is πd_1 multiplied by the distance $l n$, or $\pi d_1 p$, nearly, while the shearing surface of a single square thread is $\pi d_1 \frac{p}{2} = \frac{\pi d_1 p}{2}$

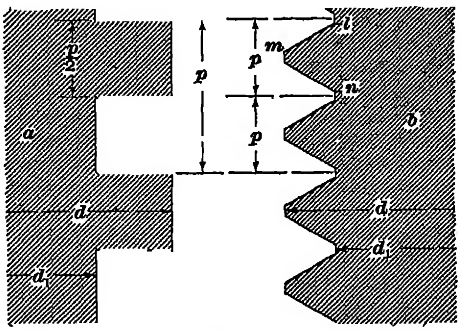


FIG 16

It follows, therefore, that the triangular thread is better for fastenings, and the square thread, for transmitting motion.

33. The trapezoidal thread combines the good features of both the triangular and the square threads, having the same shearing section as the former and the same friction as the latter. Care should be taken, however, to use the screw so that the pressure will come on the flat side of the thread, for if it is put on the inclined side, the friction and bursting force on the nut will both be greater than for a 60° triangular thread.

34. Multiple-Threaded Screws.—A screw intended to transmit motion will sometimes have a very large pitch. As the depth of the thread in a square-threaded screw is equal to one-half the pitch, it is obvious that in some cases this depth will exceed the radius of the screw. This difficulty may be overcome by reducing the dimensions of the thread and by adding one or more threads in the manner about to be explained.

Assume that Fig. 17 is a single-threaded screw having an outside diameter d, a root diameter d_1, and a pitch p. It is desired to increase the diameter d_1 to d_2 and at the same time retain the diameter d, as shown in Fig. 18. A decrease of the depth of thread from t to t_1 is necessarily followed by a decrease in its width. Hence, the original cross-section of the thread, as indicated by the square $a\,b\,d\,c$, shown dotted, is reduced to the square $a\,b_1\,d_1\,c_1$. The distance between similarly located points on the thread represents, as before, the pitch p.

When it is simply a matter of producing a desired motion, a screw with the amount of space between the threads shown in the left half of the figure would not be objectionable. But

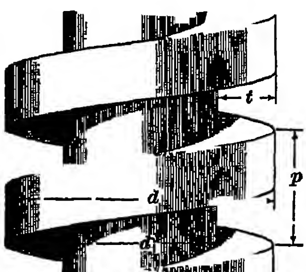


FIG. 17

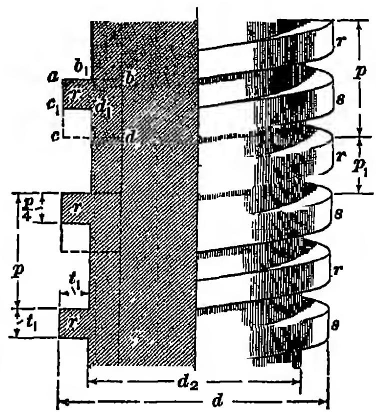


FIG. 18

when a load is to be supported, it is desirable to increase the shearing resistance of the thread as well as its bearing surface. This may be accomplished by adding another thread, as shown in the right half of the figure, where r represents the first thread and s the supplementary one.

A further increase in the diameter d_2 would make it possible to add more threads. A screw with more than one thread is termed a **multiple-threaded screw**; one with two threads, a **double-threaded screw**; with three threads, a **triple-threaded screw**; and with four threads, a **quadruple-threaded screw**.

The object of using a multiple-threaded screw, therefore, is to obtain a certain desired motion for each revolution of the screw, and at the same time to have ample strength to move the load. A single-threaded screw of the same lead would seldom have the required strength.

35. The number of threads n on a multiple-threaded screw is inversely proportional to the reduction in width of the thread of a single-threaded screw having the same pitch. For instance, if the thread of a single-threaded screw has a width of $\frac{1}{2}p$, and it is desired to increase the number of threads to n, then the width of each thread in the multiple-threaded screw will be $\frac{p}{2n}$.

In the case of multiple-threaded screws, the *lead* and the *pitch* are not equal. Instead, the pitch is equal to the lead divided by the number of threads. Thus, if the lead of a quadruple-threaded screw is 6 inches, the pitch is $6 \div 4 = 1\frac{1}{2}$ inches. In connection with multiple-threaded screws, the term *divided pitch* is often used in place of pitch. Thus, if the lead of a triple-threaded screw is $4\frac{1}{2}$ inches, the divided pitch is $4\frac{1}{2} \div 3 = 1\frac{1}{2}$ inches. In Fig. 18, the distance p is the lead and p_1 is the pitch, or, as it is sometimes called, the divided pitch. The dimensions of the thread are based on the divided pitch; that is, $t_1 = \frac{1}{2}p_1$.

36. By increasing the root diameter of a multiple-threaded screw, and consequently the number of threads, both the shearing and the helical surfaces of the threads are increased. Hence, the screw threads have a greater shearing strength, and the wear is divided over a greater area.

37. Pipe Threads.—The main difference between the pipe thread, known also as the **Briggs standard,** and the United States standard thread is that the former is built up on a conical instead of a cylindrical body. Besides being tapered, the thread must also be finer than the United States standard thread, as the depth of the latter in most cases would exceed the thickness of the pipe. For instance, the thickness of the walls in a 6-inch pipe is .280 inch, while

the depth of a 6-inch screw thread, according to the United States standard, is .289 inch, thus exceeding the thickness of the pipe by .009 inch.

In a screw thread made according to the Briggs standard, the threads have an angle of 60° and are rounded off slightly at top and bottom, so that the depth of the thread is .80 of the pitch of the thread. After cutting the thread, the outside surface has a taper to a certain distance from the end, the standard taper being such that the inclination of the outside surface toward the axis of the pipe is 1 in 32. This makes the total taper, as measured by the variation in the outside diameter, equal to 1 in 16, or ¾ inch to the foot. The total length l, Fig. 19, of the tapered part is given by the empirical formula

$$l = \frac{4.8 \text{ inches} + .8\,d}{n}$$

in which d is the actual external diameter of the tube in the straight part, in inches, and n the number of threads per inch.

Let t_1 represent the depth of the rounded thread and p the pitch, then

$$t_1 = .8\,p = \frac{.8}{n}$$

The tapered part $a\,b$ of the pipe has threads that are perfect at top and bottom. At the beginning of the straight part, from b to $b\,a_1$, there are two threads that have the correct taper at the bottom, but their tops are flattened. The remainder of the threaded part $b_1\,b_2$ consists of four threads that are imperfect at both top and bottom. The latter are not essential to the Briggs system, but are simply a result of the method of cutting the thread at a single operation.

Table V, at the end of this Section, gives the dimensions of standard pipe threads. By means of the data given under Total Length of Thread and Length of Perfect Thread, it is possible to determine the length of the part containing imperfect threads. This length can also be determined directly, when it is remembered that the number of imperfect threads are always six, and that the space occupied by them depends on the number of threads per inch for that particular diameter.

Fig. 19 shows a longitudinal section through the threaded end of a 4-inch pipe, which, as seen from Table V, has an actual outside diameter of 4.5 inches. The taper of the threaded surface does not extend beyond the point b, at which place the pipe resumes its full diameter. The inclination of the outside surface of the pipe toward its axis in this instance brings the point a at a distance of .0328 inch below the original surface. Between the points b and b_1 are located the two threads that are imperfect at the top, and between b_1 and b_2 are those that are imperfect at both top and bottom. It will be noticed that all of the perfect thread bottoms lie along a line $a_1 c$, parallel with a line $a b$ drawn through the tops of the threads. The imperfect bottoms lie along a line that passes from a_2 to b_2.

38. The cause of this peculiar thread form will be understood by an examination of Fig. 19. The pipe begins to

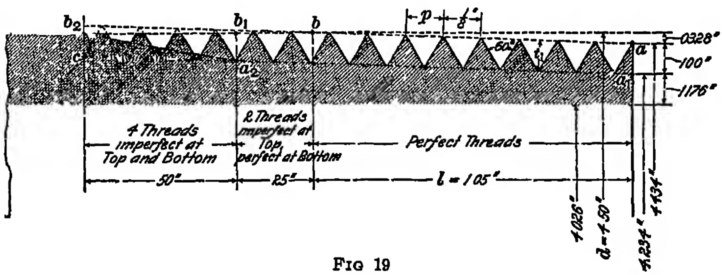


FIG 19

assume a cylindrical form at point b, and since the die forming the threads is conical, the tops of the threads from b to b_2 cannot be filled out. The reason that the bottoms of threads are incomplete from points a_2 to c is due to the fact that parts of the die threads at this place are ground away in order that the die may be easily started.

From the method of cutting the threads, it follows that if the pipe is not passed far enough into the die, the number of perfect threads will be below the requirements. On the other hand, if the pipe is passed in too far, the extreme end of the pipe will be straight instead of tapering. In either case, it will be impossible to make a strong and tight joint.

BOLTS AND NUTS

STRENGTH OF SCREW BOLTS

39. Usually the stress on a bolt acts in the direction of its axis; that is, the bolt is in tension.

Let W = load on bolt, in pounds;
S_1 = *safe* working stress, in pounds per square inch;
a = area of cross-section of bolt at root of thread;
d = nominal (outside) diameter of bolt, in inches;
d_1 = diameter at root of thread, in inches.

Then, if the bolt is in tension,

$$W = a\,S_1; \text{ or, } a = \frac{W}{S_1}$$

After finding the value of a, the corresponding values of the nominal diameter d and the diameter d_1 at bottom, or root, of the United States standard thread may be found from Table II. The relation between d and d_1 is also given in formula **4**, Art. **22**.

For bolts subjected to a constant tension, S_1 may be 8,000 pounds per square inch. More often the tension varies between zero and its maximum value; in this case, S_1 may be taken as 6,000 pounds per square inch. For cylinder-head bolts, and, in general, for bolts used to make a steam-tight joint, S_1 may vary from 3,000 pounds for small cylinders to 6,000 pounds for very large ones. Ordinarily, S_1 may be taken as 4,000 or 4,500 pounds per square inch. All the values just given are for wrought-iron bolts.

EXAMPLE.—Find the diameter of a wrought-iron bolt that is to sustain a steady load of $4\frac{1}{2}$ tons.

SOLUTION.—From the formula,

$$a = \frac{4\frac{1}{2} \times 2{,}000}{8{,}000} = 1.125 \text{ sq in.}$$

From Table II, the value of d lies between $1\frac{3}{8}$ in. and $1\frac{1}{2}$ in. The latter value should be taken. Ans.

40. For screws that have square or Acme standard threads and that are intended to transmit motion, the

following formula may be used, assuming that d_1 is approximately equal to $\frac{4}{5}d$:

$$W = 3{,}000\, d_1^2 = 1{,}920\, d^2$$

or

$$\left.\begin{aligned} d_1 &= .0183\sqrt{W} \\ d &= .0228\sqrt{W} \end{aligned}\right\} \qquad (1)$$

For screws of this character, the least number of threads in the nut that are necessary to prevent excessive wear is given by the following formula, in which n_1 is the number of threads in the nut:

$$n_1 = \frac{W}{300\, d_1^2} = .0052\frac{W}{d^2} \qquad (2)$$

Formula 2 applies to square and trapezoidal threads, and is based on the assumption that the pressure on the thread per square inch of projected area should not be greater than 700 pounds per square inch.

EXAMPLE.—A square-threaded screw $1\frac{1}{2}$ inches in diameter transmits motion to a load of 4,000 pounds. What is the least allowable number of threads in the nut?

SOLUTION.—By formula 2,

$$n_1 = \frac{.0052 \times 4{,}000}{(1\frac{1}{2})^2} = 9\tfrac{1}{4}. \text{ Ans.}$$

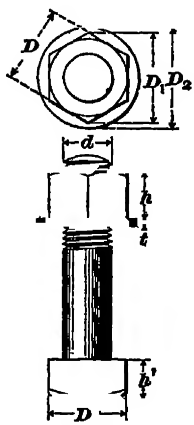


FIG. 20

PROPORTIONS OF BOLTS AND NUTS

41. The dimensions of the nut and bolt head depend on the diameter of the bolt. The United States standard form of bolt and nut, as shown in Fig. 20, has a square head and a hexagonal nut with washer, although the head may also be hexagonal. The washer is used to give a smooth seat for the nut to be screwed up against.

The following proportions are usually adopted, the dimensions being identical for both rough and finished work. In other words, the parts that are to receive a finish are made larger in the rough, so that when finished they will correspond in size with those that are to remain rough.

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The diameter of nut or head across flats is found by the formula

$$D = 1\tfrac{1}{2}d + \tfrac{1}{8} \text{ inch} \qquad (1)$$

This proportion holds for both hexagonal and square nuts. The diameter across corners D_1 may be found from the geometry of the figure. Thus, for hexagonal nuts,

$$D_1 = \frac{D}{\cos 30°} = \frac{D}{.866} = 1.73\,d + .14 \text{ inch} \qquad (2)$$

For square nuts,

$$D_1 = \frac{D}{\cos 45°} = D\sqrt{2} = 2.12\,d + .18 \text{ inch} \qquad (3)$$

For the height of nut,

$$h = d \qquad (4)$$

For the height of head,

$$h' = \frac{D}{2} = \tfrac{3}{4}d + \tfrac{1}{16} \text{ inch} \qquad (5)$$

The standard dimensions of plate washers are given in Table X at the end of this Section.

EXAMPLE.—Required, the various dimensions of a finished bolt and hexagonal nut, the bolt being $1\frac{1}{2}$ inches in diameter.

SOLUTION —By the formulas just given, the diameter of the nut across flats is equal to

$$D = 1\tfrac{1}{2}d + \tfrac{1}{8} \text{ in} = 1\tfrac{1}{2} \times 1\tfrac{1}{2} + \tfrac{1}{8} = 2\tfrac{3}{8} \text{ in. Ans.}$$

The diameter of the nut across corners is

$$D_1 = \frac{D}{866} = \frac{2\ 375}{866} = 2\ 742, \text{ or } 2\tfrac{3}{4}, \text{ in., nearly. Ans.}$$

The side of the square bolt head is $D = 2\frac{3}{8}$ in. Ans.
The height of the nut is $h = d = 1\frac{1}{2}$ in Ans.
The height of the bolt head is

$$h' = \frac{D}{2} = \frac{2\frac{3}{8}}{2} = 1\tfrac{3}{16} \text{ in} \quad \text{Ans.}$$

From Table X, the diameter of the washer is $D_2 = 3\frac{1}{2}$ in , and the thickness t is $\frac{11}{64}$ in Ans

42. Table II, which will be found at the end of this Section, gives the dimensions of the United States standard bolts and nuts, and is based on the formulas given in Arts. **21**, **22**, and **41**. In regard to the number of threads per inch for diameters between those given in the table, it is customary

to give them the number belonging to the next larger diameter. For instance, a bolt $4\frac{1}{16}$ inches in diameter will have $2\frac{7}{8}$ threads per inch.

The table, as far as it refers to threads, is generally complied with among the manufacturers, but many variations are found in the proportions of the heads and nuts. To a great extent, these parts are now made according to the *Manufacturers' standard.* In this system, the height h' of the rough hexagonal head is equal to d, and the height of the nut h, to $d + \frac{1}{8}$ inch. The dimensions of the nuts across the flats are larger than those given in Table II

43. A. L. A. M. Standard Screws and Nuts.—The pitch of the United States standard thread has proved to be too coarse for screws used in light machinery. It has also been desirable in such cases to have the dimensions of the nuts and heads somewhat reduced. As the number and variety of these special threads and nuts have caused a great deal of inconvenience, the Association of Licensed Automobile Manufacturers has adopted a new standard for screws and nuts, the dimensions of which are given in Table VI. The form of thread used, however, is identical with that of the United States standard.

It is assumed that where the screws are to be used in soft material, such as cast iron, brass, bronze, or aluminum, the existing United States standard pitches will be used.

The following terms have been adopted by the Association: *Screw* is intended to supplant the present so-called coupling bolt and capscrew; *plain hexagonal nut* is to be used in place of the present so-called United States standard nut; *castle nut* is a name given to a new nut intended to be used where a positive locking system is desired and *facing* refers to a relieved portion under the screw heads, castle nuts, and plain nuts.

The screw heads and plain nuts must be flat-chamfered, as shown in the diagrams at the top of Table VI, which also shows the form of locknut, known as a castle nut. Three diametral grooves in the latter permit a very close

adjustment, and it is locked by means of a pin inserted through a hole in the bolt. The castle nut is also chamfered and casehardened. The screws, screw heads, and plain nuts are left soft, and all heads and nuts are to be semifinished.

WRENCHES

44. The usual forms of solid wrenches are illustrated in Fig. 21, that shown at (*a*) being used for hexagonal nuts, and that at (*b*) for square nuts. The length may be from fifteen to eighteen times the diameter of the bolt for which

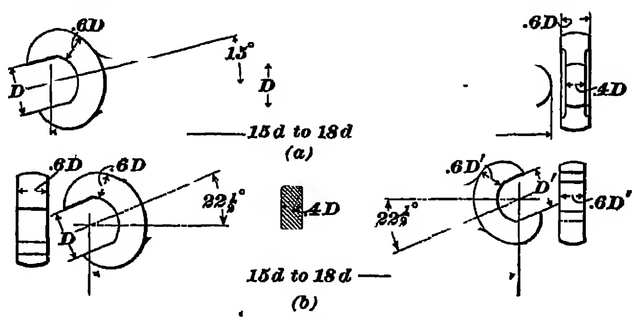


FIG 21

it is to be used. In the figures, *d* represents the diameter of the bolt. The other proportions are given in terms of the diameter across the flats of the nuts, as shown in the figure.

The advantage derived from placing the opening of a wrench at an angle with the handle is that the nut may be turned completely around in locations where the swing of the handle is limited. Thus, the wrenches (*a*) and (*b*) may turn their respective nuts a complete revolution, when the swings of the handles are limited to 30° and 45°, respectively. By turning the wrench upside down after each partial revolution, it is possible to place the handle in its initial position and thus gain room for as great an angular swing as the surrounding parts will allow.

FORMS OF BOLT HEADS

45. Common Forms.—The ordinary *square bolt head* has already been shown in Fig. 20. Other forms are shown in Figs. 22 to 30.

In Fig. 22, the **hexagonal bolt head** is similar to a hexagonal nut, and has the same dimensions, except that the height h' may be less. Usually, $h' = \frac{2}{3}d$ to d. Fig. 23 shows a hexagonal head with a collar or flange, which is added to give an increased bearing surface.

A **cylindrical head** is shown in Fig. 24, and a **hemispherical head** in Fig. 25. The height h' of the former may be from $.5\,d$ to $.8\,d$, while that of the latter is $\frac{3}{4}d$. The diameter of these heads is shown in the figures. Fig. 26 shows a bolt head with a hemispherical bearing surface

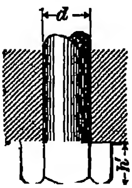


FIG 22

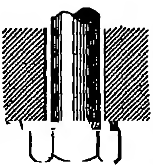


FIG 23

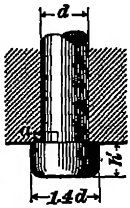


FIG 24

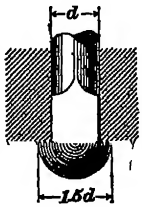


FIG. 25

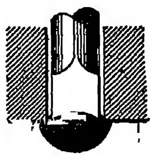


FIG. 26

resting on a seat of the same shape. This bolt may lean to one side, and the head will still remain in contact with its seat all the way around.

An **eyebolt** is shown in Fig. 27. The cross-section of the eye through the hole should either equal or exceed the area of the bolt; that is, referring to this figure,

$$2\,a\,b = .7854\,d^2;\ \text{or,}\ a\,b = .39\,d^2$$

In good practice, $a\,b$ is equal to at least $\frac{1}{2}d^2$. On calculating the diameter d_1 of the pin passing through the eye, it should be observed that the pin is in double shear, the shearing surface being twice the area of it; or,

$$2\,(.7854\,d_1^2) = 1.57\,d_1^2$$

The strength of this pin in shear should equal the strength of the eyebolt in tension; therefore, letting S_s represent the

ultimate shearing stress per square inch, S_t the *ultimate* tensile strength, and f the factor of safety,

$$1.57\, d_1^2 \frac{S_s}{f} = .7854\, d^2 \frac{S_t}{f}$$

or

$$d_1 = d\sqrt{\frac{S_t}{2\,S_s}} \qquad (1)$$

If, however, the pin is overhung, that is, if there is only one nib instead of the two shown at S, S, Fig. 27, it will be in single shear, and

$$d_1 = d\sqrt{\frac{S_t}{S_s}} \qquad (2)$$

Fig. 28 shows the head of a **hook bolt.** This form of bolt is used when it is undesirable to weaken one of the connected pieces by a bolt hole. The proportions are shown in the figure.

The **countersunk head** is shown in Fig. 29, and the **T head** in Fig. 30.

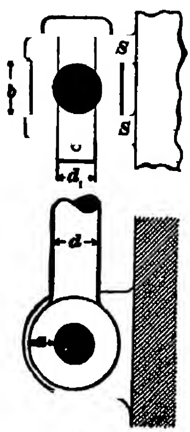


FIG. 27

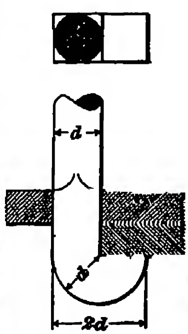


FIG. 28

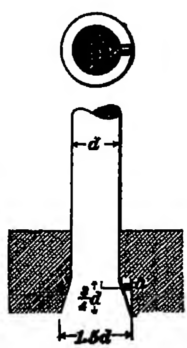


FIG. 29

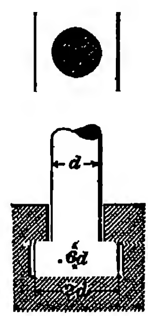


FIG 30

46. Foundation Bolts.—The ordinary method of *attaching a bolt to stonework* is illustrated in Fig. 31. The head of the bolt is long and rectangular, and is made jagged with a cold chisel, while the hole in the stone is cut larger at the bottom than at the mouth. The bolt head is placed

in the hole, and the remaining space is then filled with melted led or sulphur.

47. One method of securing the foundation bolts that fasten an engine bed to its foundation is shown in Fig. 32. These foundation bolts have no solid heads, but consist of long rods that are threaded on one end for a nut and have a slot in the other through which a *cotter C* passes. The cotter rests against a cast- or wrought-iron washer *m*, and the washer and cotter thus form the head of the bolt. The bolt head is within a recess formed in the foundation, which is arranged so as to be accessible.

The area of the washer bearing against the foundation, multiplied by the safe compressive strength of the material

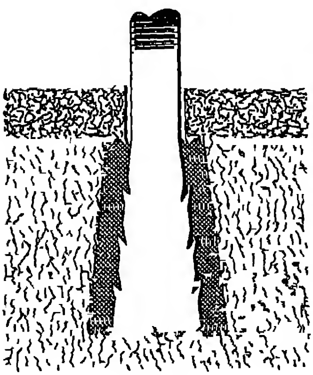


FIG 31

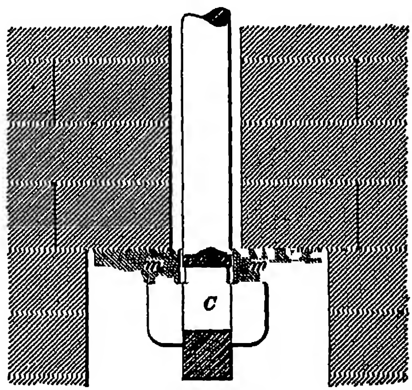


FIG 32

of the foundation, should be equal to the tension carried by the bolt. For example, the tensile strength of wrought iron is about twenty times the compressive strength of brick. Hence, the bearing area of a washer resting against a brick foundation should be about twenty times the cross-section of the bolt.

48. Methods of Preventing Bolts From Turning. Various methods are used to prevent a bolt from turning while the nut is being screwed up. A common method is to make the neck of the bolt next to the head *square*, as shown in Figs. 25, 26, and 28. The bolt hole is also made

square. Another way is to insert a pin *a* into the neck close to the head, as shown in Figs. 24 and 29. The projecting part of the pin fits into a recess cut out to receive it.

FORMS OF NUTS

49. Ordinary Forms.—The *common hexagonal nut* has been shown in Fig. 20. Ordinarily, both hexagonal and square bolt heads and nuts are chamfered off at an angle of 30° or 40°. Other forms of nuts are shown in Figs. 33 to 37.

The **flanged nut,** Fig. 33, is useful when the bolt hole is larger than the bolt, as it covers the hole and gives a greater

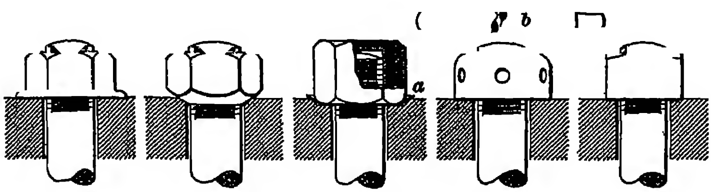


FIG 33 FIG 34 FIG. 35 FIG 36 FIG 37

bearing surface. While the same results may be obtained by means of a nut and washer, there are cases where it is preferable for the nut and washer to be in one piece.

Fig. 34 shows a nut with a **spherical bearing surface** and the seat shaped to correspond. This nut will bear on the seat all around, whether the bolt is perpendicular or inclined to the seat.

A **cap nut** is shown in Fig. 35. This form of nut is used to prevent fluids from leaking past the screw threads. To prevent leakage past the seat, the nut is screwed down on a soft, thin copper washer *a*.

Fig. 36 shows a **round nut,** and Fig. 37 an ordinary **square nut.** The round nut is provided with holes in its circumference, as shown, and is screwed up by inserting a bar *b* in one of the holes.

50. Round Slotted Nuts.—**Round slotted nuts** are used for bolts having large diameter or for shafts. They

are provided with grooves as shown in Fig. 38, and are turned by means of a spanner engaging with these grooves. If D is the short diameter of a hexagonal nut, then the diameter of a round nut will be

$$D_c = D + 2 s_2$$

s_2 being the depth of a groove. The height of the nut is $h = d$, while the width s_1 varies from 2 to 2.6 s_2. For a 1-inch bolt, $D_c = 2$ inches, $s_1 = \frac{3}{8}$ inch, and $s_2 = \frac{3}{16}$ inch.

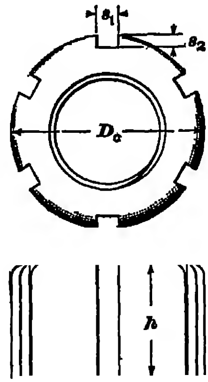


FIG 38

LOCKING DEVICES

51. Locknuts.—All nuts are slightly loose on their bolts, a small clearance being necessary to permit them to turn freely. When a nut is subject to vibration it is likely to slack back and allow the bolt to become loose. To prevent this slacking back, various locking arrangements have been devised. A common device is the **locknut,** or **jam nut,** shown in Fig. 39. Two nuts are used, one of which is about half as thick as the ordinary nut. The load is thrown

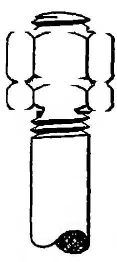


FIG 39

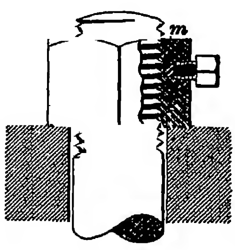


FIG 40

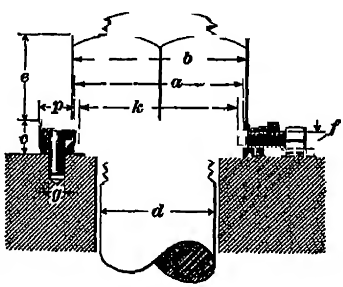


FIG 41

on the outer nut, which should therefore be the thicker one. In practice, however, the thin nut is usually placed on the outside, because the wrench is generally too thick to act on it when placed below the other. The jam nut is not always satisfactory as a locking device.

52. Locking by Means of Setscrews.—The nut may be effectively locked to the bolt by the use of a setscrew, as shown in Fig. 40. To prevent the point of the setscrew from injuring the thread, a piece of iron or steel m may be let into the nut. This piece is screwed along with the nut, and acts as a shield interposed between the setscrew and the thread.

53. A good method of locking a nut is shown in Fig. 41. The lower portion of the nut is turned down, and a groove is cut in the center of the circular portion. A collar is fastened by means of a pin to one of the pieces to be connected, and the circular part of the nut is fitted into this collar. The nut is then bound to the collar by a setscrew passing through the latter, the point of the setscrew engaging into the groove turned in the nut. The following proportions, in which the diameter of the bolt d is taken as a unit. have proved very satisfactory:

$a = 1\frac{1}{2}d - \frac{1}{16}$ in.;	$f = \frac{1}{8}d + \frac{1}{8}$ in.;
$b = 1\frac{1}{2}d + \frac{1}{8}$ in.;	$g = \frac{1}{8}d + \frac{1}{16}$ in.;
$c = \frac{1}{4}d + \frac{1}{4}$ in.;	$p = \frac{1}{4}d + \frac{1}{4}$ in.;
$e = \frac{3}{4}d$;	$k = a - \frac{1}{8}$ in.

A variation of this nut-locking device is shown in Fig. 42. By this method the collar is omitted, and the setscrew is inserted in one of the pieces to be connected. Both of these devices are very effective and are used for heavy nuts on quickly moving parts and on important bearings, as in marine engines.

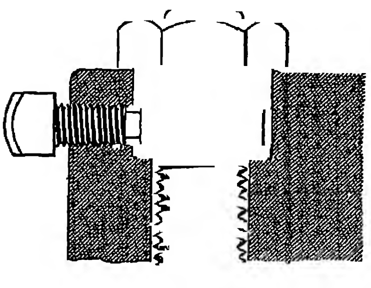


FIG. 42

54. Locking by Means of Stop-Plates.—Fig. 43 shows a device for locking a nut by means of a stop-plate. The plate is fastened to one of the pieces through which the bolt passes, and is so shaped that the bolt may be locked at intervals of $\frac{1}{12}$ revolution. Suitable proportions for this stop-plate are shown in the figure, in which d, the

diameter of the bolt, is taken as the unit, with the exception of the distance between the center of the bolt and the center of the screw, for which D, the diameter of the nut across flats, is taken as the unit. All dimensions are in inches.

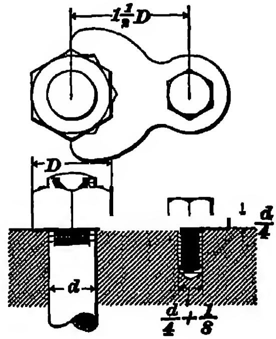


Fig 43

55. In Fig. 44 is shown another form of stop-plate, which may be conveniently used when the bolts are set in a circle, as, for example, on engine cylinder heads.

56. Slotted Locknut.—In Fig. 45 is shown a different manner of locking the nut. In this method the nut is sawed half way through, and the parts are connected by a small screw. When the nut is screwed home the small screw is tightened, thereby greatly increasing the friction between the bolt and the nut.

57. Lock Washers.—A convenient locking device, called the **positive lock washer**, is shown in Fig. 46. When not held down by the nut, the washer has the form shown in (*b*); when the nut is screwed down tightly, as in (*a*), the washer is flattened out and its body carries the load of

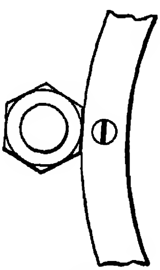


Fig. 44

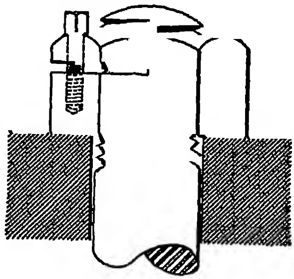
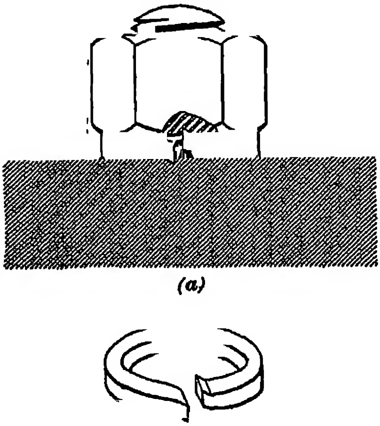


Fig 45

compression. The tapered ends are thus relieved, and the spring tension is constant. As the barbs at the ends are free to move when subjected to vibration, they force themselves gradually into the positions indicated.

Other spring washers are the **Grover spring washer,** which has no barbs but simply depends on its elasticity to keep the nut tight on the bolt, and the **National lock washer,** which has, along its inner side next to the nut contact face, a rib that upsets and forces some of the metal of the nut into the bolt thread, and thus prevents the nut from loosening.



(a)

(b)

Fig 46

Fig 47

58. Split Pins. **Split pins** may also be considered as locking devices. These pins are used where the parts occupy one position only, and from which they are seldom removed. After the nut is screwed into position, a hole is drilled through the *bolt* close to the nut, so that when the split pin is inserted it may prevent any motion of the nut. The split pin may be either straight or tapering; may be made of half-round wire and bent into the shape shown in Fig. 47; or may be circular in section and split at one end only. When in position, the projecting split ends are spread apart.

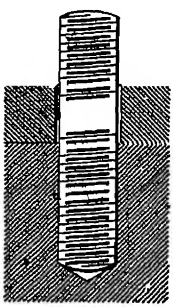


Fig 48

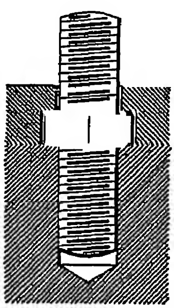


Fig. 49

FORMS OF BOLTS AND SCREWS

59. Bolts.—A **stud bolt,** or **stud,** is shown in Fig. 48. Each end of the stud has a screw thread cut on it. One end screws into one of the pieces to be connected, and the other carries a nut.

A stud having a *collar* is shown in Fig. 49. The collar, which may be either square or round, serves as a shoulder against which to screw up the stud. Also, when square, the collar is a convenient place to apply a wrench.

A **tap bolt,** shown in Fig. 50, is a bolt that does not require a nut. It is screwed directly into one of

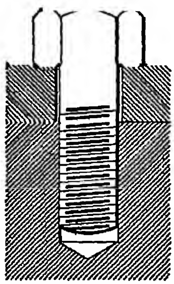


FIG. 50

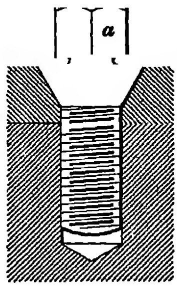


FIG 51

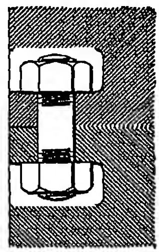


FIG 52

the pieces to be connected, while the head presses on the other piece.

Fig. 51 shows a tap bolt having a countersunk head. This style of bolt is called a **patch bolt,** and, as its name implies, is used for making patches in boilers, etc. The diameter of the neck of the projection *a* to which the wrench is applied is smaller than the root diameter of the bolt, so that, instead of the bolt breaking when too much force is applied, the projection will break off.

Fig. 52 shows a bolt having a nut at each end instead of a head at one end and a nut at the other.

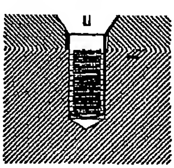


FIG 53

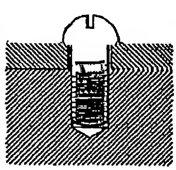


FIG 54

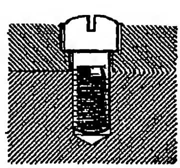


FIG 55

60. Screws.—In Figs. 53, 54, and 55 are shown several forms of machine screws slotted for a screwdriver. The one

shown in Fig. 53 is called a **countersunk-head screw,** or a **flathead machine screw;** that in Fig. 54, a **button-head screw;** and that in Fig. 55, a **fillister-head screw.** When a countersunk-head screw is used, the hole in the piece that is to be fastened is countersunk so that the head of the screw is flush, as shown.

61. Setscrews.—Screws or bolts that are used to press against a piece, and thus, by friction, prevent it from moving or rotating relatively to another piece are called **setscrews.** For example, a setscrew may be screwed through the hub of

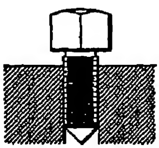


FIG. 56

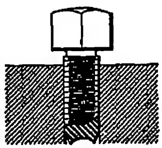


FIG. 57

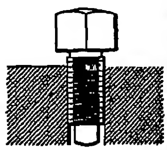


FIG. 58

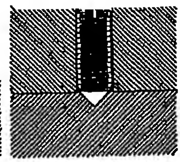


FIG. 59

a pulley, and by its pressure against the shaft will prevent the pulley from turning on the shaft.

Various forms of setscrews are shown in Figs. 56 to 59. The one shown in Fig. 56 is called a **cone-point setscrew;** in Fig. 57, a **cupped setscrew;** in Fig. 58, a **round pivot-point setscrew;** and in Fig. 59, a **headless cone-point setscrew.**

62. Bolts in Shear.—Usually bolts are in direct tension, but constructions occur in which a bolt may be placed in shear. The relative values of the tensile and shearing strengths of a material may vary. For wrought iron, these values may be equal, as indicated in *Strength of Materials.* But in order to be on the safe side, it may be supposed that in the material used for bolts the ultimate shearing strength is less than the tensile strength. Assuming that the shearing strength of wrought iron is about four-fifths of the tensile strength, the strength of a bolt in shear is about four-fifths that of a bolt in tension. Hence, the diameter of a bolt in single shear should be $\sqrt{\frac{5}{4}} = 1.1$ that of a bolt in

tension under the same load, and the diameter of a bolt in double shear should be $\sqrt{\frac{5}{4 \times 2}} = .8$ that of a bolt in tension under the same load.

63. Knuckle Joint.—The **knuckle joint**, as shown in Fig. 60, is an example of a bolt in shear. Since the bolt is

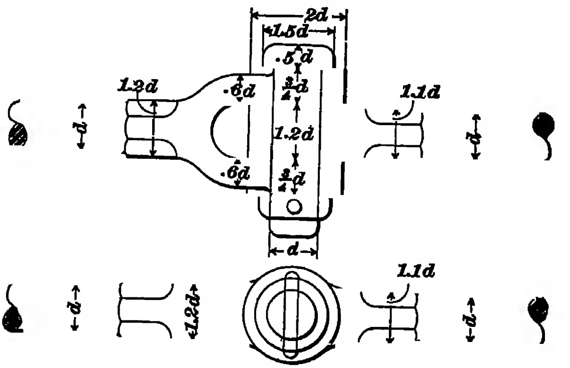


FIG 60

in double shear, it need be theoretically only .8 the diameter of the rod. The bolt wears, however, and since it should at no time be less than .8 the diameter of the rod, the bolt and rod are made equal in diameter. The other proportions are in terms of the diameter of the bolt, all dimensions being in inches.

Other examples of bolts in shear may be seen in pin-connected iron bridges.

EXAMPLES FOR PRACTICE

1. Calculate the diameter of a wrought-iron bolt that is to sustain a varying load of 2,300 pounds.
Ans. $\frac{7}{8}$ in.

2. What steady load may be safely sustained by five bolts $1\frac{1}{8}$ inches in diameter?
Ans 13.85 tons

3. A screw with square threads transmits motion to a load of 1,500 pounds Calculate the diameter of the screw, the number of threads per inch, and the necessary number of threads in the nut.

$$\text{Ans.}\begin{cases}\text{Diameter} = \frac{7}{8}\text{ in.}\\ \text{Threads per inch} = 4\frac{1}{2}\\ \text{Threads in nut} = 10\end{cases}$$

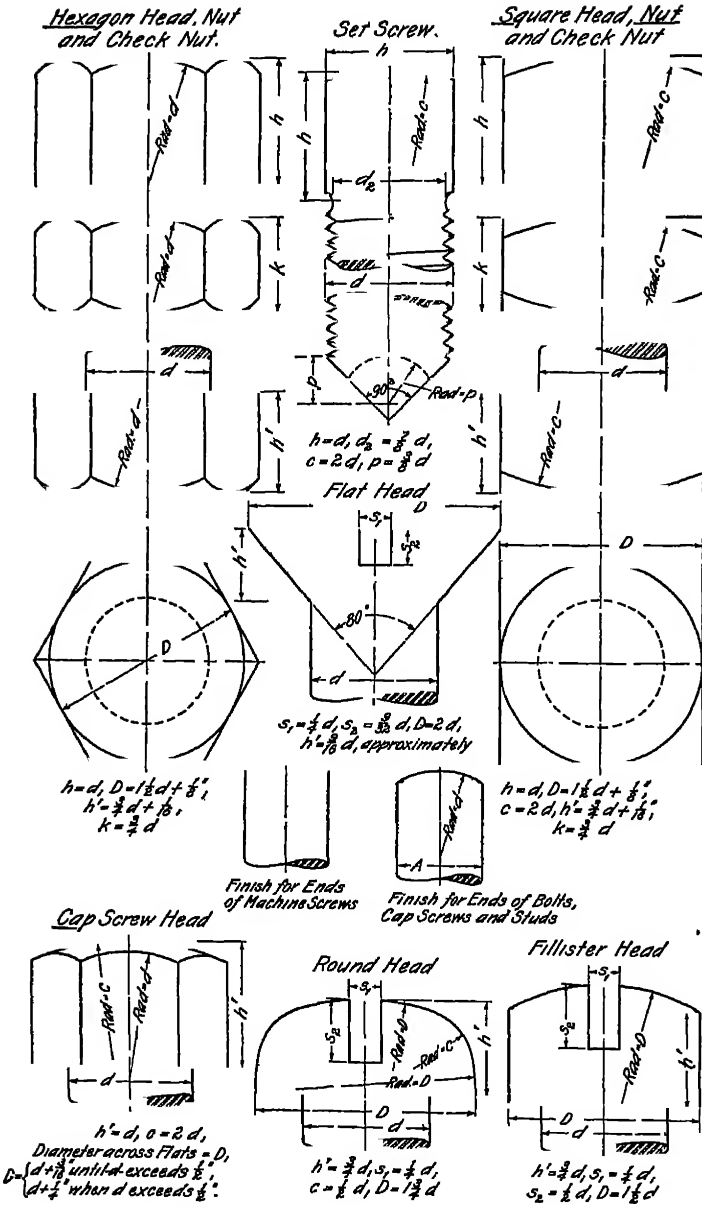

Hexagon Head, Nut and Check Nut.
Set Screw.
Square Head, Nut and Check Nut
Rad.=d
Rad.=c
h=d, d₂ = 7/8 d,
c=2d, p=3/8 d
Flat Head
Rad.=p
s₁=1/4 d, s₂=3/32 d, D=2d,
h'=3/16 d, approximately
h=d, D=1 1/2 d+1/8",
h'=3/4 d+1/16,
k=3/4 d
h=d, D=1 1/2 d+1/8",
c=2d, h'=3/4 d+1/16",
k=3/4 d
Finish for Ends of Machine Screws
Finish for Ends of Bolts, Cap Screws and Studs
Cap Screw Head
Round Head
Fillister Head
h'=d, c=2d,
Diameter across Flats = D,
D={d+1/16" until d exceeds 1/2", d+1/4" when d exceeds 1/2".
h'=3/4 d, s₁=1/4 d,
c=1/2 d, D=1 3/4 d
h'=3/4 d, s₁=1/4 d,
s₂=1/2 d, D=1 1/2 d

Fig 61

64. Details of Construction.—The additional information required for the construction of the standard forms of bolt heads, nuts, and the principal screw heads is given in Fig. 61. The diagrams of the standard hexagon and square heads and nuts have been repeated from Table II, for the purpose of giving the radii of the curved outlines of some of the faces. The various dimensions are indicated by letters in the diagrams, and the relations between these dimensions are stated either below each separate diagram or below each series of diagrams. It will be seen that all dimensions are based on the diameters of the bolts or screws.

In order to avoid the calculations required for the use of the diagrams in regard to standard bolts and nuts, the values given in Table II may be used, the same letters in each case referring to the same dimensions. The dimensions of the parts not referred to in Table II, such as tap bolts, capscrews, and setscrews, are given in Table I. The diagrams at the top of the table contain the letters that represent the various dimensions inserted in the same. The values given in Table I are not according to any generally accepted standard, and the number of threads per inch will vary with different manufacturers.

65. Dimensions of small standard machine or wood screws are given in Table VII. The sizes of these screws are indicated by gauge numbers instead of by diameters, but for the purpose of comparison, the table also gives the diameter, in inches, corresponding to any of the gauge numbers. To draw the head of any one of these screws, find its diameter in the table and lay out the head according to the instructions given in Fig. 61.

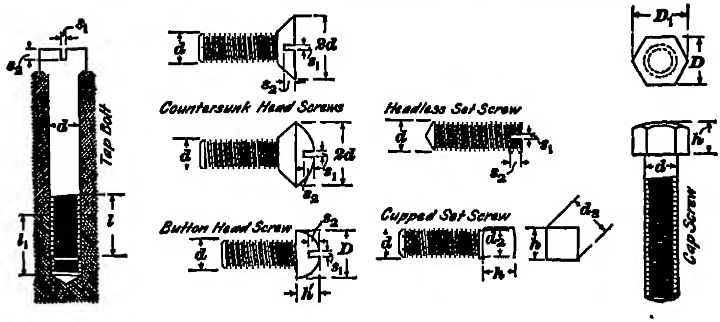
66. Diameters of Small Holes.—The sizes of small holes in general are also indicated by gauge numbers instead of by their diameters. In this case, they receive their gauge number from the drill used for drilling them. Table VIII gives the gauge numbers and corresponding diameters of the Morse twist drills. This table will also serve to indicate the gauge numbers of ordinary steel wire according to the

Morse system, but, as will be observed, it differs from other wire gauges, such as the Stubs, Brown & Sharpe, etc.

67. Decimal Equivalents of Parts of 1 Inch. Table IX is intended to assist in finding the nearest 64th inch corresponding in value to a dimension given in thousandths of an inch.

The decimal fractions are printed in two sizes of type, the large size giving the exact value of the corresponding fractional part of an inch to the fourth decimal place. A given decimal fraction of an inch is rarely exactly equal to any of these values, but is either above or below, and the question then is to decide to which of two values it is the nearest, whether to the preceding or the succeeding one. For instance, it is desired to lay off the fraction .1330 inch in 64ths of an inch. The nearest decimal fractions are .1250 and .1406, and the question is which of these to choose. By means of the decimal fractions, printed in smaller type, this question may be answered at once, as the value of any one of these is the mean of the two adjacent fractions printed in larger type. If, therefore, any of the given decimal fractions is *above* the mean in value, it belongs to the succeeding decimal fraction; if *below*, to the preceding one. In this instance the mean fraction is .1328, and as .1330 is greater than this, .1406 inch, or $\frac{9}{64}$ inch will be chosen. In the same manner the nearest 64th inch corresponding to the decimal fractions .3670 inch and .8979 inch are found to be $\frac{23}{64}$ inch and $\frac{57}{64}$ inch, respectively.

TABLE I.—DIMENSIONS OF TAP BOLTS AND SETSCREWS

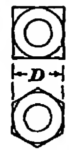
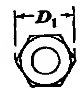
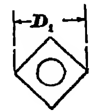
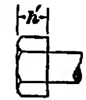


Diameter of Bolt	Number of Threads per Inch	Tap Bolts	Setscrews							Capscrews		
			Button Heads, Countersunk, and Headless				Cupped					
		Tap Hole l_1 Length of Thread l_2	Diameter	Depth of Head	Width	Depth						
d	n	l_2	D	h'	s_1	s_2	h	d_2	d_3	D	D_1	h'
$\frac{1}{4}$	20	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{16}$	$\frac{3}{64}$	$\frac{3}{32}$	$\frac{1}{4}$	$\frac{7}{32}$	$\frac{23}{64}$			
$\frac{5}{16}$	18	$\frac{5}{8}$	$\frac{15}{32}$	$\frac{15}{64}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{5}{16}$	$\frac{9}{32}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{37}{64}$	$\frac{5}{16}$
$\frac{3}{8}$	16	$\frac{3}{4}$	$\frac{9}{16}$	$\frac{9}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{21}{64}$	$\frac{17}{32}$	$\frac{9}{16}$	$\frac{21}{32}$	$\frac{3}{8}$
$\frac{7}{16}$	14	$\frac{7}{8}$	$\frac{5}{8}$	$\frac{21}{64}$	$\frac{3}{32}$	$\frac{3}{16}$						
$\frac{1}{2}$	13	1	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{32}$	$\frac{3}{16}$	$\frac{1}{2}$	$\frac{7}{16}$	$\frac{45}{64}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{1}{2}$
$\frac{9}{16}$	12	$1\frac{1}{16}$	$\frac{27}{32}$	$\frac{27}{64}$	$\frac{3}{32}$	$\frac{3}{16}$						
$\frac{5}{8}$	11	$1\frac{3}{16}$	$\frac{15}{16}$	$\frac{15}{32}$	$\frac{3}{32}$	$\frac{3}{16}$	$\frac{5}{8}$	$\frac{35}{64}$	$\frac{7}{8}$	$\frac{7}{8}$	1	$\frac{5}{8}$
$\frac{3}{4}$	10	$1\frac{3}{8}$	$1\frac{1}{16}$	$\frac{9}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{21}{32}$	$1\frac{1}{16}$	1	$1\frac{5}{32}$	$\frac{3}{4}$
$\frac{7}{8}$	9	$1\frac{9}{16}$	$1\frac{1}{4}$	$\frac{21}{32}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{7}{8}$	$\frac{49}{64}$	$1\frac{15}{64}$	$1\frac{1}{8}$	$1\frac{5}{16}$	$\frac{7}{8}$
1	8	$1\frac{3}{4}$	$1\frac{7}{16}$	$\frac{3}{4}$	$\frac{5}{32}$	$\frac{5}{16}$	1	$\frac{7}{8}$	$1\frac{13}{32}$	$1\frac{1}{4}$	$1\frac{7}{16}$	1
$1\frac{1}{8}$	7	$1\frac{15}{16}$	$1\frac{5}{8}$	$\frac{27}{32}$	$\frac{5}{32}$	$\frac{5}{16}$						
$1\frac{1}{4}$	7	$2\frac{1}{8}$	$1\frac{13}{16}$	$\frac{15}{16}$	$\frac{3}{16}$	$\frac{3}{8}$				$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{1}{4}$
$1\frac{3}{8}$	6	$2\frac{5}{16}$	2	$1\frac{1}{32}$	$\frac{3}{16}$	$\frac{3}{8}$						
$1\frac{1}{2}$	6	$2\frac{1}{2}$	$2\frac{3}{16}$	$1\frac{1}{8}$	$\frac{7}{32}$	$\frac{7}{16}$						
$1\frac{5}{8}$	$5\frac{1}{2}$	$2\frac{11}{16}$										
$1\frac{3}{4}$	5	$2\frac{7}{8}$										
$1\frac{7}{8}$	5	$3\frac{1}{16}$										
2	$4\frac{1}{2}$	$3\frac{1}{4}$										
$2\frac{1}{4}$	$4\frac{1}{2}$	$3\frac{5}{8}$										
$2\frac{1}{2}$	4	4										
$2\frac{3}{4}$	4	$4\frac{3}{8}$										
3	4	$4\frac{3}{4}$										

NOTE.—All dimensions are given in inches.

TABLE II

DIMENSIONS OF UNITED STATES STANDARD BOLTS AND NUTS

Bolts					Nuts					Heads
Diameter of Bolt	Number of Threads per Inch	Diameter at Bottom of Threads	Area at Bottom of Threads	Area of Body of Bolt	Diameter of Bolt, $d = h$ 				Diameter of Tap Drill	D and D_1 the Same as for Nuts 
Inches d	n	Inches d_1	Square Inches a	Square Inches a_1	Rough Inches	Rough Inches	Rough Inches	Rough Inches	Inches	Rough Inches
$\frac{1}{4}$	20	.185	.0269	.0491	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{37}{64}$	$\frac{45}{64}$	$\frac{3}{16}$	$\frac{1}{4}$
$\frac{5}{16}$	18	.240	.0452	.0767	$\frac{5}{16}$	$\frac{19}{32}$	$\frac{11}{16}$	$\frac{27}{32}$	$\frac{1}{4}$	$\frac{19}{64}$
$\frac{3}{8}$	16	.294	.0679	.1105	$\frac{3}{8}$	$\frac{11}{16}$	$\frac{51}{64}$	$\frac{31}{32}$	$\frac{5}{16}$	$\frac{11}{32}$
$\frac{7}{16}$	14	.345	.0935	.1503	$\frac{7}{16}$	$\frac{25}{32}$	$\frac{29}{32}$	$1\frac{7}{64}$	$\frac{23}{64}$	$\frac{25}{64}$
$\frac{1}{2}$	13	.400	.1257	.1964	$\frac{1}{2}$	$\frac{7}{8}$	$1\frac{1}{64}$	$1\frac{15}{64}$	$\frac{13}{32}$	$\frac{7}{16}$
$\frac{9}{16}$	12	.454	.1619	.2485	$\frac{9}{16}$	$\frac{31}{32}$	$1\frac{1}{8}$	$1\frac{3}{8}$	$\frac{15}{32}$	$\frac{31}{64}$
$\frac{5}{8}$	11	.507	2019	.3068	$\frac{5}{8}$	$1\frac{1}{16}$	$1\frac{15}{64}$	$1\frac{1}{2}$	$\frac{17}{32}$	$\frac{17}{32}$
$\frac{3}{4}$	10	.620	.3019	.4418	$\frac{3}{4}$	$1\frac{1}{4}$	$1\frac{7}{16}$	$1\frac{3}{4}$	$\frac{5}{8}$	$\frac{5}{8}$
$\frac{7}{8}$	9	.731	.4197	.6013	$\frac{7}{8}$	$1\frac{7}{16}$	$1\frac{21}{32}$	$2\frac{1}{32}$	$\frac{3}{4}$	$\frac{23}{32}$
1	8	.838	.5515	.7854	1	$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{19}{64}$	$\frac{27}{32}$	$\frac{13}{16}$
$1\frac{1}{8}$	7	.939	.6925	.9940	$1\frac{1}{8}$	$1\frac{13}{16}$	$2\frac{3}{32}$	$2\frac{9}{16}$	$\frac{31}{32}$	$\frac{29}{32}$

$1\frac{1}{4}$	7	1 064	8892	1 2272	$1\frac{1}{4}$	2	$2\frac{5}{16}$	$2\frac{53}{64}$	$1\frac{3}{32}$	1
$1\frac{3}{8}$	6	1.158	1 0532	1 4849	$1\frac{3}{8}$	$2\frac{3}{16}$	$2\frac{17}{32}$	$3\frac{3}{32}$	$1\frac{3}{16}$	$1\frac{3}{32}$
$1\frac{1}{2}$	6	1 283	1 2928	1.7671	$1\frac{1}{2}$	$2\frac{3}{8}$	$2\frac{3}{4}$	$3\frac{23}{64}$	$1\frac{9}{32}$	$1\frac{3}{16}$
$1\frac{5}{8}$	$5\frac{1}{2}$	1 389	1 5153	2 0739	$1\frac{5}{8}$	$2\frac{9}{16}$	$2\frac{61}{64}$	$3\frac{5}{8}$	$1\frac{13}{32}$	$1\frac{9}{32}$
$1\frac{3}{4}$	5	1 490	1 7437	2 4053	$1\frac{3}{4}$	$2\frac{3}{4}$	$3\frac{11}{64}$	$3\frac{57}{64}$	$1\frac{1}{2}$	$1\frac{3}{8}$
$1\frac{7}{8}$	5	1 615	2 0485	2 7612	$1\frac{7}{8}$	$2\frac{15}{16}$	$3\frac{25}{64}$	$4\frac{5}{32}$	$1\frac{5}{8}$	$1\frac{15}{32}$
2	$4\frac{1}{2}$	1 711	2 2993	3 1416	2	$3\frac{1}{8}$	$3\frac{39}{64}$	$4\frac{27}{64}$	$1\frac{3}{4}$	$1\frac{9}{16}$
$2\frac{1}{4}$	$4\frac{1}{2}$	1 961	3 0203	3 9761	$2\frac{1}{4}$	$3\frac{1}{2}$	$4\frac{3}{64}$	$4\frac{61}{64}$	$1\frac{31}{32}$	$1\frac{3}{4}$
$2\frac{1}{2}$	4	2 175	3 7154	4 9087	$2\frac{1}{2}$	$3\frac{7}{8}$	$4\frac{15}{16}$	$5\frac{31}{64}$	$2\frac{3}{16}$	$1\frac{15}{16}$
$2\frac{3}{4}$	4	2 425	4.6186	5 9396	$2\frac{3}{4}$	$4\frac{1}{4}$	$4\frac{29}{32}$	$6\frac{1}{64}$	$2\frac{7}{16}$	$2\frac{1}{8}$
3	$3\frac{1}{2}$	2 629	5 4284	7 0686	3	$4\frac{5}{8}$	$5\frac{11}{32}$	$6\frac{35}{64}$	$2\frac{5}{8}$	$2\frac{5}{16}$
$3\frac{1}{4}$	$3\frac{1}{2}$	2 879	6 5099	8 2958	$3\frac{1}{4}$	5	$5\frac{25}{32}$	$7\frac{5}{64}$	$2\frac{29}{32}$	$2\frac{1}{2}$
$3\frac{1}{2}$	$3\frac{1}{4}$	3 100	7 5477	9 6211	$3\frac{1}{2}$	$5\frac{3}{8}$	$6\frac{13}{64}$	$7\frac{19}{32}$	$3\frac{1}{32}$	$2\frac{11}{16}$
$3\frac{3}{4}$	3	3 317	8 6414	11.0447	$3\frac{3}{4}$	$5\frac{3}{4}$	$6\frac{41}{64}$	$8\frac{1}{8}$	$3\frac{11}{32}$	$2\frac{7}{8}$
4	3	3 567	9 9930	12 5664	4	$6\frac{1}{8}$	$7\frac{5}{64}$	$8\frac{21}{32}$	$3\frac{19}{32}$	$3\frac{1}{16}$
$4\frac{1}{4}$	$2\frac{7}{8}$	3 798	11 3292	14 1863	$4\frac{1}{4}$	$6\frac{1}{2}$	$7\frac{1}{2}$	$9\frac{3}{16}$	$3\frac{13}{16}$	$3\frac{1}{4}$
$4\frac{1}{2}$	$2\frac{3}{4}$	4 027	12.7366	15 9044	$4\frac{1}{2}$	$6\frac{7}{8}$	$7\frac{15}{16}$	$9\frac{23}{32}$	$4\frac{1}{32}$	$3\frac{7}{16}$
$4\frac{3}{4}$	$2\frac{5}{8}$	4 255	14.2197	17 7206	$4\frac{3}{4}$	$7\frac{1}{4}$	$8\frac{3}{8}$	$10\frac{1}{4}$	$4\frac{9}{32}$	$3\frac{5}{8}$
5	$2\frac{1}{2}$	4 480	15 7633	19 6350	5	$7\frac{5}{8}$	$8\frac{13}{16}$	$10\frac{25}{32}$	$4\frac{1}{2}$	$3\frac{13}{16}$
$5\frac{1}{4}$	$2\frac{1}{2}$	4 730	17 5717	21 6476	$5\frac{1}{4}$	8	$9\frac{15}{64}$	$11\frac{5}{16}$	$4\frac{3}{4}$	4
$5\frac{1}{2}$	$2\frac{3}{8}$	4 953	19 2676	23 7584	$5\frac{1}{2}$	$8\frac{3}{8}$	$9\frac{43}{64}$	$11\frac{27}{32}$	$4\frac{31}{32}$	$4\frac{3}{16}$
$5\frac{3}{4}$	$2\frac{3}{8}$	5 203	21 2617	25 9673	$5\frac{3}{4}$	$8\frac{3}{4}$	$10\frac{7}{64}$	$12\frac{3}{8}$	$5\frac{7}{32}$	$4\frac{3}{8}$
6	$2\frac{1}{4}$	5 423	23.0978	28 2744	6	$9\frac{1}{8}$	$10\frac{17}{32}$	$12\frac{29}{32}$	$5\frac{7}{16}$	$4\frac{9}{16}$

TABLE III

INTERNATIONAL STANDARD SCREW THREAD

Bolt Diameter d		Diameter at Bottom of Thread d_1	Pitch p	Number of Threads per Inch	Bolt Diameter d		Diameter at Bottom of Thread d_1	Pitch p	Number of Threads per Inch
Mm.	Inches	Inches	Mm.		Mm.	Inches	Inches	Mm	
6	.236	.181	1 00	25 4	33	1 299	1 106	3 50	7 26
7	.276	.220	1.00	25 4	36	1 417	1 196	4 00	6 35
8	.315	.246	1 25	20 3	39	1 535	1 314	4.00	6 35
9	.354	.285	1.25	20 3	42	1 654	1.404	4 50	5 64
10	.394	.311	1.50	16 9	45	1.772	1.523	4 50	5 64
11	.433	.350	1 50	16.9	48	1.890	1.613	5 00	5 08
12	.472	.376	1 75	14 5	52	2.047	1.770	5.00	5 08
14	551	.441	2.00	12.7	56	2 205	1 900	5 50	4 62
16	.630	.519	2.00	12 7	60	2 362	2 058	5 50	4 62
18	.709	.571	2.50	10 16	64	2.520	2 187	6 00	4 23
20	.787	.649	2.50	10.16	68	2 677	2.345	6 00	4 23
22	.866	.728	2 50	10.16	72	2 835	2 474	6 50	3 91
24	945	.779	3 00	8.47	76	2 992	2 632	6 50	3 91
27	1.063	.897	3 00	8 47	80	3 150	2 762	7 00	3 63
30	1 181	.987	3 50	7 26					

TABLE IV

ACME STANDARD SCREW THREAD

Threads per Inch n	Depth of Thread Inch t	Thickness at Top of Thread Inch a	Width of Space at Bottom of Thread Inch b	Width of Space at Top of Thread Inch c	Thickness at Root of Thread Inch e
1	5100	3707	3655	6293	6345
1⅓	.3850	2780	2728	4720	4772
1½	3433	2471	2419	4195	.4247
2	2600	1853	1801	3147	3199
3	.1767	1235	1183	2098	2150
4	.1350	.0927	0875	1573	1625
5	1100	0741	0689	1259	1311
6	0933	0618	0566	1049	1101
7	.0814	0529	0478	0899	0951
8	0725	0463	0411	0787	0839
9	.0655	.0413	0361	0699	0751
10	.0600	0371	0319	0629	0681

TABLE V

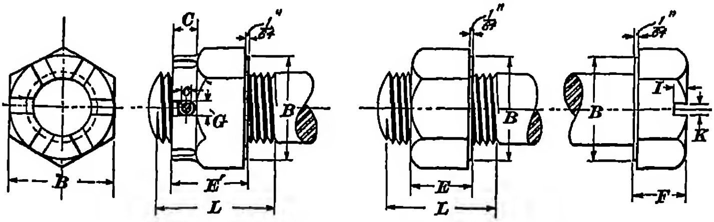
UNITED STATES STANDARD STEAM, GAS, AND WATER PIPE

Sizes of Pipes	Threads per Inch	Actual External Diameter	Actual Internal Diameter	Total Length of Thread	Length of Perfect Thread	Diameter of Tap Drill	Diameter at End of Pipe Inches	
Inches		Inches	Inches	Inches	Inches	Inches	Outside	At Bottom of Thread
$\frac{1}{8}$	27	405	270	41	19	$\frac{21}{64}$	393	334
$\frac{1}{4}$	18	540	364	62	29	$\frac{29}{64}$	522	.433
$\frac{3}{8}$	18	675	494	.63	.30	$\frac{19}{32}$	.656	568
$\frac{1}{2}$	14	840	623	83	.39	$\frac{23}{32}$	815	701
$\frac{3}{4}$	14	1 050	824	84	.40	$\frac{15}{16}$	1 025	.911
1	$11\frac{1}{2}$	1 315	1 048	1 03	.51	$1\frac{3}{16}$	1 283	1 144
$1\frac{1}{4}$	$11\frac{1}{2}$	1 660	1 380	1 06	54	$1\frac{15}{32}$	1 626	1 488
$1\frac{1}{2}$	$11\frac{1}{2}$	1 900	1 611	1 07	.55	$1\frac{23}{32}$	1 866	1 727
2	$11\frac{1}{2}$	2 375	2 067	1 10	58	$2\frac{3}{16}$	2 339	2 200
$2\frac{1}{2}$	8	2 875	2 468	1 64	89	$2\frac{11}{16}$	2 819	2 620
3	8	3 500	3 067	1.70	95	$3\frac{5}{16}$	3 441	3 241
$3\frac{1}{2}$	8	4 000	3 548	1 75	1 00	$3\frac{13}{16}$	3 938	3 738
4	8	4 500	4 026	1 80	1 05	$4\frac{7}{32}$	4 434	4 234
$4\frac{1}{2}$	8	5 000	4 508	1 85	1 10	$4\frac{21}{32}$	4 931	4 731
5	8	5 563	5 045	1 91	1.16	$5\frac{15}{64}$	5 491	5 291
6	8	6 625	6 065	2 01	1 26	$6\frac{1}{4}$	6 546	6.346
7	8	7 625	7 023	2 11	1 36	$7\frac{5}{16}$	7 540	7 340
8	8	8 625	7 982	2 21	1 46	$8\frac{5}{16}$	8 534	8 334
9	8	9 625	8 937	2 32	1 57	$9\frac{5}{16}$	9 527	9 327
10	8	10 750	10 019	2 43	1 68	$10\frac{5}{16}$	10 645	10.445

NOTE —The taper of the threaded part is $\frac{3}{4}$ inch per foot.

TABLE VI

ASSOCIATION OF LICENSED AUTOMOBILE MANUFACTURERS' STANDARD SCREWS AND NUTS



Diameter of Screw d	Threads per Inch n	Length of Threaded Portion L	Short Diameter of Nut B	Height of Castle C	Thickness of Nut E	Thickness of Castle Nut E'	Thickness of Head F	Width of Slot for Cotter Pin G	Depth of Slot in Head I	Width of Slot in Head K	Diameter of Cotter Pin c	Diameter of Cotter-Pin Hole G
$\frac{1}{4}$	28	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{32}$	$\frac{7}{32}$	$\frac{9}{32}$	$\frac{3}{16}$	$\frac{5}{64}$	$\frac{3}{32}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{5}{64}$
$\frac{5}{16}$	24	$\frac{15}{32}$	$\frac{1}{2}$	$\frac{3}{32}$	$\frac{17}{64}$	$\frac{21}{64}$	$\frac{15}{64}$	$\frac{5}{64}$	$\frac{7}{64}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{5}{64}$
$\frac{3}{8}$	24	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{1}{8}$	$\frac{21}{64}$	$\frac{13}{32}$	$\frac{9}{32}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{1}{8}$
$\frac{7}{16}$	20	$\frac{21}{32}$	$\frac{11}{16}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{29}{64}$	$\frac{21}{64}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{1}{8}$
$\frac{1}{2}$	20	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{16}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{1}{8}$
$\frac{9}{16}$	18	$\frac{27}{32}$	$\frac{7}{8}$	$\frac{3}{16}$	$\frac{31}{64}$	$\frac{39}{64}$	$\frac{27}{64}$	$\frac{5}{32}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{5}{32}$
$\frac{5}{8}$	18	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{1}{4}$	$\frac{35}{64}$	$\frac{23}{32}$	$\frac{15}{32}$	$\frac{5}{32}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{5}{32}$
$\frac{11}{16}$	16	$1\frac{1}{32}$	1	$\frac{1}{4}$	$\frac{19}{32}$	$\frac{49}{64}$	$\frac{33}{64}$	$\frac{5}{32}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{5}{32}$
$\frac{3}{4}$	16	$1\frac{1}{8}$	$1\frac{1}{8}$	$\frac{1}{4}$	$\frac{21}{32}$	$\frac{13}{16}$	$\frac{9}{16}$	$\frac{5}{32}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{5}{32}$
$\frac{7}{8}$	14	$1\frac{5}{16}$	$1\frac{1}{4}$	$\frac{1}{4}$	$\frac{49}{64}$	$\frac{29}{32}$	$\frac{21}{32}$	$\frac{5}{32}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{5}{32}$
1	14	$1\frac{1}{2}$	$1\frac{7}{16}$	$\frac{1}{4}$	$\frac{7}{8}$	1	$\frac{3}{4}$	$\frac{5}{32}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{1}{8}$	$\frac{5}{32}$

NOTE —The material is steel with an ultimate tensile strength not less than 100,000 pounds per square inch, the elastic limit is not less than 60,000 pounds per square inch, length of threaded portion is one and one-half times body diameter All dimensions are given in inches.

TABLE VII

MACHINE AND WOOD SCREWS

Gauge No	Diameter Inch	Threads per Inch	Diameter of Round Head Inch	Diameter of Filister Head Inch	Diameter of Flat Head Inch
000	.0315				
00	.0447				
0	.0578	104			
1	.0710	72			
2	.0842	64	.1544	.1332	.1631
3	.0973	48	.1786	.1545	.1894
4	.1105	36	.2028	.1747	.2158
5	.1236	32	.2270	.1985	.2421
6	.1368	32	.2510	.2175	2684
7	.1500	32	.2754	.2392	.2947
8	.1631	32	.2936	.2610	.3210
9	.1763	32	.3238	.2805	.3474
10	.1894	32	.3480	.3035	.3737
11	.2026	24			
12	.2158	24	.3922	.3445	.4263
13	.2289	22			
14	.2421	20	.4364	.3885	.4790
15	.2552	20			
16	.2684	18	.4866	.4300	.5316
17	.2816	18			
18	.2947	18	.5248	.4710	.5842
19	.3079	18			
20	.3210	16	.5690	.5200	.6308
21	.3342				
22	.3474	16	.6106	.5557	.6894
23	.3605				
24	.3737	16	6522	.6005	.7420

TABLE VIII

MORSE TWIST-DRILL AND STEEL-WIRE GAUGE

Gauge No	Diameter Inch	Gauge No	Diameter Inch	Gauge No.	Diameter Inch
1	.2280	28	.1405	55	.0520
2	.2210	29	1360	56	.0465
3	2130	30	.1285	57	.0430
4	.2090	31	.1200	58	.0420
5	.2055	32	1160	59	.0410
6	.2040	33	1130	60	.0400
7	.2010	34	1110	61	.0390
8	.1990	35	.1100	62	.0380
9	.1960	36	.1065	63	.0370
10	.1935	37	1040	64	.0360
11	.1910	38	.1015	65	.0350
12	.1890	39	.0995	66	.0330
13	.1850	40	.0980	67	.0320
14	1820	41	0960	68	.0310
15	1800	42	.0935	69	.02925
16	1770	43	0890	70	.0280
17	1730	44	.0860	71	.0260
18	.1695	45	.0820	72	.0250
19	1660	46	.0810	73	.0240
20	.1610	47	.0785	74	.0225
21	.1590	48	.0760	75	.0210
22	.1570	49	0730	76	.0200
23	1540	50	.0700	77	0180
24	1520	51	.0670	78	0160
25	.1495	52	.0635	79	.0145
26	.1470	53	.0595	80	0135
27	.1440	54	0550		

TABLE IX

DECIMAL EQUIVALENTS OF PARTS OF 1 INCH

Fraction	Decimal	Fraction	Decimal	Fraction	Decimal	Fraction	Decimal
	0078		.2578		5078		.7578
$\frac{1}{64}$	.0156	$\frac{17}{64}$	.2656	$\frac{33}{64}$	.5156	$\frac{49}{64}$	7656
	.0235		.2735		5235		7735
$\frac{1}{32}$	.0313	$\frac{9}{32}$	.2813	$\frac{17}{32}$	.5313	$\frac{25}{32}$	.7813
	.0391		.2891		5391		7891
$\frac{3}{64}$	.0469	$\frac{19}{64}$	.2969	$\frac{35}{64}$	.5469	$\frac{51}{64}$	.7969
	.0547		.3047		.5547		.8047
$\frac{1}{16}$	.0625	$\frac{5}{16}$	.3125	$\frac{9}{16}$	.5625	$\frac{13}{16}$	.8125
	.0703		3203		5703		8203
$\frac{5}{64}$	.0781	$\frac{21}{64}$	.3281	$\frac{37}{64}$	.5781	$\frac{53}{64}$	.8281
	0860		3360		.5860		.8360
$\frac{3}{32}$	.0938	$\frac{11}{32}$	.3438	$\frac{19}{32}$	.5938	$\frac{27}{32}$	.8438
	.1016		.3516		.6016		.8516
$\frac{7}{64}$	.1094	$\frac{23}{64}$	.3594	$\frac{39}{64}$	.6094	$\frac{55}{64}$	.8594
	1172		.3672		6172		8672
$\frac{1}{8}$	.1250	$\frac{3}{8}$	.3750	$\frac{5}{8}$	.6250	$\frac{7}{8}$	8750
	1328		.3828		.6328		8828
$\frac{9}{64}$	.1406	$\frac{25}{64}$	.3906	$\frac{41}{64}$	.6406	$\frac{57}{64}$	.8906
	.1485		3985		6485		8985
$\frac{5}{32}$	.1563	$\frac{13}{32}$	.4063	$\frac{21}{32}$	.6563	$\frac{29}{32}$	9063
	.1641		.4141		6641		9141
$\frac{11}{64}$	.1719	$\frac{27}{64}$	.4219	$\frac{43}{64}$	.6719	$\frac{59}{64}$	.9219
	1797		.4297		.6797		9297
$\frac{3}{16}$	.1875	$\frac{7}{16}$	.4375	$\frac{11}{16}$	.6875	$\frac{15}{16}$	.9375
	1953		4453		.6953		9453
$\frac{13}{64}$	.2031	$\frac{29}{64}$	.4531	$\frac{45}{64}$	.7031	$\frac{61}{64}$	.9531
	.2110		.4610		.7110		9610
$\frac{7}{32}$	.2188	$\frac{15}{32}$	.4688	$\frac{23}{32}$	.7188	$\frac{31}{32}$	9688
	.2266		.4766		7266		9766
$\frac{15}{64}$	.2344	$\frac{31}{64}$	.4844	$\frac{47}{64}$	.7344	$\frac{63}{64}$	.9844
	.2422		.4922		.7422		.9922
$\frac{1}{4}$	.2500	$\frac{1}{2}$	.5000	$\frac{3}{4}$	.7500	1	1 0000
	.2578		.5078		.7578		1 0078

TABLE X

DIMENSIONS OF PLATE WASHERS

Diameter of Bolt d Inch	Diameter of Washer D_s Inches	Diameter of Hole Inches	Thickness of Washer t		Diameter of Bolt d Inches	Diameter of Washer D_s Inches	Diameter of Hole Inches	Thickness of Washer t	
			Birmingham Wire Gauge No.	Approximate Value Inch				Birmingham Wire Gauge No.	Approximate Value Inch
$\frac{3}{16}$	$\frac{9}{16}$	$\frac{1}{4}$	18	$\frac{3}{64}$	$1\frac{1}{8}$	$2\frac{3}{4}$	$1\frac{1}{4}$	9	$\frac{5}{32}$
$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{16}$	16	$\frac{1}{16}$	$1\frac{1}{4}$	3	$1\frac{3}{8}$	8	$\frac{11}{64}$
$\frac{5}{16}$	$\frac{7}{8}$	$\frac{3}{8}$	16	$\frac{1}{16}$	$1\frac{3}{8}$	$3\frac{1}{4}$	$1\frac{1}{2}$	8	$\frac{11}{64}$
$\frac{3}{8}$	1	$\frac{7}{16}$	14	$\frac{5}{64}$	$1\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{5}{8}$	8	$\frac{11}{64}$
$\frac{7}{16}$	$1\frac{1}{4}$	$\frac{1}{2}$	14	$\frac{5}{64}$	$1\frac{5}{8}$	$3\frac{3}{4}$	$1\frac{3}{4}$	8	$\frac{11}{64}$
$\frac{1}{2}$	$1\frac{3}{8}$	$\frac{9}{16}$	12	$\frac{7}{64}$	$1\frac{3}{4}$	4	$1\frac{7}{8}$	8	$\frac{11}{64}$
$\frac{9}{16}$	$1\frac{1}{2}$	$\frac{5}{8}$	12	$\frac{7}{64}$	$1\frac{7}{8}$	$4\frac{1}{4}$	2	8	$\frac{11}{64}$
$\frac{5}{8}$	$1\frac{3}{4}$	$\frac{11}{16}$	10	$\frac{9}{64}$	2	$4\frac{1}{2}$	$2\frac{1}{8}$	8	$\frac{11}{64}$
$\frac{3}{4}$	2	$\frac{13}{16}$	10	$\frac{9}{64}$	$2\frac{1}{4}$	$4\frac{3}{4}$	$2\frac{3}{8}$	6	$\frac{13}{64}$
$\frac{7}{8}$	$2\frac{1}{4}$	$\frac{15}{16}$	9	$\frac{5}{32}$	$2\frac{1}{2}$	5	$2\frac{5}{8}$	5	$\frac{7}{64}$
1	$2\frac{1}{2}$	$1\frac{1}{16}$	9	$\frac{5}{32}$					

MACHINE DESIGN

(PART 2)

Serial 997B Edition 1

KEYS AND COTTERS

KEYS

1. **Keys** are iron or steel wedges used to secure wheels, cranks, and pulleys to shafts. It is the duty of the key to prevent the relative rotation of the pieces connected; if, for example, the pieces in question are a pulley and shaft, the function of the key is to prevent the pulley from turning on the shaft. Generally, the key will also prevent a wheel or pulley from moving lengthwise along the shaft.

FORMS OF KEYS

2. Keys may be divided into three classes: *sunk*, *concave*, and *flat*.

Sunk keys, by their resistance to shear, prevent motion between the shaft and the attached part. To this class

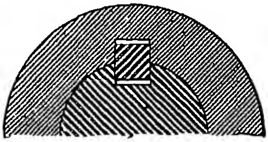


FIG. 1

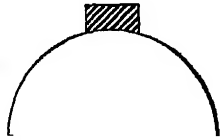


FIG. 2

belong keys that are fitted in slots cut both in the shaft and in the hub of the attached member. Fig. 1 shows a shaft with a sunk key.

Concave, or saddle, keys depend on the friction between them and the shaft to prevent relative rotation. In this case, a slot is provided only in the attached part, while the key is hollowed out to fit the shaft. Such keys are suitable for light work only. A concave key is shown in Fig. 2.

Flat keys use a combination of the means just mentioned to prevent the attached part from changing position. Also, in this key, only one slot is used, but instead of making the key to fit the shaft, a flat surface is cut on the latter to receive the key. This key is consequently more effective than the concave key. Fig. 3 shows a flat key, also called *key on the flat*.

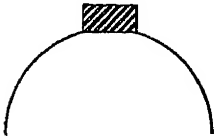


FIG 3

The slot that is cut lengthwise in the shaft, or in the hub of the pulley, gear, etc., is called a **keyway, or key seat.**

3. There are two forms of sunk keys in use, each of which requires a different method for fitting the key in the key seat.

1. The **rectangular key,** shown in Fig. 4, is driven in light and usually fits at the top and bottom as well as at the sides. In order to insure a good fit and to make the driving in and the removal of the key less difficult, it is slightly tapered; that is, the top and bottom planes of the key are not parallel. As a key of this shape is liable to spring the parts out of true, it is not suitable for conditions where exact fitting is required; in general, it is used for fastening gear-wheels, pulleys, etc. to shafts.

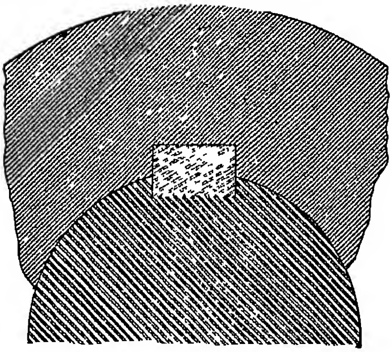


FIG 4

NOTE —To prevent any misunderstanding, the word *taper*, when used in this subject, will mean the gradual diminution in size of a slender object Thus, should it be stated that a certain conical piece has a taper of 2 inches per foot, it would be meant that were the conical piece 1 foot long, the diameter at one end would be 2 inches larger than at the other.

The keys shown in Figs. 2 and 3 are also tapered in the manner just described. The taper varies from $\frac{3}{16}$ to $\frac{1}{8}$ or $\frac{3}{32}$ inch per foot, the smaller tapers being used in the most accurate work.

2. The **square key,** Fig. 1, fits tightly at the sides, but not at the top and bottom. The parts will therefore not be strained, which is an important consideration when accurate fitting is required, as in the case of machine tools. As a square key has no taper, it does not always prevent axial movement of the attached piece unless the latter is forced on the shaft. Otherwise, additional means, such as setscrews, end collars, etc., must be used for holding the piece in position.

4. In cases where a piece has to be adjusted on the shaft in various positions, a key seat of sufficient length is milled out in the shaft and a square key inserted. By the use of setscrews, which force the key toward the shaft, the piece may be held in any required position. It will thus be possible to move the piece without removing the key. The square key requires no taper.

5. Large wheels or pulleys may be fastened to the shaft by using two, three, or four keys. Fig. 5 shows a method

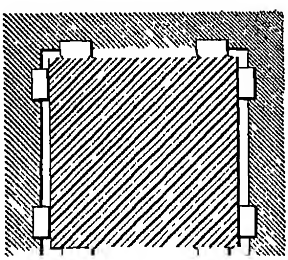


Fig. 5

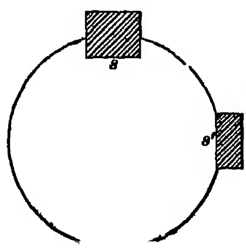


Fig 6

of keying a piece to a square shaft. When several keys are thus used the wheel or pulley may be centered on the shaft by means of the keys.

If a pulley is accidentally bored a little too large for the shaft, it may be prevented from rocking by using both a sunk key and a flat key, as shown in Fig. 6. The flat key s' is placed at a distance around the shaft of about 90° from the sunk key s, and the pulley is thus supported at three points.

FIG 7

When a key cannot be conveniently driven out from the small end, it should be made with a gib head *a*, as shown in Fig. 7. The head forms a shoulder to drive against.

6. Sliding, or feather, keys, or splines, are used to prevent a piece from rotating on a shaft, and at the same time allow it to slide lengthwise. The key, or spline, is usually fastened to the piece, and is free to slide in the key-

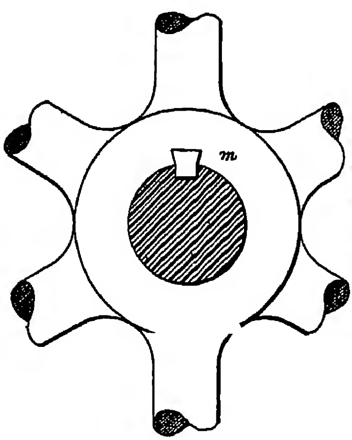


FIG. 8

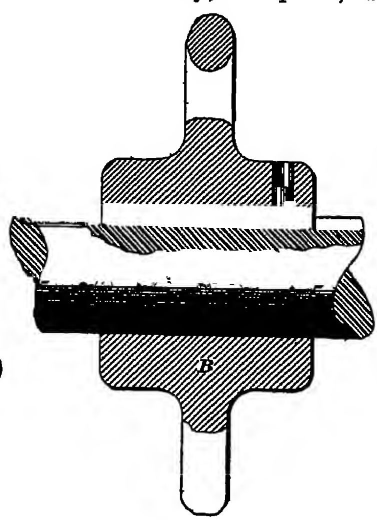


FIG. 9

way of the shaft, though the operation is sometimes reversed and the key is fastened to the shaft. This form of key is made without any taper. Various methods of fixing the key to the wheel or pulley are shown in Figs. 8, 9 and 10. In Fig. 8, the key is dovetailed and driven tightly into the hub *m*.

When the hub of a wheel in which there is a feather abuts against a collar or a bearing, it is evident that the feather must not project. In such a case the feather key, Fig. 8,

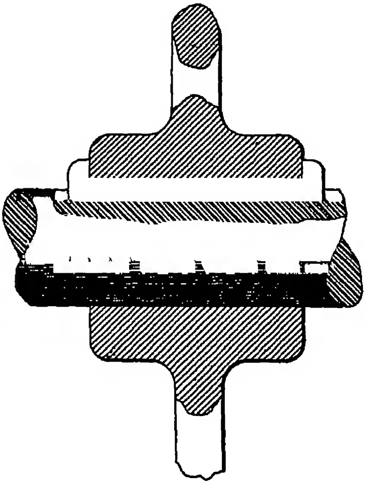


FIG 10

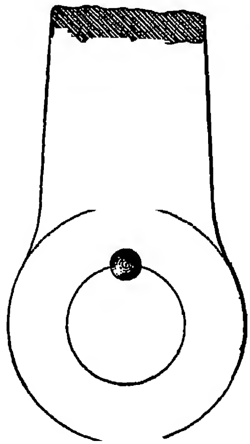


FIG. 11

or the flush feather key, Fig. 9, may be used; otherwise, the key may have gib heads, as shown in Fig. 10.

7. Round, or pin, keys may be used when the piece is shrunk on to the shaft, as for example, a small crank, as shown in Fig. 11 A hole is drilled partly in the shaft and partly in the crank, and a round pin is driven into the hole, as shown in the figure.

8. Woodruff Key.—The **Woodruff key,** shown in Fig. 12, is used to some extent on machine tools. This key is

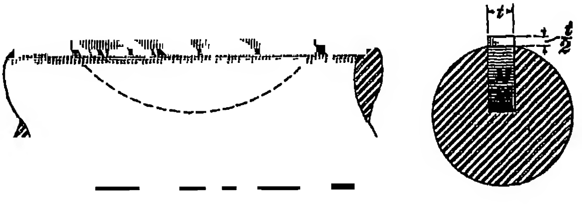


FIG 12

nearly semicircular in outline and rests in a seat, the bottom

of which is curved in conformity with the key. This arrangement will allow the key to adjust itself by a partial rotation until a perfect contact is obtained between its flat edge and the key seat in the piece to be fastened. By reason of its form, the lower key seat is readily made by a milling cutter having a diameter equal to that of the curved portion of the key, thus effecting a great saving of labor in making the seat and in fitting the key.

As shown in Fig. 12, the key is of great depth relative to its width, and is therefore amply insured against tipping. The amount that the key projects above the shaft is equal to $\frac{1}{2}t$, t being the thickness of the key. If the shearing resistance of one key is insufficient, more keys may be added by arranging them lengthwise.

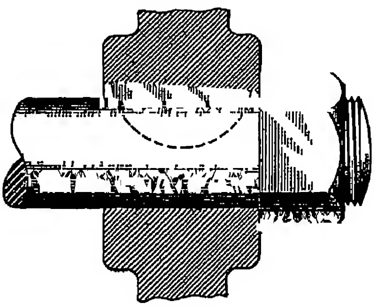


Fig. 13

Fig. 13 shows a hub in which the flat key seat inclines toward the axis of the hub. In this case, while the hub is being placed in position, the key will adjust itself by a partial rotation until it bears against the hub throughout the length of its flat edge.

STRENGTH AND PROPORTIONS OF KEYS

9. A sunk key is subjected to two kinds of stresses. The twisting of the piece on the shaft tends to shear the key, and also to crush it by compression.

Let b = width of key, in inches;
t = thickness of key, in inches;
l = length of key, in inches;
S_s = safe shearing stress allowable, in pounds per square inch;
S_s = safe crushing stress allowable, in pounds per square inch;
P = force, in pounds, acting at rim of wheel or pulley;

R = radius of wheel or pulley, in inches;
d = diameter of shaft, in inches.

The shearing area of the key is $b\,l$; hence, the safe resistance of the key to shearing is $b\,l\,S_s$. Taking moments about the center of the shaft,

$$b\,l\,S_s \times \tfrac{1}{2} d = P \times R$$

or
$$b\,l = \frac{2\,P\,R}{S_s\,d} \qquad (1)$$

Supposing the key to be half bedded in the shaft, the crushing area is $\frac{1}{2}\,t\,l$, and the resistance to crushing is $\frac{1}{2}\,t\,l\,S_c$. If the key is designed to be equally strong against shearing and crushing, the shearing resistance must equal the crushing resistance, or

$$b\,l\,S_s = \tfrac{1}{2}\,t\,l\,S_c$$

or
$$b = \tfrac{1}{2}\,t\,\frac{S_c}{S_s} \qquad (2)$$

If, now, the crushing strength of the material is assumed to be double the shearing strength, $\frac{S_c}{S_s} = 2$, and $b = t$. With square and feather keys, S_c may be nearly equal to $2\,S_s$; but for flat, tapered keys the crushing action is smaller. The reason for this difference is that the key, in its tendency to tip, will act as a strut and thus exert a certain radial bursting pressure on the hub. This bursting pressure will to some extent reduce the side pressure on the key, and, as the danger of crushing is smaller, a smaller factor of safety may be used. In any case, b is rarely less than t, and is frequently made greater.

For shearing, a factor of safety of about 10 may be used, giving a safe shearing stress S_s of 5,000 pounds per square inch for wrought iron, and 7,000 pounds for steel. Then, formula **1** becomes

$$\left.\begin{aligned} b\,l &= \frac{P\,R}{2{,}500\,d} \text{ for wrought iron} \\ b\,l &= \frac{P\,R}{3{,}500\,d} \text{ for steel} \end{aligned}\right\} \qquad (3)$$

Instead of the twisting moment $P\,R$ of formula **3**, it may be more convenient to use the horsepower transmitted by the shaft and also its number of revolutions.

Let N = number of revolutions per minute;
H = horsepower.

Then, a point on the circumference of the wheel or pulley moves $2\pi R N$ inches, or $\frac{2\pi R N}{12}$ feet, per minute.

Hence, if a force P constantly acts at the circumference of the wheel, the work done per minute is

$$\frac{2\pi R N}{12} \times P \text{ foot-pounds}$$

Therefore, $\frac{2\pi R N P}{12} \div 33{,}000 = H$, the horsepower.

$$\frac{12 \times 33{,}000 \times H}{2\pi N} = P R = 63{,}025 \frac{H}{N} \qquad (4)$$

Substituting the value of PR from formula **4**, in formula **3**,

$$\left.\begin{aligned} b\,l &= \frac{25\,H}{d\,N} \text{ for wrought iron} \\ b\,l &= \frac{18\,H}{d\,N} \text{ for steel} \end{aligned}\right\} \qquad (5)$$

Formulas **3** and **5** may be used in calculating the sizes of keys for large work. For small shafts, the sizes given by formulas **3** and **5** are much smaller than are used in actual practice.

10. Designers frequently adopt some standard ratio between the depth and the width of the key, the ratio varying from $\frac{1}{2}$ to $\frac{5}{8}$. In this Section, the ratio $\frac{2}{3}$ is adopted; that is, $t = \frac{2}{3}\,b$.

EXAMPLE.—The maximum pressure on an engine crankpin is 12,500 pounds, and the length of crank is 10 inches Assume the diameter of the shaft to be 5 inches, and the length of the key the same. What should be the dimensions of a wrought-iron key to hold the crank to the shaft? The crank is not to be shrunk on the shaft.

SOLUTION —Transposing formula **3**,

$$b = \frac{PR}{2{,}500\,d\,l} = \frac{12{,}500 \times 10}{2{,}500 \times 5 \times 5} = 2 \text{ in} \quad \text{Ans.}$$

$$t = \tfrac{2}{3}\,b = \tfrac{2}{3} \times 2 = 1\tfrac{1}{3}, \text{ say } 1\tfrac{11}{32}, \text{ in.} \quad \text{Ans.}$$

11. In ordinary designing, the sizes of keys are determined by empirical formulas that give an excess of strength. For ordinary keys in cranks, driving pulleys, etc. the following proportion may be adopted:

$$\left.\begin{aligned} b &= .21\,d + .04 \text{ inch} \\ t &= .12\,d + .04 \text{ inch} \end{aligned}\right\} \qquad (1)$$

When d is less than $1\frac{5}{16}$ inches, then

$$\left.\begin{aligned} b &= \frac{d}{3} \\ t &= \frac{d}{5} \end{aligned}\right\} \qquad (2)$$

Using formula **1** in the example of the crank-shaft, Art **10**,

$b = .21\,d + .04 = .21 \times 5 + .04 = 1\frac{1}{8}$ inches, nearly

$t = .12\,d + .04 = .12 \times 5 + .04 = \frac{5}{8}$ inch, nearly

Unless otherwise stated, formula **1** may be used in solving problems.

As an aid in ordinary designing, Table I, at the end of this Section, may be used. This table gives dimensions closely approximating those formed by formulas **1** and **2**.

Table II, also at the end of this Section, gives the dimensions of square keys, as used by William Sellers & Company. In this case, the keys are supposed to fit tightly sidewise, but not necessarily so on top and bottom. On the other hand, the hub is supposed to fit very tightly on the shaft.

For sliding feather keys, the following formulas give the proportions used in ordinary practice:

$$\left.\begin{aligned} b &= \frac{d}{8} + \tfrac{1}{16} \text{ inch} \\ t &= \frac{3\,d}{16} + \tfrac{1}{16} \text{ inch} \end{aligned}\right\} \qquad (3)$$

12. In some instances, pulleys may be keyed to a large shaft and yet transmit a small amount of power. In such cases the dimensions of a key, if based on the actual diameter of the shaft, will be much too large. The proportions may therefore be based on the diameter of a shaft that would be necessary to transmit the power of the pulley in question, and no more. Letting H represent the horsepower transmitted by the pulley, and N the revolutions per minute of the shaft,

$$d = 5\sqrt[3]{\frac{H}{N}}$$

and this value of d may be used in formula **2**, Art. **11**.

EXAMPLE.—A pulley transmitting 2 horsepower is keyed to a shaft 4 inches in diameter and making 120 revolutions per minute. Determine the dimensions of the key.

SOLUTION.—The diameter of a shaft to transmit 2 H P. is

$$d = 5\sqrt[3]{\frac{H}{N}} = 5\sqrt[3]{\frac{2}{120}} = 1.28 \text{ in.} \quad \text{Ans.}$$

From formula **2**, Art. **11**,

$$b = \frac{1.28}{3} = \tfrac{7}{16} \text{ in.} \quad \text{Ans.}$$

$$t = \frac{1.28}{5} = \tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Calculate the dimensions of an ordinary sunk key for a shaft $3\frac{1}{2}$ inches in diameter. Ans $\frac{1}{2}$ in $\times \frac{3}{4}$ in.

2. Calculate the dimensions of a feather key for a shaft $2\frac{3}{4}$ inches in diameter. Ans. $\frac{13}{32}$ in. $\times \frac{9}{16}$ in.

3. A wrought-iron key is used to fasten a flywheel on a 6-inch shaft. If the maximum pressure on the crankpin is 15,000 pounds, and the

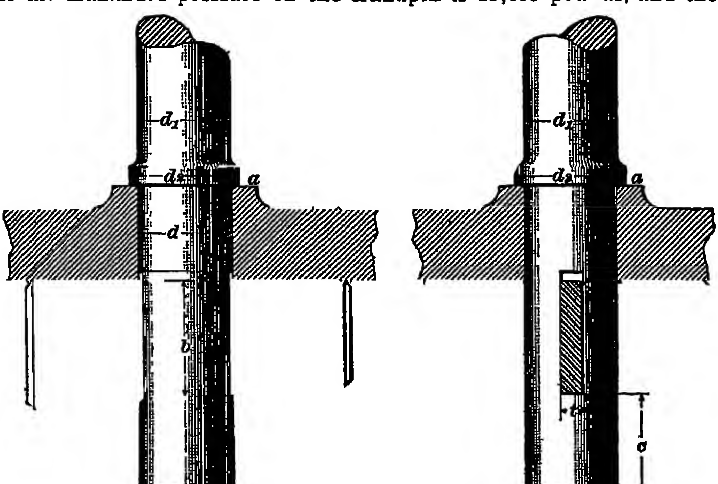


FIG 14

crank radius is 16 inches, what should be the dimensions of the key, its length being 8 inches? Ans. 2 in. $\times 1\frac{11}{32}$ in.

4. A pulley transmits $3\frac{1}{2}$ horsepower, and is keyed to a shaft 5 inches in diameter and making 150 revolutions per minute Calculate the dimensions of the key. Ans. $\frac{1}{2}$ in. $\times \frac{5}{16}$ in.

COTTERS

13. A **cotter** is an iron or steel bar that is driven through one or both of two pieces to be connected, and holds them together by its resistance to shearing at two *transverse* cross-sections. A simple form of a cotter is shown in Fig. 14. The cotter passes through the rod only, and acts when the rod is in tension. The enlargement, or collar, *a* on the rod prevents any downward movement, and therefore resists thrust.

Fig. 15 shows a cotter with gib ends. Since in this case the rod is not provided with a collar, this arrangement will resist tension only.

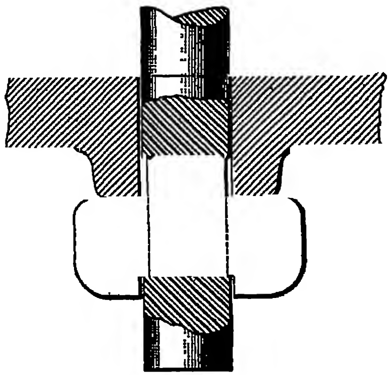


FIG. 15

In the arrangement shown in Fig. 16, the cotter is divided into two parts, the one with hooked ends being called the

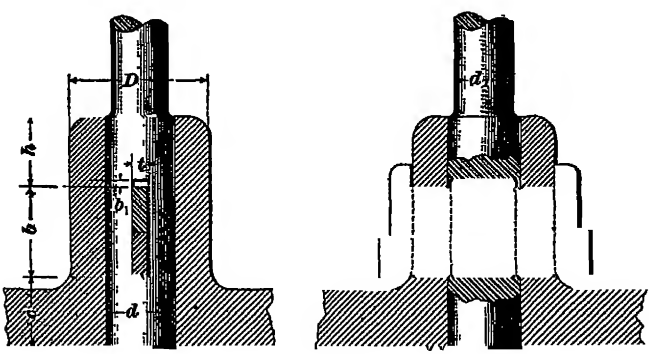


FIG 16

gib, and the other the cotter. In this construction, the rod should be placed in tension only. If it is necessary to provide for thrust, the end of the rod should be tapered as shown in Fig. 17.

Fig. 18 shows an arrangement in which a rod is cottered into a socket. As shown in Figs. 16 and 18, the cotter is long and tapered; it serves, therefore, as a means of adjusting the length of the connected pieces. By driving the cotter farther in, the total length of the two pieces is lessened, and vice versa.

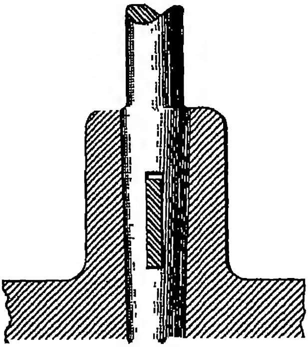


FIG 17

NOTE.—The dimensions *b* and *c*, Fig 18, run to the intersection of the slant side of the cotter and the center line of the rod.

A cotter may be used to connect two straps *m* and *n* to a rod *l*, as shown in Fig 19. When driven down, the friction between the cotter and the lower strap may cause the latter to open out, as shown by the dotted lines; hence, it is desirable in

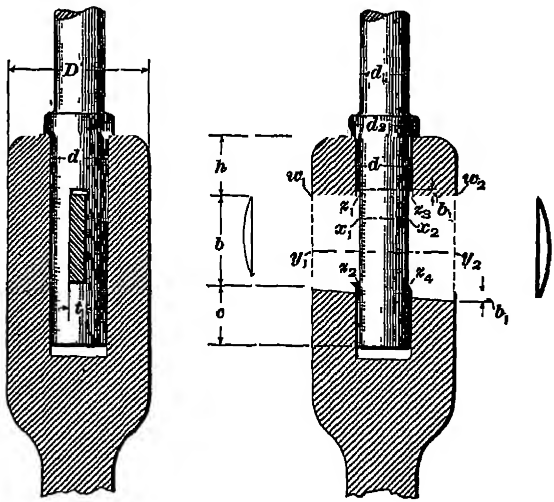


FIG. 18

such a case to use a cotter combined with a gib, as shown in Fig. 20, or with two gibs, as shown in Fig. 21.

The gibs serve to keep the straps from spreading. In Fig. 22, the side *a b* of the gib and *c d* of the cotter are

parallel to each other and perpendicular to the axis of the rod, the taper coming between the gib and the cotter.

14. Strength and Proportions of Cotters.—In designing a cotter connection, the following points should be taken into account. Referring to the illustration of the cotter, Fig. 14, (1) the cross-section $b\,t$ of the cotter must be sufficient to withstand the shearing force; (2) the thickness t must be great enough to provide against failure by crushing;

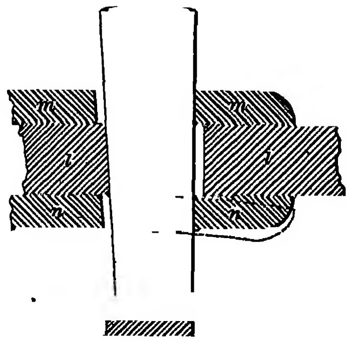


FIG. 19

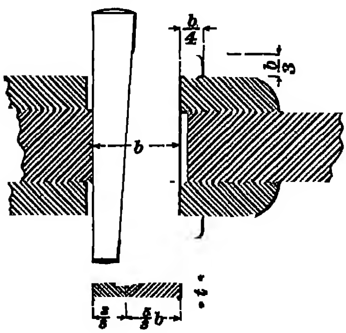


FIG. 20

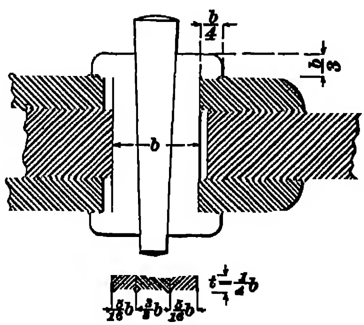


FIG. 21

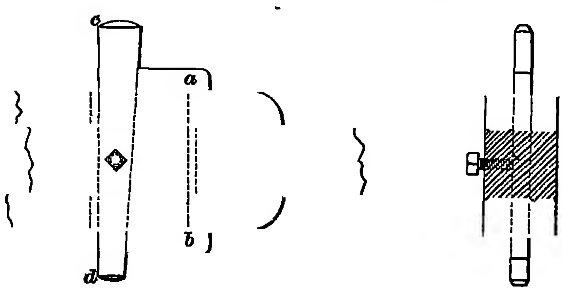


FIG. 22

and (3) the two diameters d_1 and d should be so designed that the rod is of uniform strength throughout.

Let P = force, in pounds, exerted on the rod;
S_1 = safe tensile strength of rod, in pounds per square inch;
S_2 = safe shearing strength of cotter, in pounds per square inch;
S_3 = safe compressive strength of cotter or rod, in pounds per square inch.

The various diameters and other dimensions are indicated in the figures.

Consider the arrangement shown in Fig. 18, and conceive a transverse section taken through the cotter hole. The net area of the rod is $.7854\,d^2 - d\,t$, very nearly; the shearing area of the cotter is $2\,b\,t$; the area of the cotter subject to crushing against the rod is $d\,t$, and against the socket $(D - d)\,t$, very nearly. The area of the socket subject to tension is $.7854\,(D^2 - d^2) - (D - d)\,t$, and the area of the smaller part of the rod is $.7854\,d_1^2$. Finally, the cotter may be considered as a beam uniformly loaded, in which case the bending moment will be $\frac{P\,d}{8} = \frac{I}{c}S_1$. From the table in *Strength of Materials*, Part 2, $I = \frac{1}{12}\,b\,d^3$ and $c = \frac{1}{2}\,d$. In the moment-of-inertia formula, b and d represent the dimensions t and b, respectively, of the key. Hence, $\frac{P\,d}{8} = \frac{t\,b^2}{6}S_1$, and solving for P, then

$$P = \frac{4}{3}\frac{t\,b^2}{d}S_1 \qquad (a)$$

The other formulas representing the tensile, shearing, and crushing stresses are as follows:

At the section $x_1 x_2$, Fig. 18, for tension in the rod,

$$P = (.7854\,d^2 - d\,t)\,S_1 \qquad (b)$$

At the section $y_1 y_2$, for tension in the socket,

$$P = [.7854\,(D^2 - d^2) - (D - d)\,t]\,S_1 \qquad (c)$$

Considering the tension in the rod of diameter d_1,

$$P = .7854\,d_1^2\,S_1 \qquad (d)$$

At the sections $z_1 z_2$ and $z_3 z_4$, for shearing stresses in the cotter,

$$P = 2\,b\,t\,S_2 \qquad (e)$$

At the surface $z_3 z_4$, for crushing stresses between the rod and the cotter,

$$P = d\,t\,S_3 \qquad (f)$$

At the surfaces $w_1 z_1$ and $z_2 w_2$, for crushing stresses between the socket and the cotter,

$$P = (D - d)\,t\,S_3 \qquad (g)$$

It is supposed that the rod and the cotter are made of the same material, either wrought iron or steel. Taking $\frac{S_2}{S_1} = \frac{4}{5}$, as in *Machine Design*, Part 1,

$$S_2 = \tfrac{4}{5} S_1$$

Substituting this value of S_2 in formula (e),

$$P = 2\,b\,t\,\tfrac{4}{5} S_1 = 1.6\,b\,t\,S_1$$

Placing this value of P equal to that found by means of formula (a),

$$1.6\,b\,t\,S_1 = \tfrac{4}{5}\frac{t\,b^2}{d} S_1$$

whence $\qquad b = 1.2\,d$

If this value of b is inserted in formula (e) and the values of P in formulas (e) and (b) are then equated,

$$2 \times 1.2\,d\,t\,S_2 = (.7854\,d^2 - d\,t)\,S_1$$

Inserting for S_2 the value $\frac{4}{5} S_1$,

$$2.4\,d\,t\,\tfrac{4}{5} S_1 = (.7854\,d^2 - d\,t)\,S_1$$

$$1.92\,d\,t = .7854\,d^2 - d\,t$$

$$t = \frac{.7854\,d}{2.92} = .27\,d, \text{ or } \tfrac{1}{4}\,d, \text{ nearly}$$

To prevent failure from crushing by the cotter, the bearing surface of the socket should equal that of the rod. Equating the values of P in formulas (g) and (f),

$$(D - d)\,t\,S_3 = d\,t\,S_3$$

Then, $\qquad D = 2\,d$

Equating the values of P in formulas (b) and (f),

$$(.7854\,d^2 - d\,t)\,S_1 = d\,t\,S_3,$$

and making $t = \frac{1}{4}\,d$,

$$\frac{S_3}{S_1} = 2.14, \text{ or } 2, \text{ approximately}$$

Experience has shown that this ratio may be found where materials and workmanship are good.

Equating the values of P in formulas (b) and (d), and taking $t = \frac{1}{4}d$,

$$.7854\,d^2 - d\,t = .7854\,d_1^2$$

or

$$.7854\,d^2 - \frac{d^2}{4} = .7854\,d_1^2$$

and

$$d_1 = .83\,d$$

The diameter d_2 of the collar, Fig. 18, should be such that its bearing surface is at least equal to that of the cotter in the rod. Hence,

$$.7854\,(d_2^2 - d^2) = d\,t = \frac{d^2}{4}$$

and

$$.7854\,d_2^2 = 1.0354\,d^2,$$

$$d_2 = d\sqrt{\frac{1.0354}{.7854}} = 1.15\,d$$

A clearance b_1 is shown in Figs. 16 and 18 between the cotter and the socket, and the cotter and the rod. The amount of this clearance depends on the taper of the cotter and on the distance through which the latter may be moved. In general, this clearance is not less than $\frac{1}{8}$ inch.

Collecting the foregoing results,

$b = 1.2\,d$	$d_2 = 1.15\,d$
$t = \frac{1}{4}\,d$	$h = c = \frac{3}{4}\,d$ to $1\frac{1}{4}\,d$
$D = 2\,d$	$b_1 = \frac{1}{8}$ inch, or more
$d_1 = .83\,d$	

15. If a steel cotter is used in a wrought-iron rod, b may be made equal to d, the other dimensions remaining the same as just given.

For a cotter of the form shown in Fig. 20 or 21, it is good practice to make $t = \frac{1}{4}\,b$, and $b\,t = \frac{5}{4} \times$ sectional area of strap. The width b is the same whether a single cotter, a gib and cotter, or two gibs and cotter are used. The other proportions are shown in the figures.

EXAMPLE.—Suppose that in Fig. 22 the strap is $\frac{7}{8}$ in. $\times$ $3\frac{1}{2}$ in., find the thickness of the cotter, and the combined width of gibs and cotter.

SOLUTION.— $b\,t = \frac{5}{4} \times \frac{7}{8} \times 3\frac{1}{2} = \frac{245}{64}$. Making $t = \frac{1}{4}\,b$, as just stated, $b\,t = \frac{1}{4}\,b^2 = \frac{245}{64}$, whence

$$b = \sqrt{\tfrac{245}{16}} = 3.91 = 3\tfrac{15}{16}\text{ in. Ans.}$$

$$t = \tfrac{1}{4}\,b = 1\text{ in., nearly. Ans.}$$

The width of cotter is, then, $\frac{3}{8}\,b$, or $1\frac{1}{2}$ inches, and the width of each gib is $\frac{5}{16}\,b$, or $1\frac{7}{32}$ inches, according to the dimensions given in Fig. 21.

16. Taper of Cotters.—The greatest allowable taper that a cotter may have without danger of slacking back is about $1\frac{3}{4}$ inches per foot. Usually, when the cotter is not secured, the taper is from $\frac{1}{4}$ to $\frac{1}{2}$ inch per foot. If fastened by a setscrew or bolt and nut, as in Figs. 22 and 23, the cotter may have a taper of $1\frac{1}{2}$ to 2 inches per foot. The taper per foot of a cotter is found by means of proportion from its increase in width. For instance, if a cotter is 2 inches wide at one end, $2\frac{3}{8}$ inches wide at the other, and 9 inches long, the increase in width is $2\frac{3}{8} - 2 = \frac{3}{8}$ inch. Then, if x is the taper per foot, the proportion $\frac{3}{8} : x = 9 : 12$, will give the value of x. Thus,

$$x = \frac{3}{8} \times \frac{12}{9} = \frac{1}{2} \text{ inch per foot}$$

17. Locking Arrangements.—In Figs. 22 and 23 are shown **locking arrangements for cotters.** In Fig. 22, the cotter is held by a setscrew, the point of which fits into a groove cut into the cotter. The diameter of the setscrew may be $\frac{1}{3}\,b + \frac{1}{8}$ inch. In the

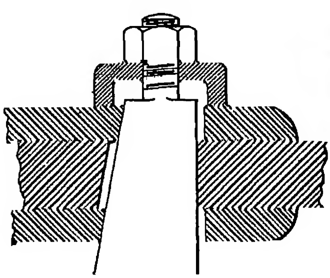


FIG 23

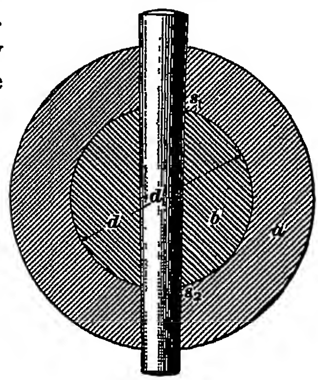


FIG 24

arrangement shown in Fig. 23, the end of the cotter is a screw, and the cotter is secured by a nut on an extra seat. This method is used where the cotter has an excessive taper.

18. Taper Pins.—A **taper pin** may be classed either as a key or as a cotter, but as its action is more like that of

a cotter it will be described at this time. As shown in Fig. 24, the taper pin passes through both the hub *a* and the shaft *b* and is given a certain amount of taper to insure a good fit and to facilitate its removal. This pin is used mostly for light work, as it cannot be made large enough in diameter to resist a relatively large amount of shear without weakening the cross-sectional area of the shaft. In many cases a taper pin is inserted more for the purpose of preventing motion of the hub along the shaft than for the purpose of driving.

Let P = force, in pounds, acting at rim of wheel or pulley;
R = radius of wheel or pulley, in inches;
d = diameter of shaft, in inches;
d_1 = average diameter of pin, in inches;
S_s = safe shearing stress.

As the shear will take place at the points s_1 and s_2, Fig. 24, there will be two shearing areas. The total shearing area of the pin, then, is $2 \times .7854\, d_1^2$, and the safe resistance to shearing is $2 \times .7854\, d_1^2 S_s$. Taking moments about the center of the shaft,

$$2 \times .7854\, d_1^2 S_s \frac{d}{2} = PR$$

and

$$d_1 = 1.13\sqrt{\frac{PR}{d S_s}}$$

The pins are tapered according to certain adopted standards corresponding to the Morse standard reamers. Table III, at the end of this Section, gives the standard diameters of the pins, the length varying from ¾ inch to 6 inches. The taper is ¼ inch per foot.

EXAMPLE.—The hub of a lever 10 inches long is to be fastened by means of a taper pin to a shaft 2 inches in diameter. If the end of the lever is to support a weight of 180 pounds and the pin is made of steel having a safe fiber stress S_s of 7,000 pounds, what will be the diameter of the pin?

SOLUTION.—From the formula of this article, $d_1 = 1.13\sqrt{\frac{180 \times 10}{2 \times 7{,}000}}$ = .405 in., corresponding to a No. 7 taper pin. Ans.

EXAMPLES FOR PRACTICE

1. Find the dimensions of a rod and socket of the form shown in Fig. 18, assuming that S_1 is equal to 6,000 pounds. The load or pull on the rod is 4,600 pounds.

$$\text{Ans} \begin{cases} d = 1\tfrac{3}{16} \text{ in.}; & b = 1\tfrac{1}{2} \text{ in.}; \\ d_1 = 1 \text{ in}; & t = \tfrac{19}{64} \text{ in}; \\ d_2 = 1\tfrac{3}{8} \text{ in.}; & D = 2\tfrac{3}{8} \text{ in.} \\ h = 1 \text{ in.}; & \end{cases}$$

2. A cotter of the form shown in Fig. 15 resists a pull of 3,200 pounds. Find the necessary breadth and thickness, assuming that S_s is 4,000 pounds and that the thickness is one-fourth the breadth.

$$\text{Ans.} \begin{cases} b = 1.27 = 1\tfrac{1}{4} \text{ in.} \\ t = \tfrac{5}{16} \text{ in.} \end{cases}$$

3. A cotter and two gibs connect two straps to a rod, as shown in Fig. 21. Supposing the pull on the rod to be 9,000 pounds and taking S_s = 5,400 for steel, find the dimensions of cotter and gibs.

$$\text{Ans.} \begin{cases} t = \tfrac{15}{32} \text{ in.} \\ \text{Width of cotter} = \tfrac{11}{16} \text{ in.} \\ \text{Width of gibs} = \tfrac{9}{16} \text{ in.} \end{cases}$$

4. Calculate the dimensions of a steel cotter that fastens a wrought-iron rod $2\frac{3}{8}$ inches in diameter. Ans. $2\frac{3}{8}$ in. $\times \frac{19}{32}$ in.

5. A cotter is $1\frac{3}{4}$ inches wide in the middle and tapers on each side. If the cotter is 18 inches long, what is its width at each end? Assume that the taper is $\frac{1}{2}$ inch to the foot. Ans. $2\frac{1}{8}$ in. and $1\frac{3}{8}$ in.

RIVETED JOINTS

RIVETS

19. Distinctive Features.—Rivets differ from bolts in the following respects: (1) Instead of a detachable nut, the rivet has a supplementary head, which is formed from the body of the rivet after it has been inserted in its proper place. (2) The diameter of the rivet body is increased in size after the rivet is placed in position and while the new head is being formed.

As rivets are provided with two heads, they are used to unite permanently two or more parts, mostly steel or iron plates. As the safe union of the combined parts in this case does not depend on the strength of screw threads, as in the case of bolts and nuts, but on the cross-sectional area of the

rivet body, some metals that would be less serviceable for bolts are available for rivets. Among these metals may be mentioned copper, brass, and aluminum, although steel and wrought iron are most commonly used.

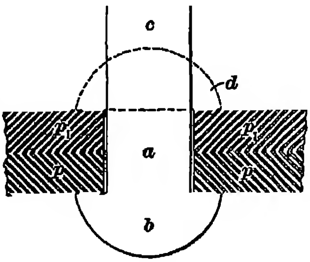


FIG. 25

20. Elements of a Rivet. A rivet in the form furnished by the manufacturer, that is, before being provided with a second head, is termed a **blank**. In Fig. 25 is shown a blank, in which *a* is the **shank**, *b* the **head**, and *c* the **point**. The head, which is made subsequently from the point and is shown by dotted lines at *d*, is termed the **point head**, or, simply, **point**. As will be noticed, the shank does not fit the hole tightly; this difference in diameter between the hole and the shank generally amounts to $\frac{1}{16}$ inch. The shank is slightly tapered toward the point. To form the point head, the part of the shank projecting beyond the plate p_1 must have sufficient material to form not only the point head *d*, but also to provide the additional material required for the shank itself, as the diameter of the latter increases to that of the hole. In general, the protruding part of the shank is of

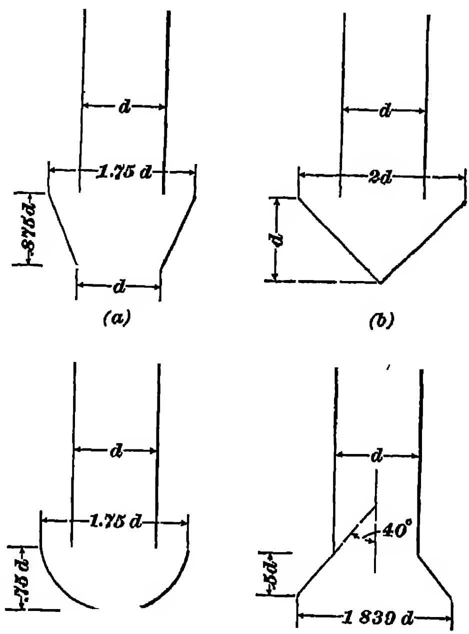


FIG. 26

a length amounting to from 1.3 to 1.7 times the diameter of shank.

21. Forms of Rivet Heads.—The forms of rivet heads in general use are shown in Fig. 26. The one shown at (*a*) is known as the *conical head;* at (*b*), the *steeple head,* at (*c*), the *button head,* and at (*d*), the *countersunk head.*

The proportions ordinarily given to these rivet heads are indicated in the various views in terms of the diameter *d*; but these dimensions are not always adopted. The countersunk head shows the most variation from the proportions given, both in regard to the height and to the diameter of head. Consequently, the angle between the side of the head and the axis of the rivet will also vary, and may be as small as 30° instead of 40°, as shown.

22. Rivet Holes.—The holes for rivets may be either drilled or punched, the former method being adopted where accuracy and strength are required. Accuracy cannot always be insured by punching, partly because the spacing of the holes may vary and also because the holes themselves are not truly cylindrical, but conical. The reason for this inaccuracy is that the diameter of the hole in the die is larger than that of the punch; hence, the hole in the plate has a larger diameter at the bottom than at the top. Punching is also believed to injure the plates, particularly if they are at all hard or brittle. Another objection to punching is that a ragged edge is produced, which has a tendency to decrease the shearing resistance of the rivet. Many makers, therefore, prefer to punch the hole smaller than its intended diameter and then ream it out. This operation cuts away the injured metal around the hole and makes it clean-cut and cylindrical. Annealing after punching will partly remove the injury caused by punching, but the better practice, where the greatest strength is required, is to punch the hole small and to ream to size.

23. Riveting.—The riveting, that is, the formation of the point head, may be done either by hand or by machine; in modern practice, the latter method is used to a greater

extent. In either case the rivet is heated to a red heat and then inserted in the hole prepared for it. When the riveting is done by machine, the rivet is subjected to the pressure of two dies having the shape, or form, that the point head is to receive. As a result of the hammering by hand or of the pressure of the machine, the shank swells out and completely fills the hole. It is advantageous to have the shank hotter at the head end, thereby insuring that the swelling is complete before the point head is formed. When the rivet cools and contracts, the two heads are drawn firmly against the intervening plates.

In riveting, the rivet holes are often brought into line by means of a *drift pin*, which is a long, round taper pin that is small enough at one end to enter two rivet holes not well matched, and large enough at the other end to bring the holes into alinement. In boiler construction, the use of drift pins for enlarging the holes is prohibited by the best authorities.

24. Examples of Riveting.—In Figs. 27, 28, 29, 30, and 31 are shown some examples of the proportions of rivet shanks and the finished point heads. The dimensions of some of the heads differ from those given in Fig. 26, for the reason that there is no uniformly adopted standard, the rivet forms being left, to some extent, to the preference of the various manufacturers. The original form of the shank is shown by dotted lines and its dimensions are also indicated.

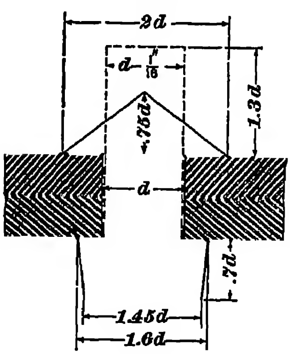


FIG. 27

The point heads shown in Figs. 27 and 28 are made by hand; this form of head is termed a **snap** or **cup head**, and is produced by hammering the shank down roughly and finishing the head with a cup-shaped die, or set. The rivets shown in Figs. 29 and 30 are examples of machine riveting. A rivet with a countersunk head is shown in Fig. 31;

such heads are sometimes necessary where a smooth surface is needed, as, for instance, when attaching boiler mountings.

Sometimes, especially on iron ships, the edges of the rivet hole are slightly chamfered, as shown in Figs. 30

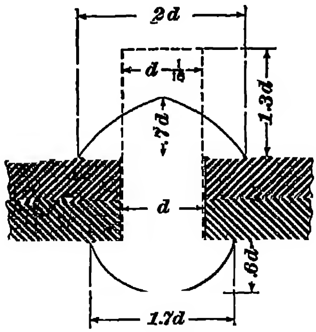


FIG 28

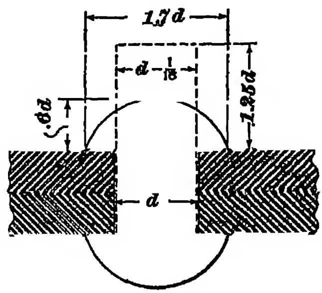


FIG. 29

and 31. A rivet driven in a hole of this kind assumes a corresponding form, resulting in fillets near the heads. Thus, the change in diameter from shank to head is made

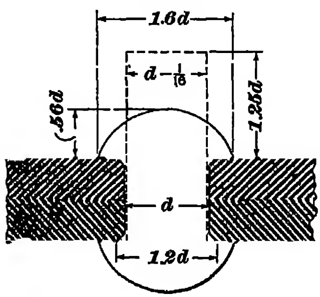


FIG. 30

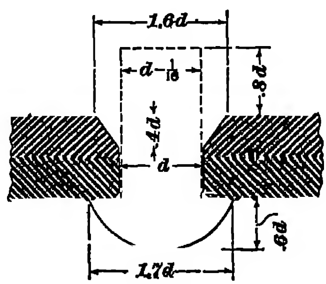


FIG 31

more gradual, and the head is less liable to shear off than it would if subjected to the cutting action of the sharp and ragged edge of a drilled or punched hole.

CLASSIFICATION OF RIVETED JOINTS

25. Lap Joints.—The simplest form of joint is one in which the plates to be riveted are made to overlap. A joint

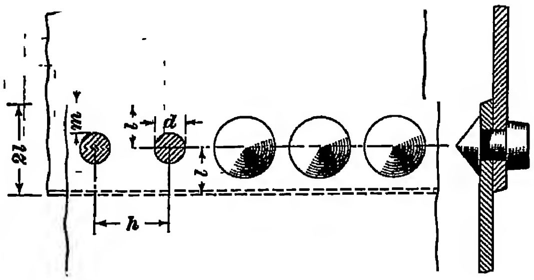


FIG. 32

of this kind is shown in Fig. 32, and is known as a **lap joint.** When a joint is made with only one row of rivets, as in Fig. 32, it is called a *single-riveted lap joint;* with two rows, as in Figs. 33 and 34, a *double-riveted lap joint;* with three rows, a *triple-riveted lap joint;* and with four rows, a *quadruple-riveted lap joint.*

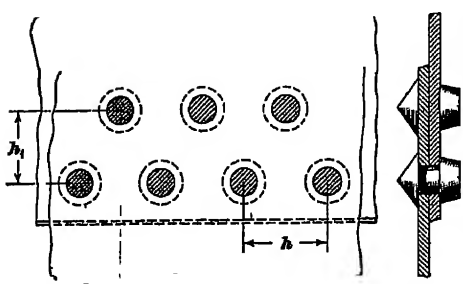


FIG. 33

When more than one row of rivets is used, distinction is made in regard to the relative arrangement of rivets in the adjoining rows. For instance, if all the centers of adjoining rivets in different rows lie in lines perpendicular to the joint, as in Fig. 34, it is called **chain riveting;** but if the rivets in one

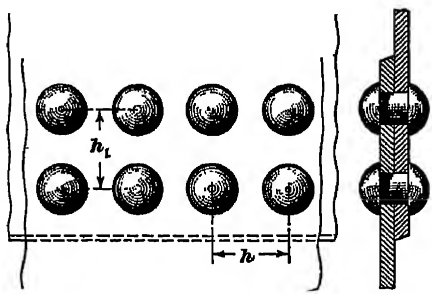


FIG. 34

row are opposite the spaces in the adjoining row, as in Fig. 33, it is called **staggered, or zigzag, riveting.**

A single-riveted lap joint may be provided with a *cover-plate*, or *strap*, as at b, b', Fig. 35. Fig. 36 shows a double-riveted lap joint with cover-plate. Both joints are used extensively in boiler work, and if they are compared with the joints shown in Figs. 32 and 33, it will be seen that they are based on the latter joints, with the addition to each joint of a cover-plate and two rows of rivets spaced farther apart.

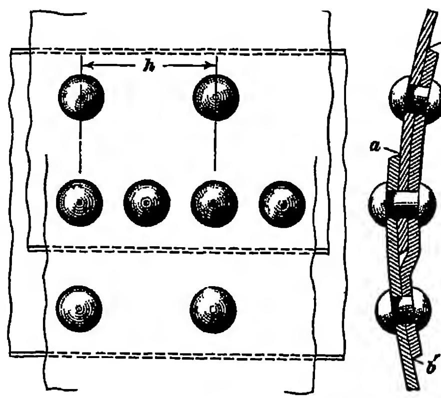


FIG 35

26. Butt Joints.—In **butt joints,** the ends of the plates abut against each other. To effect a union, one or more cover-plates, or straps, are required. With one cover-plate, as in Fig. 37, it constitutes a *single-strap butt joint,*

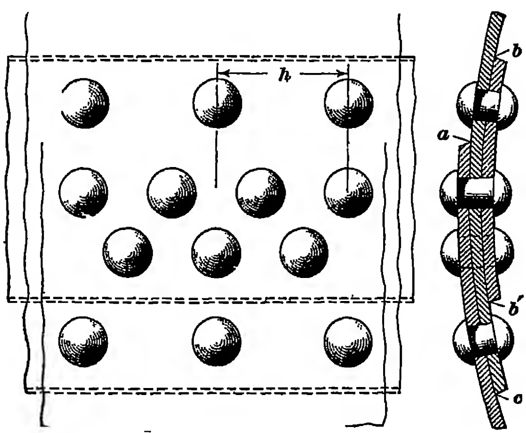


FIG 36

and with two cover-plates, as in Fig. 38, a *double-strap butt joint.* Butt joints are *single-riveted* when there is only one row on *each side* of the joint; with two, three, and four rows on each side, they are called *double-*, *triple-*, and *quadruple-riveted butt joints*, respectively.

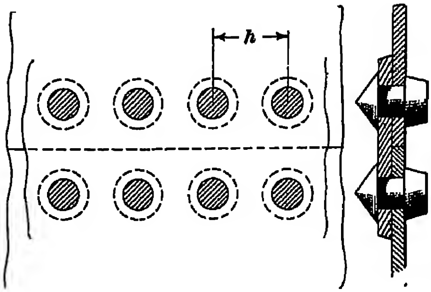


Fig 37

Fig. 37 shows a single-riveted, and Fig. 38 a double-riveted, butt joint. The two cover-plates may differ in width, and the rivets on the outer rows may be spaced farther apart, as shown in Fig. 39.

A triple-riveted double-strap butt joint is shown in Fig. 40. The rivets through both straps are staggered, as in the double-riveted lap joint, Fig. 33.

Fig. 41 shows a quadruple-riveted double-strap butt joint

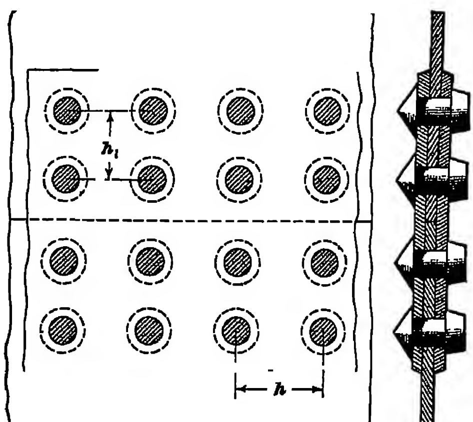


Fig 38

that is used for plates $\frac{5}{8}$ inch or over in thickness. This joint is very expensive, owing to the large number of holes to be drilled and the large number of rivets to be used; but it is

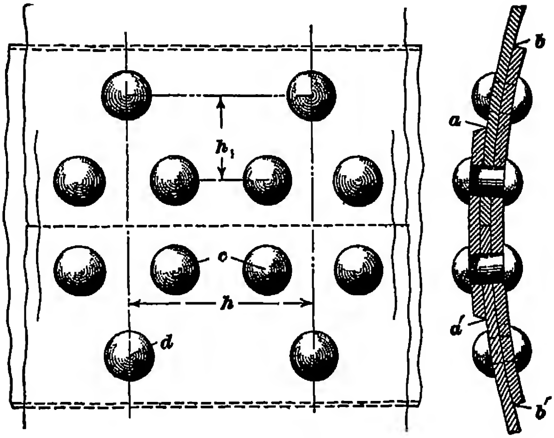


FIG. 39

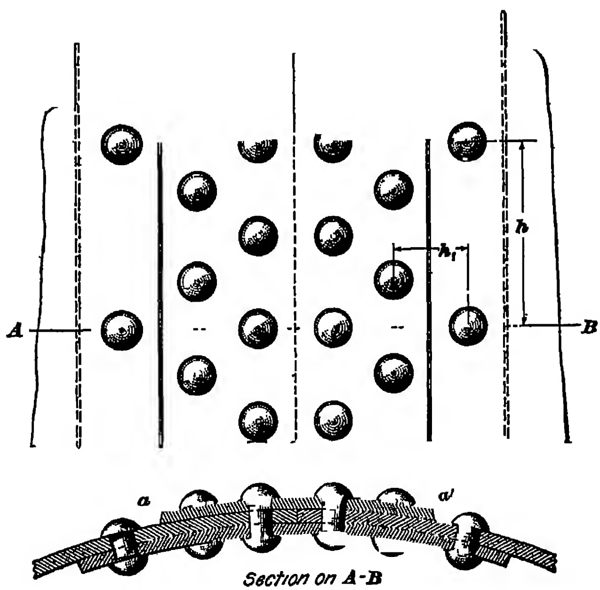


FIG. 40

one of the strongest joints to be found in practice and is also of the highest efficiency.

27. Pitch, Margin, and Lap.—The distance *h*, Fig. 32, between the centers of adjacent rivets in one row is called the **pitch** of the rivets. This pitch may vary in different

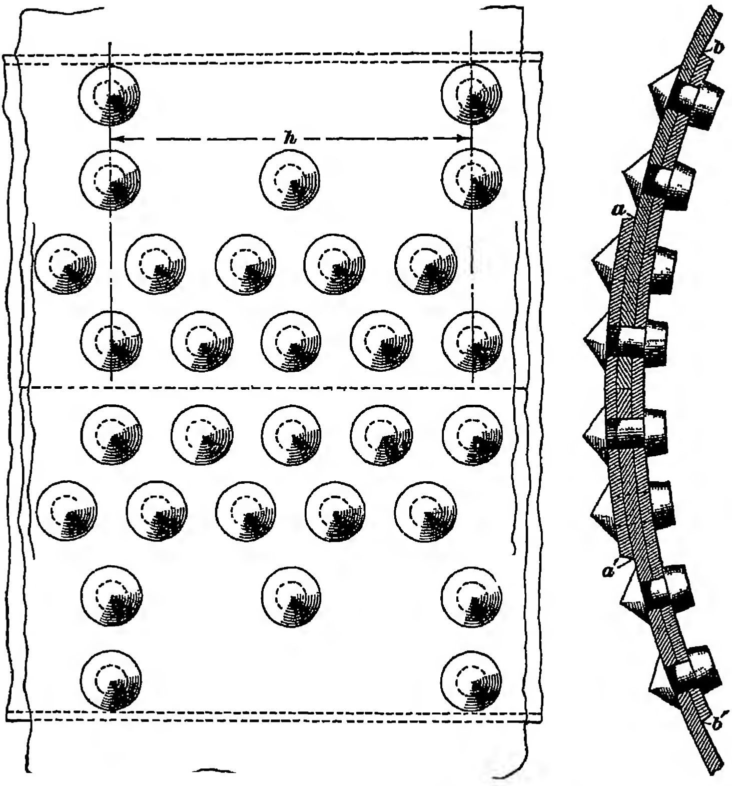


FIG 41

rows, as shown in Fig. 35, in which the greatest distance, as shown at *h*, is considered as the pitch

The distance *m*, Fig 32, between the edge of the rivet hole and the edge of the plate is the **margin**, and the length 2 *l* is the **lap**. The distance *l* is usually made one and one-half times the diameter *d* of the rivet hole. Recent

experiments, however, have shown that when l is made only $1\frac{1}{2}d$ the margin may tear before the joint will fail in any other way. For this reason, some engineers prefer to make l equal to $2d$. The distance h_1 between two rows of rivets is called the **transverse pitch.**

28. Calking.—After the shank of the rivet has contracted, the high pressure produced on the parts between the heads is not always sufficient to insure a steam or water-tight joint where this is required. An additional treatment, termed **calking,** is then necessary. This treatment consists in forming a burr along the edge of one of the plates by means of a suitable calking tool, as shown in Fig. 42. A round-nose calking tool is driven against the beveled edge of the upper plate, forcing the metal in close contact with the lower plate and effectually closing the seam. A tool with a sharp edge should never be used, as it is liable to score the under plate and cause grooving. Such grooves may act as starting points for cracks or bends.

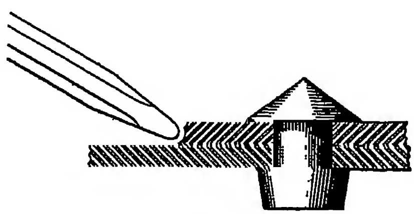


FIG. 42

On boiler joints, the calking may be done either on the inside or on the outside, or on both, the most effective calking being done on the outside. The calking edge of a plate should be beveled to make an angle of 15° with a line normal to the contact surface of the plate. To insure a good calking edge, the lap of the joint should not be excessive, as the plate will buckle instead of having its edge upset. Likewise, the pitch of the rivets should be such as to make it possible to produce a tight joint by calking.

In some of the preceding examples of riveted joints, the calking edges are indicated by the letter a. For instance, in Fig. 35, the joint is calked along the edge a of the outside plate. If the calking were done along the cover-plate, it would not only be necessary to calk two edges b, b', but the

difficulty in securing a tight joint would be greater—in some cases too great—on account of the wide spacing of the adjoining rivets.

In the joint shown in Fig. 36, the calking is done at the edge *a* of the outer plate, as it would be useless to calk the cover-plate at *b*, *b'*, while at *c* the rivets are spaced too far apart to insure a tight joint.

The butt joints in Figs. 39, 40, and 41 are calked at the edges *a*, *a'*, of the outside cover-plates, as the other cover-plates are inside and have rivets spaced too far apart.

STRENGTH OF RIVETED JOINTS

29. Failure of Riveted Joints.—When the load applied to the rivet joint acts in a direction at right angles to the axes of the rivets, the joint may fail in one of the following five ways, which are illustrated in Fig. 43.

1. The plate may fracture along the center line of the rivets, as shown in (*a*).
2. The rivet may shear, as shown in (*b*).
3. The rivet or the plate in front of it may be crushed, as shown in (*c*).
4. The plate may shear in front of the rivets, as shown in (*d*).
5. The plate may break across the margin, as shown in (*e*).

To ascertain which part of a rivet joint is the weakest and where, therefore, a failure is likely to occur, each part of the joint will be considered separately and a formula representing the strength at this particular point will be given.

Let d = diameter of rivet *after* riveting, or diameter of rivet hole;
h = pitch of rivets, in inches; where the pitch varies, the largest pitch is taken;
t = thickness of plate, in inches;
f = factor of safety;
n = number of rivets for a length of joint equal to h,

l = distance from center of rivet to edge of plate, in inches, usually taken as $1\frac{1}{2}\,d$;

S_t = ultimate tensile strength, in pounds per square inch of plate = 55,000 pounds for steel and 50,000 pounds for wrought iron;

S_c = ultimate crushing strength per square inch of the plate = 90,000 pounds for steel and 80,000 pounds for wrought iron;

S_s = ultimate shearing strength per square inch of rivets = 45,000 pounds for steel and 40,000 pounds for wrought iron;

R_t, R_c, and R_s = ultimate strength of riveted joint for a width equal to h, for tension, compression, and shearing, respectively.

The values of the ultimate strengths just given are average values for materials used in riveted joints. Whenever possible these values should be replaced by values found by actual tests of the material to be used.

30. Strength of Lap Joints.—Beginning with the first case, illustrated in Fig. 43 (a), the possible modes of failure mentioned in Art. **29** will be considered in detail. By referring to Fig. 32, it will be seen that the area of metal between two rivet holes is $(h - d)t$; its resistance to rupture is $(h - d)\,t\,S_t$; therefore,

$$R_t = (h - d)\,t\,S_t \qquad \textbf{(1)}$$

In the second case, Fig. 43 (b), the resistance of the rivets to shear is the next point to be considered as a possible cause of failure. The shearing area of a rivet is $.7854\,d^2$, and as there is only one rivet in the length h, the shearing resistance of the rivets is equal to $.7854\,d^2\,S_s$, or,

$$R_s = .7854\,d^2\,S_s \qquad \textbf{(2)}$$

In the third case, Fig. 43 (c), the resistance to crushing of the parts located in front of each rivet will depend on the *projected crushing area* of the rivet; that is, the area of the projection of the rivet shank on a plane parallel to its axis. In the present case, this area is $d\,t$. The resistance to crushing for the length h is $d\,t\,S_c$, and

$$R_c = d\,t\,S_c \qquad \textbf{(3)}$$

In the fourth case, Fig. 43 (*d*), the resistance to shear of those parts of the plate situated in front of the rivet holes is considered. Here, the distance l, Fig 32, between the center of the rivet hole and the edge of the plate must be taken into consideration. The part exposed to shear is nearly of prismatic form, having a width nearly equal to d, with a length l. As shearing will take place along two sides

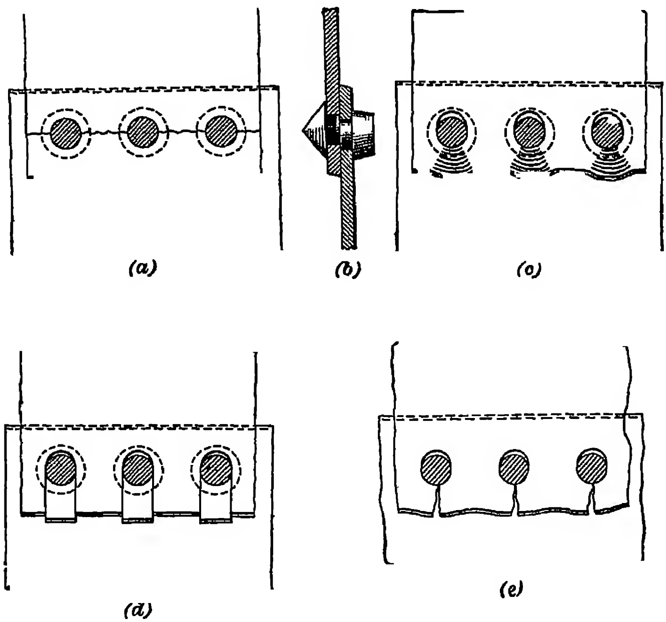


FIG. 43

of this prism, the total shearing area is $2\,l\,t$, and the resistance to shear is

$$R_s = 2\,l\,t\,S_s \qquad (4)$$

The requirement necessary to prevent the fracture considered in the fifth case, Fig. 43 (*e*), will depend somewhat on the nature of the material and whether the holes were punched or drilled. Considering the part in front of the rivet as a beam subjected to bending, calculations show that the margin m, Fig. 32, should be equal to $.4\,d$, in order that its resistance to fracture will equal the shearing strength of the

rivet. As it is common practice to make m at least equal to d, the resistance to fracture at this place is reasonably ample.

31. If the margin of a joint is ample, it is customary to consider only the first, second, and third cases, represented by formulas **1**, **2**, and **3**, Art. **30**. In order that each part of the joint will be equal in strength, the values of R, as found by these formulas, should be equal. The suitable values of d, t, and h may then be found by equating the values of R_t, R_s, and R_c.

32. In determining the diameter of the rivets, it is desirable to find one that will give the rivet equal resistance against shearing and crushing. This may be done by equating R_s and R_c in formulas **2** and **3**, Art. **30**. Thus,

$$.7854\, d^2 S_s = d\, t\, S_c$$

and

$$d = \frac{t\, S_c}{.7854\, S_s} = 1.27 \frac{t\, S_c}{S_s} \qquad (1)$$

As the shearing resistance of a rivet varies as the square of its diameter, and the resistance to crushing varies directly as the diameter, it follows that if the diameter is made smaller than that found by formula **1**, the resistance to crushing will exceed that of shearing, and if the diameter is greater, its shearing resistance will be the greater.

To find the distance h that will give the plate a tensile strength between rivet holes equal to the shearing resistance of the rivets, combine formulas **1** and **2**, Art. **30**; or,

$$(h - d)\, t\, S_t = .7854\, d^2 S_s$$

Then,

$$h = .7854\, d^2 \frac{S_s}{t\, S_t} + d \qquad (2)$$

In this instance, it is assumed that the diameter of the rivet is equal to or smaller than that found by formula **1**, and that, therefore, its resistance to shearing equals or is smaller than that to crushing. If the rivet has a diameter *larger* than that found by formula **1**, its crushing resistance must be taken into account, and the values of R_t and R_c in formulas **1** and **3**, Art. **30**, must be equated. Then,

$$(h - d)\, t\, S_t = d\, t\, S_c$$

and

$$h = \frac{d(S_t + S_c)}{S_t} \qquad (3)$$

33. In double-riveted joints, such as shown in Figs. 33 and 34, all rivets contained in a length h of the joint (see Fig. 34) are supposed to participate in the shearing and crushing stresses. Therefore, in Figs. 33 and 34, there will be two rivets subjected to shearing, and the crushing stress will be distributed along a length of plate corresponding to that located on front of two rivets. In determining the tensile strength of the plate, only that part included between two rivets in one row is considered, and formula **1**, Art. **30**, may therefore be used for this joint also.

The shearing resistance of the rivets is

$$R_s = 2 \times .7854\, d^2 S_s = 1.57\, d^2 S_s \qquad (1)$$

The resistance of the joint to crushing is

$$R_c = 2\, d\, t\, S_c \qquad (2)$$

If the plates have the same thickness as those of a single-riveted lap joint, the rivets may either be placed farther apart or they can be of smaller diameter.

Equating the values of R_s and R_c,

$$d = 1.27 \frac{t\, S_c}{S_s} \qquad (3)$$

Equating the values of R_t and R_s,

$$h = \frac{1.57\, d^2 S_s}{t\, S_t} + d \qquad (4)$$

If d is taken *larger* than the value found by formula **3**, then, following the method given in Art. **32**,

$$h = \frac{d(2\, S_c + S_t)}{S_t} \qquad (5)$$

The transverse pitch h_1, Fig. 33, should have a minimum value of $.4\,h$; it is generally $.5\,h$ to $.65\,h$. For the joint, Fig. 34, h_1 should not be less than $2\,d$ and is preferably $2.5\,d$ for boilers and $3\,d$ for structural work.

EXAMPLE 1.—A single-riveted joint is to be made with $\frac{5}{16}$-inch wrought-iron plates. Find the rivet pitch and the diameter of the rivet hole.

SOLUTION.—From formula **1**, Art. **32**, $d = 1.27 \frac{t\, S_c}{S_s}$. From Art. **29**, $S_s = 40{,}000$ lb., and $S_c = 80{,}000$ lb.; hence,

$$d = 1.27 \times \tfrac{5}{16} \times \frac{80{,}000}{40{,}000} = .79 \text{ in.}$$

If the nearest smaller size available is $\frac{3}{4}$ in., then formula **2**, Art. **32**, is used for ascertaining h, if the next larger size, such as $\frac{13}{16}$ in., then formula **3**, Art **32**, is used. As $d = \frac{3}{4}$ in.,

$$h = .7854 \times (\tfrac{3}{4})^2 \times \frac{40{,}000}{\frac{5}{16} \times 50{,}000} + \tfrac{3}{4} = 1.88, \text{ or } 1\tfrac{7}{8}, \text{ in., nearly}$$

If $d = \frac{13}{16}$ in.,

$$h = \frac{\frac{13}{16} \times (50{,}000 + 80{,}000)}{50{,}000} = 2.113, \text{ or } 2\tfrac{1}{8}, \text{ in., nearly. Ans.}$$

EXAMPLE 2.—If a double-riveted lap joint is to be made with the plates in example 1, what will be the pitch?

SOLUTION.—From formula **3**, Art **33**, it is seen that the rivet diameter remains the same. From formula **4**, Art. **33**,

$$h = \frac{1.57 \times (\frac{3}{4})^2 \times 40{,}000}{\frac{5}{16} \times 50{,}000} + \tfrac{3}{4} = 3 \text{ in. Ans.}$$

34. Strength of Butt Joints.—The butt joint with a single cover-plate, shown in Fig. 37, consists practically of two lap joints, each being a joint between the strap and a plate. The formulas applying to a single lap joint may therefore be used for each half of the butt joint. To prevent the cover-plate from bending, its thickness is increased by at least one-eighth that of the plates, and its tensile and crushing resistances will be greater.

The butt joint with two cover-plates, shown in Fig. 38, has an advantage over a lap joint or over a butt joint with one cover-plate, in that it is free from the tendency to bend the plates. In addition to this, the rivets are subjected to double shear. This style of joint is therefore considered very strong, but it is expensive on account of the work required to drill so many holes and to drive so many rivets. In first-class work, however, the required strength of joint, and not the cost, should determine the type of joint to be used.

The tensile resistance of the plate along the outer row of rivets is the same as that of a lap joint, and is found by means of formula **1**, Art. **30**,

$$R_t = (h - d)\, t\, S_t \qquad \textbf{(1)}$$

There are two rivets in a length h subjected to double shear. The fact that these rivets are in double shear does not make them twice as strong as rivets in single shear. The ratio between the strength of a rivet in double shear to

that in single shear is 1.75, according to the British Board of Trade and the Canadian rules, while the Hartford Steam Boiler Insurance and Inspection Company uses and advocates a ratio of 1.85. In the succeeding calculations, a value of 1.75 will be used. The total resistance of the rivets is therefore taken as

$$R_s = 2 \times 1.75 \times .7854\, d^2 S_s = 2.75\, d^2 S_s \qquad (2)$$

The resistance of the plate to crushing will be

$$R_c = 2\, d\, t\, S_c \qquad (3)$$

The value of d that gives the rivet an equal shearing and crushing resistance is found by equating formulas **2** and **3**, or

$$2.75\, d^2 S_s = 2\, d\, t\, S_c$$

when

$$d = .73\, t\, \frac{S_c}{S_s} \qquad (4)$$

To ascertain the pitch that will give the joint equal strength against rupture and crushing, formulas **1** and **3** may be equated as follows:

$$(h - d)\, t\, S_t = 2\, d\, t\, S_c$$

when

$$h = \frac{2\, d\, S_c}{S_t} + d \qquad (5)$$

The combined thickness of the butt straps should be more than that of the plate. Single butt straps should be at least $\frac{9}{8}\, t$. For double straps, the minimum thickness should be $\frac{5}{8}\, t$, if no rivets are omitted in the outer row. In the latter case, the straps should be increased in thickness in the ratio $\frac{h - d}{h - 2\, d}$. For the joint in Fig. 41, calculations show that the straps should be $\frac{3}{4}\, t$. When the joint must be calked, the outer strap should not be made less than from $\frac{1}{4}$ to $\frac{5}{16}$ inch thick, and both straps are usually made of the same thickness. If the straps are of unequal thickness, it is customary to calculate the thickness of the thinner strap as if both straps were of the same thickness.

35. The double-riveted butt joint shown in Fig. 39 differs from the joint shown in Fig. 38 in that the rivets in the outer rows do not pass through the outside cover-plate, and that they have a greater pitch than those in the inner

rows. In this instance, the distance h between the rivets in the outer rows is generally taken as the pitch and as the width of the unit strip.

The joint may fail by tearing along the outer rows of rivets, and the strength of the plate required to resist this failure may therefore be determined by formula **1**, Art. **30**, that is,

$$R_t = (h - d)\,t\,S_t \qquad (1)$$

The failure might occur by shearing two rivets c, Fig. 39, in double shear and one rivet d in single shear. As in Fig. 38, the resistance of a rivet in double shear is taken as 1.75 times its resistance in single shear. The resistance of the joint to shear will therefore be that of two rivets in double shear, equal to $2 \times 1.75 = 3.5$ in single shear, plus one rivet in single shear, or of 4.5 rivets in single shear. Then, the resistance to shear is

$$R_s = 4.5 \times .7854\,d^2 S_s = 3.53\,d^2 S_s \qquad (2)$$

The failure might be caused by tearing the plate along the inner row of rivets and shearing the outer row of rivets in single shear. Then, if R_{ts} represents the resistance to tension and shearing combined,

$$R_{ts} = (h - 2\,d)\,t\,S_t + .7854\,d^2 S_s \qquad (3)$$

The failure might be from crushing the plate in front of the three rivets in the distance h. Then,

$$R_c = 3\,d\,t\,S_c \qquad (4)$$

Finally, the failure might be from crushing the plate in front of two inner rivets and shearing one of the outer rivets in single shear. Then, letting R_{cs} represent the resistance to crushing and shearing combined,

$$R_{cs} = 2\,d\,t\,S_c + .7854\,d^2 S_s \qquad (5)$$

Equating the values of R_s and R_c, then

$$3.53\,d^2 S_s = 3\,d\,t\,S_c$$

and

$$d = .85\,t\frac{S_c}{S_s} \qquad (6)$$

Equating the values of R_t and R_s, then

$$(h - d)\,t\,S_t = 3.53\,d^2 S_s$$

and

$$h = \frac{3.53\,d^2 S_s}{t\,S_t} + d \qquad (7)$$

For double-riveted butt joints with zigzag riveting, and for chain riveting with alternate rivets omitted in the outer row, the transverse pitch is usually

$$h_1 = \frac{4d + 1}{2}$$

For zigzag riveting with alternate rivets omitted in the outer row, similar to Fig. 39, the least value of

$$h_1 = \frac{\sqrt{(11h + 20d)(h + 20d)}}{20} \qquad (8)$$

36. Efficiency of a Riveted Joint.—By **efficiency** of a riveted joint is meant the ratio of its strength to that of the solid plate. In the preceding calculations of strength of joints, the pitch and the dimensions of rivets and plates were chosen so as to make the joint of equal strength against tensile, shearing, and crushing stresses. It is then immaterial which one of these strengths is compared with the strength of the solid plate in order to ascertain the efficiency, though it is customary to consider the strength of the plate between the two rivets in the outer row. Considering a width h, Fig. 32, the area of the solid plate is $h\,t$, and that of the metal between the rivet holes is $(h - d)\,t$. If y is the efficiency of the joint, then

$$y = \frac{(h - d)\,t}{h\,t}$$

or

$$y = \frac{h - d}{h}$$

The efficiency of the riveted joint should be considered for all cases of riveting.

EXAMPLE.—In a single-riveted lap joint, the rivet holes are $\frac{11}{16}$ inch in diameter and the pitch of the rivets is 2 inches. What is the efficiency of the joint?

SOLUTION.—Applying the formula $y = \frac{h - d}{h}$,

$$y = \frac{2 - \frac{11}{16}}{2} = .66, \text{ nearly. Ans.}$$

37. Practical considerations often demand that some of the dimensions be changed; consequently, the three values

of R will no longer be equal. For instance, a riveted joint of sufficient strength may have a distance between rivets so great that it will be difficult to secure a tight joint. Smaller rivets, more closely spaced, may then be required. Likewise, it is possible that the commercial sizes of plates and rivets may not correspond to those demanded by the formulas, and that other sizes must be substituted. In such cases, the formula $y = \frac{h-d}{h}$ would not represent the efficiency of the joint, and it would be necessary to ascertain also the other efficiencies. The lowest one of these efficiencies would then be considered as that of the joint.

By increasing the number of rivet rows in combination with an increased pitch in the outer row, the efficiency of a joint is generally increased. For this reason, Fig. 41 represents one of the most efficient joints in practical use, but it is also an expensive one on account of the number of holes and rivets required.

38. Strength of Cylindrical Shells.—Before showing the application of the preceding formulas, it will be necessary to state briefly the requirements as to the thickness of shells exposed to internal pressure, such as boilers, reservoirs, etc. The ultimate strength of such shells may be computed by using the formula $p\,D = 2\,t\,S_t$, the derivation of which was shown in *Strength of Materials*, Part 2. In this formula,

P = pressure, in pounds per square inch;
D = diameter of shell, in inches;
t = thickness of shell, in inches;
S_t = ultimate tensile strength, in pounds per square inch;
f = factor of safety.

Then, the safe working stress is $\frac{S_t}{f}$ and

$$p\,D = \frac{2\,t\,S_t}{f}$$

For shells subjected to internal pressure, the factor f is generally taken from 4 to 6.

Solving for S_t in the formula just given,

$$S_t = \frac{p D f}{2 t} \qquad (1)$$

Solving for t,

$$t = \frac{p D f}{2 S_t} \qquad (2)$$

In the riveted joints used on a shell, distinction is made between those that run parallel with the axis and others that run around the shell. The first are termed **longitudinal seams** and the latter **girth seams.**

The length of a girth seam is equal to the circumference of the shell, or 3.1416 D, and the sectional area of the shell along a girth seam is 3.1416 $D t$. Its ultimate resistance to rupture is 3.1416 $D t S_t$, and its safe working strength 3.1416 $D t \frac{S_t}{f}$.

The area on which the pressure p acts to produce a girth rupture is .7854 D^2; hence, the pressure tending to produce a rupture is .7854 $D^2 p$. For the area located along the line where rupture would take place, the two expressions indicating pressure and resistance must be equal. Therefore,

$$.7854 D^2 p = 3.1416 D t \frac{S_t}{f}$$

or

$$S_t = \frac{p D f}{4 t} \qquad (3)$$

Formulas **1** and **3** prove that for a given pressure the stress in the shell is twice as great in the longitudinal seam as it is in the girth seam. For this reason, the efficiency of a girth seam need be only one-half as great as the efficiency of the longitudinal seam, in order to furnish equal resisting strength to a given internal pressure; this explains why girth seams are single-riveted, while longitudinal seams of the same shell are double-riveted.

39. Design of Riveted Joints.—As an example of the application of the formulas relating to riveted joints, let it be required to design the riveting and to find the thickness of the shell intended for a compressed-air reservoir 3 feet 9 inches

in diameter and working under a pressure of 160 pounds per square inch, assuming a factor of safety of 6. The longitudinal seam is to be a double-riveted butt joint, as shown in Fig. 39, and the girth seam, a single-riveted lap joint. The factor of safety refers to the solid plate. In a perforated plate, the factor will be smaller, depending on the efficiency of the joint. If this efficiency is about 80 per cent., the actual factor of safety will be nearer 5.

It is assumed that the material to be used is steel, and that its properties have been ascertained by special tests, whereby the following values were found: $S_t = 55{,}000$ pounds; $S_c = 90{,}000$ pounds; and $S_s = 45{,}000$ pounds.

The thickness of the shell will be found from formula **2**, Art. **38**; thus,

$$t = \frac{160 \times 45 \times 6}{2 \times 55{,}000} = .39 \text{ inch}$$

The nearest commercial size is $\frac{7}{16}$ inch, which is the value given t.

By equating the values of R_s and R_c in formulas **2** and **4**, Art. **35**, the smallest allowable value for the diameter d for this style of joint will be obtained. Thus,

$$3.53\, d^2 S_s = 3\, d\, t\, S_c$$

On solving for d,

$$d = \frac{3\, t\, S_c}{3.53\, S_s} = \frac{3 \times .4375 \times 90{,}000}{3.53 \times 45{,}000} = .74 \text{ inch}$$

As this is the smallest allowable size, a $\frac{3}{4}$-inch rivet would do, but the driven size had better be taken larger; a $\frac{13}{16}$-inch rivet hole is therefore preferable.

By equating the values of R_{ts} and R_{cs} in formulas **3** and **5**, Art. **35**, a value for h can be obtained. Thus,

$$(h - 2\,d)\, t\, S_t + .7854\, d^2 S_s = 2\, d\, t\, S_c + .7854\, d^2 S_s$$

Then, $$(h - 2\,d)\, t\, S_t = 2\, d\, t\, S_c$$

and $$h - 2\,d = \frac{2\, d\, S_c}{S_t}$$

from which

$$h = 2\,d\left(1 + \frac{S_c}{S_t}\right) = 1\tfrac{5}{8} \times \left(1 + \frac{90{,}000}{55{,}000}\right)$$
$$= 4.284 = 4\tfrac{5}{16} \text{ in., nearly}$$

From formula **8**, Art. **35**,

$$h_1 = \frac{\sqrt{(11 \times 4\tfrac{5}{16} + 20 \times \tfrac{13}{16}) \times (4\tfrac{5}{16} + 20 \times \tfrac{13}{16})}}{20}$$

$$= 1.81, \text{ or } 1\tfrac{13}{16}, \text{ inches, nearly}$$

40. From the formulas given in Art. **35**, the values R_t, R_s, R_{st}, R_c, and R_{cs} may now be calculated. The smallest value will determine the efficiency of the joint.

From formula **1**, Art. **35**,

$$R_t = (4\tfrac{5}{16} - \tfrac{13}{16}) \times \tfrac{7}{16} \times 55{,}000 = 84{,}219 \text{ pounds}$$

From formula **2**, Art. **35**,

$$R_s = 3.53 \times (\tfrac{13}{16})^2 \times 45{,}000 = 104{,}860 \text{ pounds}$$

From formula **3**, Art. **35**,

$$R_{st} = (4\tfrac{5}{16} - 1\tfrac{5}{8}) \times \tfrac{7}{16} \times 55{,}000 + .7854 \times (\tfrac{13}{16})^2 \times 45{,}000$$
$$= 88{,}000 \text{ pounds}$$

From formula **4**, Art. **35**,

$$R_c = 3 \times \tfrac{13}{16} \times \tfrac{7}{16} \times 90{,}000 = 95{,}977 \text{ pounds}$$

From formula **5**, Art. **35**,

$$R_{cs} = 2 \times \tfrac{13}{16} \times \tfrac{7}{16} \times 90{,}000 + .7854 \times (\tfrac{13}{16})^2 \times 45{,}000$$
$$= 87{,}316 \text{ pounds}$$

Of these values, R_t is the smallest, and will therefore determine the efficiency of the joint. The strength of the solid plate is

$$h\,t\,S_t = 4\tfrac{5}{16} \times \tfrac{7}{16} \times 55{,}000 = 103{,}770 \text{ pounds}$$

Hence, the efficiency of the joint is

$$y = \frac{84{,}219}{103{,}770} = .81$$

From Art. **38**, the efficiency of the girth seam need not be more than one-half that of the longitudinal seam. Retaining rivets of the same size, and making the pitch h in the single-riveted lap joint one-half that of the longitudinal seam,

$$h = \frac{4\tfrac{5}{16}}{2} = 2\tfrac{5}{32} \text{ inch}$$

The efficiency is

$$y = \frac{2\tfrac{5}{32} - \tfrac{13}{16}}{2\tfrac{5}{32}} = .62$$

As this is considerably more than half the efficiency of the longitudinal seam, the girth seam will be amply strong.

The real factor of safety of the shell is affected by the efficiency of the riveted joint. The safe strength of the shell at the longitudinal joint for a width h is $(h-d)\,t\frac{S_t}{f}$, and for the two sides of the shell $2(h-d)\,t\frac{S_t}{f}$. This value must be equal to the internal pressure for the length h, which is $h\,p\,D$. Hence,

$$h\,p\,D = 2(h-d)\,t\frac{S_t}{f}$$

and

$$f = \frac{2(h-d)\,t\,S_t}{h\,p\,D} = \frac{2(4\frac{5}{16} - \frac{13}{16}) \times \frac{7}{16} \times 55{,}000}{4\frac{5}{16} \times 160 \times 45} = 5.42$$

If the joint is perfect in design and workmanship, the strengths at the various possible places of failure should be equal; or, to be on the safe side, the value R_s should be greater, as it is not always practicable to make the rivet holes in the shell plates and cover-plates come absolutely fair; consequently, the rivet sections would be reduced. As a rule, perfect equality of the strengths is difficult to obtain, because the commercial sizes of plates and rivets must be used; and these do not always correspond to the sizes required by the formulas.

41. In the joint under consideration, it is assumed that the cover-plates are of such thickness as to prevent their failure before the steel plate. To insure this condition, the thickness of each cover-plate should generally be at least five-eighths that of the shell. In this instance, therefore, they should be at least $\frac{7}{16} \times \frac{5}{8} = \frac{35}{128}$ inch. The nearest value to this in sixteenths is $\frac{5}{16}$ inch.

The mode of failure of the cover most likely to occur is the tearing of the inside cover-plate at the inner row of rivets, the shell plate freeing itself by shearing the rivets in the outside cover-plate.

The resistance of the cover-plate is

$$(h - 2d)\,t\,S_t = (4\tfrac{5}{16} - 1\tfrac{5}{8}) \times \tfrac{5}{16} \times 55{,}000 = 46{,}191 \text{ pounds}$$

The resistance of the two rivets in shear is

$$2 \times .7854\,d^2\,S_s = 2 \times .7854 \times (\tfrac{13}{16})^2 \times 45{,}000 = 46{,}664 \text{ pounds}$$

The total resistance is 46,191 + 46,664 = 92,855 pounds, which is greater than R_t.

For a quadruple-riveted joint, such as is shown in Fig. 41, the thickness of the cover-plate must be at least three-fourths as thick as the shell plate; otherwise, the efficiency will be lowered by the mode of failure just mentioned.

It should always be remembered that the pitch and rivet diameter found efficient for a shell plate of a given thickness and strength may be entirely unsuited for a plate having different thickness and strength.

42. Tables of Riveted Joints.—Table IV, at the end of this Section, gives the proportions of riveted joints used by a number of the leading manufacturers in the United States. The table is applicable to lap joints, and butt joints with a single cover-plate. For boiler plates more than $\frac{1}{2}$ inch thick, butt joints with two cover-plates are recommended. The efficiencies y in this table are obtained from the formula of Art. **36.** The terms *single* and *double* refer to single- and double-riveted joints, respectively. For example, the efficiency of a double-riveted joint made of $\frac{7}{16}$-inch plates is .74, and the pitch h is $3\frac{3}{8}$ inches. In calculating the efficiencies given in Table IV, it is supposed that the proportions of the joints and the properties of the material are such that the single-riveted joints will fail by tearing along the single row of rivets, and the double-riveted joints by tearing along the outer row of rivets.

Table V, at the end of this Section, gives the size, arrangement of rivets, and width of lap for various thicknesses of sheets in single-riveted lap joints, such as used on tanks, stacks, etc. For rectangular tanks, this table also gives the distance between the center of rivet and outside of tank, likewise the radius of curvature of the corners in same.

STRUCTURAL RIVETING

43. Distinctive Features.—Structural riveting deals with the fastening of rolled shapes, such as beams, angles, etc., to one another or to plates by means of rivets. This class of riveting has much in common with that used where internal pressure must be resisted, both in regard to the kinds of joints and the shapes of the rivets. The difference between them lies in the fact that structural riveting allows more freedom in the selection of pitch; the distance between the rivets is of secondary importance, the main object being to employ a sufficient number of rivets to gain the requisite strength. The pitch, however, should not be so great as to allow moisture to enter the joint, resulting in the production of rust, and should rarely exceed 6 inches. The minimum pitch is determined by the necessary clearance of the riveting tool, and by the danger of causing cracks in the plate between the holes during the punching or riveting process.

44. Rivet Heads.—The forms of rivets found in structural work are of less variety then those used in shells and tanks. The principal forms are: (1) the **button head,** also called **round head,** or **full head;** (2) the **countersunk head;** and (3) the **flattened head,** which is made by flattening the button head, and is used in extreme cases where a round head would be too high.

45. Shop and Field Rivets.—Distinction is made between *shop rivets* and *field rivets*. **Shop rivets** are those driven in place in the shop, and **field rivets,** those that have to be inserted at the place of erection. Shop rivets are made of soft steel, while field rivets are of wrought iron, because this material is less likely to be damaged in heating, and also because considerable heat is lost in passing the rivets from the forge to the riveters, thus reducing them to a temperature that would be too low to work on steel.

46. Diameter of Rivets.—Rivets used in structural work vary in diameter from $\frac{3}{8}$ inch to 1 inch, being made in

$\frac{3}{8}$-, $\frac{1}{2}$-, $\frac{5}{8}$-, $\frac{3}{4}$-, $\frac{7}{8}$-, and 1-inch sizes. The $\frac{3}{4}$- and $\frac{7}{8}$-inch rivets are most frequently used, the smaller sizes being employed only on unimportant details, or where the clearance will not allow the insertion of larger ones. Rivets larger than $\frac{7}{8}$ inch are used only where the plates are very thick or the stresses very great. Field rivets should never be more than $\frac{7}{8}$ inch.

On account of the difficulty in punching holes of smaller diameter than the thickness of the plate, the diameter of the rivet should not be less than the thickness of the thickest plate through which it passes. Rolled sections with small flanges necessitate the use of small rivets, so as to allow enough margin and sufficient space for the tool in driving the rivet. When the size is not determined by the conditions just mentioned, a rivet should be chosen that will resist the existing stresses and at the same time be the least expensive. It is cheaper to use one kind of rivet throughout a piece, and the smaller rivets are less expensive, as they reduce the weight and the cost of punching and driving. The standard sizes used by the Cambria Steel Company in **I** beams, channels, and angles are given in Table VI, at the end of this Section.

As to the relations between the diameters of rivets and the corresponding rivet holes, the explanations given in Art. **20** hold good. Holes are not drilled except in very important work, or when hard steel is used.

47. Dimensions of Heads.—The height and diameter of a rivet head bear the following relations to the diameter of the rivet:

For button heads, when d is the diameter of the rivet, the standard height of the head is $.6\,d$, and the radius of the head is $\frac{3}{4}d + \frac{1}{16}$ inch. These dimensions are shown in Fig. 44.

For countersunk heads, the diameter of the head is the same as for a button head, and its bevel is 60°. A simple rule is to make the depth equal to $\frac{1}{2}d$. Countersunk rivets should not be used in plates having a thickness less

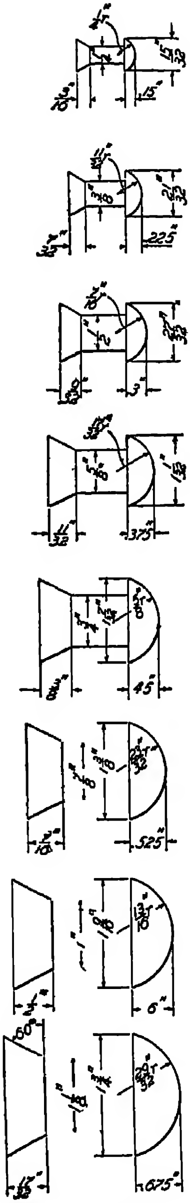


FIG. 44

than $\frac{1}{2}d$. Where economy is to be considered, their use should be avoided if possible, as the extra work in countersinking makes them expensive.

For flattened heads, the height is never less than $\frac{5}{16}$ inch.

48. Length, or Grip.—The maximum **grip**, or distance between heads, is generally accepted as not more than four times the diameter of the rivet. In determining the length to be given in ordering rivets, first find the grip, which is considered as the thickness of the parts joined plus $\frac{1}{32}$ inch for each joint between plates; this $\frac{1}{32}$ inch is to allow for unevenness in the surfaces of the plates, which might prevent them from being in close contact. This grip must be increased in the ratio of the area of the hole to the area of the rivet section, to allow for upsetting, the hole being $\frac{1}{16}$ inch larger in diameter than the rivet. To this result must be added the height required for the head, and the sum is the length of rivet under the formed head.

The values given in Table VII, at the end of this Section, have been calculated in the manner just described. To illustrate the application of this table, two examples are given.

It is desired to find the shank length of a $\frac{3}{4}$-inch button-head rivet with a grip of $2\frac{1}{8}$ inches. Proceed horizontally from the value $2\frac{1}{8}$ in the first column of the table to the vertical column headed $\frac{3}{4}$; in this column the value $3\frac{5}{8}$ is found, which is the required length of the shank.

If the shank length of a countersunk rivet is to be ascertained, a certain amount

has to be deducted from the value given in the table. The amount of this deduction is found at the foot of the table. For instance, if the shank mentioned in the preceding example is that of a countersunk rivet, then the amount $\frac{5}{8}$, which is found at the foot of the vertical column headed $\frac{3}{4}$, is to be deducted. The shank length of a $\frac{3}{4}$-inch countersunk rivet is therefore $3\frac{5}{8} - \frac{5}{8} = 3$ inches.

NOTE.—The right-hand illustration at the head of Table VII indicates the length that is considered as the grip of a countersunk rivet.

49. Pitch.—The **pitch**, or distance from center to center of rivets, should be at least three diameters of the rivet. If spaced closer, the material is likely to fracture or become otherwise injured. For members in compression, the pitch in the direction of the stress should not be greater than 6 inches, or sixteen times the thickness of the thinnest outside plate. If possible, the pitch should be kept in even inches, making it either 2, 3, 4, 5, or 6 inches, as the case may require, especially when the rows are long.

In Table VIII, at the end of this Section, are given the maximum and minimum pitches for rivets of different sizes and the minimum and usual distances from the end.

50. Distance of Rivet From Edge of Plate.—In order to avoid the danger of fracturing the material, rivets should not be placed too near the edge or end of the plate. Owing to the greater strength of wrought iron in the direction of the fiber, however, rivets in wrought iron may be placed nearer the edge than the end. According to practice, the distance between the edge of any piece and the center of the rivet hole should not be less than $1\frac{1}{4}$ inches for $\frac{3}{4}$- and $\frac{7}{8}$-inch rivets, except in bars less than $2\frac{1}{2}$ inches wide; when practicable, this distance should be at least two diameters of the rivet for all sizes and should not exceed eight times the thickness of the plate. A simple rule is to make the end distance twice the diameter of the rivets and the side distance $\frac{1}{4}$ inch less than this.

51. Clearance.—A certain amount of **clearance** is required for tools in forming the heads of rivets in shapes

having short flanges. For a $\frac{7}{8}$-inch rivet, the distance from the center of the rivet to the back of the angle should not be less than $1\frac{1}{4}$ inches; and for a $\frac{3}{4}$-inch rivet, this distance should not be less than $1\frac{1}{8}$ inches. The perpendicular distance between the center line of a rivet in one leg of an angle and the top of a rivet head on the other leg should not be less than $1\frac{1}{8}$ inches.

52. Beam Connections.—In Figs. 45, 46, and 47 are shown the details for I-beam connections of several sizes used in structural steel work. They are designed so as to

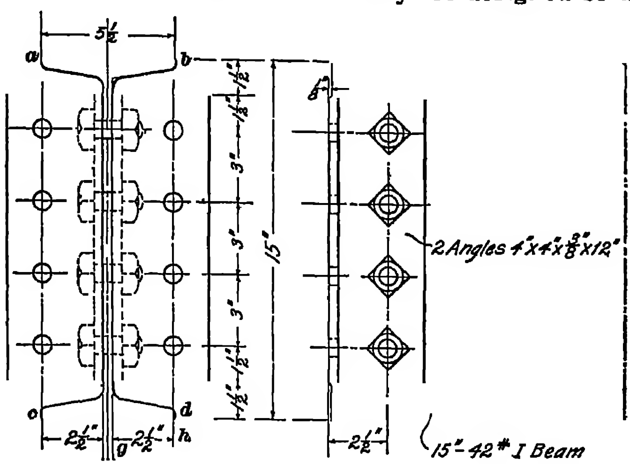


FIG 45

be strong enough to resist the downward pressure at the ends, due to the safe load supported by the beam, provided the span is not unusually short. In that case, the beam would be able to sustain a greater load than the connection.

Fig. 45 shows *connection angles* bolted to a 15-inch 42-pound beam, the mark # being a conventional sign for pounds. The several dimensions of the beam section *a b c d* are not given, for, in practice, they are determined by the weight of the beam per foot. The necessary dimensions will be found in the catalog of the maker whose beams are to be used, in which the dimensions of the connections for a given size of beam are also given.

The distance gh, locating the vertical center line of the rivet holes from the back of the angles, is the standard

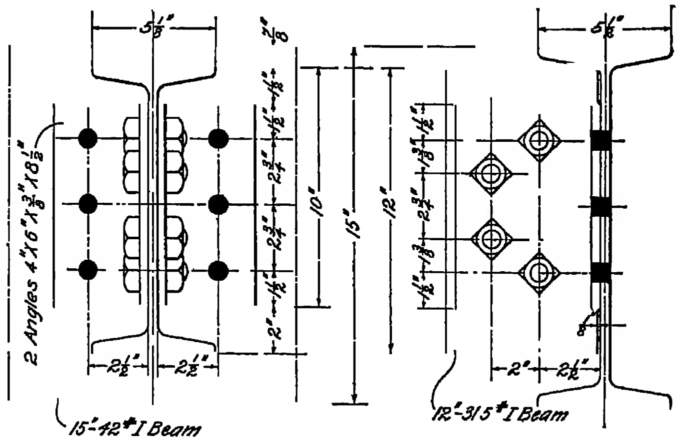


FIG 46

distance employed by most steel mills for this size of angle. It will be noticed from the right-hand view that the connec-

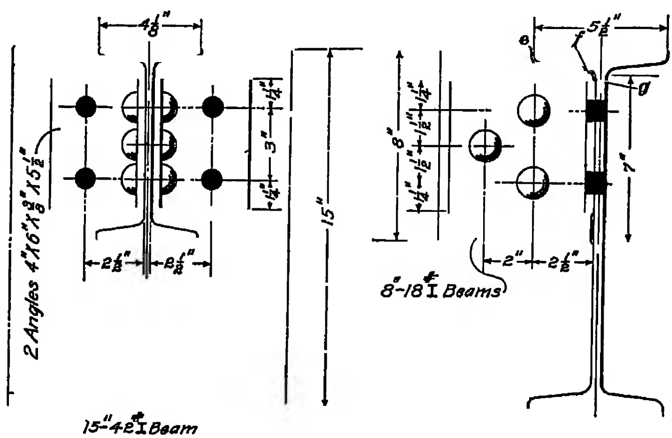


FIG 47

tion angles are extended outwards $\frac{1}{8}$ inch from the end of the beam, so that when the angles are bolted against the

supporting beam, they will fit against it square and not be kept away by the projecting end of the beam, which may be cut somewhat roughly.

Fig. 46 shows the connection of a 12-inch I beam to one 15 inches in depth. This type of joint is always used in the construction of steel-beam floor systems, and is the standard connection for the sizes shown. In Figs. 45 and 46, bolts are shown instead of rivets, but either may be used. The bolt holes shown in solid black in Fig. 46 indicate *field bolts* or *rivets*.

Fig. 47 shows the connection of an 8-inch beam to one 15 inches in depth. This connection differs somewhat from the construction shown in Fig. 46, in that the beams are flush on top. When a beam connection has the flanges of the two beams flush, the auxiliary beam must be cut as shown at *e*, *f*, *g*, so that its web will fit under the flange of the principal beam and the angle connection can be made rigid.

In the notes on the angles indicating their dimensions, as 2 Angles $4'' \times 4'' \times \frac{3}{8}'' \times 12''$, in Fig. 45, the last value $12''$ refers to the length of the angle. All the rivets or bolts in these views are $\frac{3}{4}$ inch in diameter.

NOTE.—When beams are intended for the support of heavy loads and for long spans, they are built up of plates and angles securely riveted together. By this means, the material may be distributed in the most advantageous manner, to obtain the requisite strength with a minimum of weight. Beams constructed in this manner are called *girders*. The long, straight, parallel upper and lower members are called *flanges*, and the vertical plate connecting them is the *web*. If the latter is replaced by braces, the horizontal members are termed *chords*. A chord usually consists of several angles and plates. Chords and braces are shown in Figs. 48 and 49.

53. Rivets in Single Shapes.—There is some difficulty in riveting single shapes, such as bars, angles, etc., to other parts so as to provide proper connections at the ends. Therefore, in order that the efficiency of the metal will be a maximum, the rivets should be arranged so as to make the net cross-sectional area of the bars, etc., as great as possible. The rivets should be placed symmetrically with respect to the axis of the piece, and its cross-section should be reduced as little as possible.

In Fig. 48 (*a*), the cross-section of the plate taken along or parallel with the line *a b* is reduced by only one or two rivet holes. Where the rivets are arranged as shown in Fig. 48 (*b*), a section at right angles to the axis of the member in most cases cuts three rivet holes; consequently, this design is not so efficient as the one shown in (*a*).

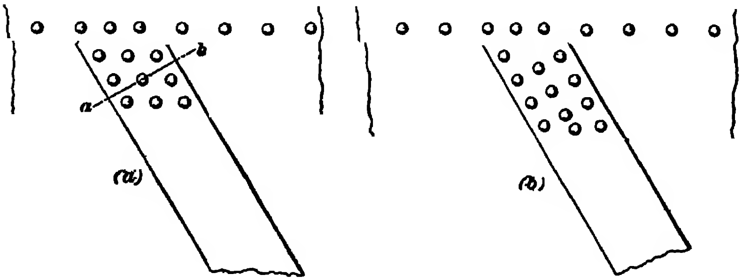


FIG 48

To give the greatest strength, angle irons should be attached through both legs, and the rivets should be arranged symmetrically, as in plates. Fig. 49 shows the detail as it is usually made. These joints are all supposed to be in tension.

54. Joints in Compression.—If it were possible to fit the single shapes in Fig. 48 so as to abut against a projecting angle of the horizontal member, a compressive stress could be transmitted entirely by the single bars, and the only use of the rivets would be to hold the parts together. But as a perfect fit cannot be insured, the entire stress is assumed to be transmitted by the rivets, and the calculations are the same as for rivets in tension joints. The only difference is that there is no deduction made from the cross-section for rivet holes, as the rivet completely fills the hole and transmits the stress.

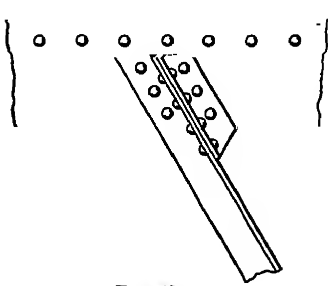


FIG. 49

Where several bars under compression are joined together to transmit the stress, it is necessary sometimes to have the

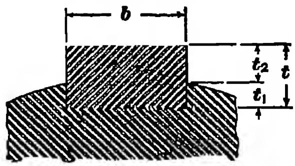
FIG. 50

longitudinal pitch less than in tension joints, so as to avoid the danger of buckling the plates between the rivets, as shown in Fig. 50.

TABLE I

TAPERED KEYS AND KEY SEATS FOR MARINE ENGINES

(Newport News Shipbuilding and Dry Dock Company)



Diameter of Shaft Inches d	Inch b	Inch t	Inch t_1	Inch t_2	Diameter of Shaft. Inches d	Inches b	Inches t	Inch t_1	Inch t_2
$\frac{1}{2}$	$\frac{7}{32}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	5	$1\frac{1}{16}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{3}{8}$
$\frac{5}{8}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$5\frac{1}{4}$	$1\frac{1}{8}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{3}{8}$
$\frac{3}{4}$	$\frac{9}{32}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$5\frac{1}{2}$	$1\frac{3}{16}$	$\frac{5}{8}$	$\frac{1}{4}$	$\frac{3}{8}$
$\frac{7}{8}$	$\frac{9}{32}$	$\frac{7}{32}$	$\frac{3}{32}$	$\frac{1}{8}$	$5\frac{3}{4}$	$1\frac{3}{16}$	$\frac{11}{16}$	$\frac{5}{16}$	$\frac{3}{8}$
1	$\frac{5}{16}$	$\frac{7}{32}$	$\frac{3}{32}$	$\frac{1}{8}$	6	$1\frac{1}{4}$	$\frac{11}{16}$	$\frac{5}{16}$	$\frac{3}{8}$
$1\frac{1}{8}$	$\frac{11}{32}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{5}{32}$	$6\frac{1}{4}$	$1\frac{5}{16}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{7}{16}$
$1\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{5}{32}$	$6\frac{1}{2}$	$1\frac{3}{8}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{7}{16}$
$1\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{5}{32}$	$6\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{4}$	$\frac{5}{16}$	$\frac{7}{16}$
$1\frac{1}{2}$	$\frac{13}{32}$	$\frac{1}{4}$	$\frac{3}{32}$	$\frac{5}{32}$	7	$1\frac{7}{16}$	$\frac{13}{16}$	$\frac{5}{16}$	$\frac{1}{2}$
$1\frac{3}{4}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$7\frac{1}{4}$	$1\frac{1}{2}$	$\frac{13}{16}$	$\frac{5}{16}$	$\frac{1}{2}$
2	$\frac{1}{2}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$7\frac{1}{2}$	$1\frac{9}{16}$	$\frac{7}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
$2\frac{1}{4}$	$\frac{9}{16}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$7\frac{3}{4}$	$1\frac{9}{16}$	$\frac{7}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
$2\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	8	$1\frac{5}{8}$	$\frac{7}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$8\frac{1}{4}$	$1\frac{5}{8}$	$\frac{15}{16}$	$\frac{3}{8}$	$\frac{9}{16}$
3	$\frac{11}{16}$	$\frac{7}{16}$	$\frac{3}{16}$	$\frac{1}{4}$	$8\frac{1}{2}$	$1\frac{3}{4}$	$\frac{15}{16}$	$\frac{3}{8}$	$\frac{9}{16}$
$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{7}{16}$	$\frac{3}{16}$	$\frac{1}{4}$	$8\frac{3}{4}$	$1\frac{3}{4}$	$\frac{15}{16}$	$\frac{3}{8}$	$\frac{9}{16}$
$3\frac{1}{2}$	$\frac{13}{16}$	$\frac{7}{16}$	$\frac{3}{16}$	$\frac{1}{4}$	9	$1\frac{3}{4}$	1	$\frac{7}{16}$	$\frac{9}{16}$
$3\frac{3}{4}$	$\frac{13}{16}$	$\frac{1}{2}$	$\frac{3}{16}$	$\frac{5}{16}$	$9\frac{1}{4}$	$1\frac{7}{8}$	1	$\frac{7}{16}$	$\frac{9}{16}$
4	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{3}{16}$	$\frac{5}{16}$	$9\frac{1}{2}$	$1\frac{7}{8}$	$1\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{8}$
$4\frac{1}{4}$	$\frac{15}{16}$	$\frac{9}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$9\frac{3}{4}$	2	$1\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{8}$
$4\frac{1}{2}$	1	$\frac{9}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	10	2	$1\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{8}$
$4\frac{3}{4}$	1	$\frac{9}{16}$	$\frac{1}{4}$	$\frac{5}{16}$					

NOTE.—Taper of key = $\frac{1}{8}$ inch per foot.

TABLE II

SQUARE KEYS AND KEY SEATS FOR MACHINE TOOLS

(*Wm. Sellers & Company*)

Diameter of Shaft Inches	Size of Key Inches		Size of Key Seat Inches	
	b	t	b	$\frac{t}{2}$
$\frac{1}{2}$ and under	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{32}$
$\frac{1}{2}$ to $\frac{15}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$
1 to $1\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{32}$
$1\frac{1}{4}$ to $1\frac{7}{16}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$
$1\frac{1}{2}$ to $1\frac{11}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{32}$
$1\frac{3}{4}$ to $2\frac{3}{16}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{7}{32}$
$2\frac{1}{4}$ to $2\frac{11}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{16}$	$\frac{9}{32}$
$2\frac{3}{4}$ to $3\frac{15}{16}$	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{11}{32}$
4 to $5\frac{7}{16}$	$\frac{13}{16}$	$\frac{13}{16}$	$\frac{13}{16}$	$\frac{13}{32}$
$5\frac{1}{2}$ to $6\frac{15}{16}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{15}{32}$
7 to $8\frac{15}{16}$	$1\frac{1}{16}$	$1\frac{1}{16}$	$1\frac{1}{16}$	$\frac{17}{32}$

NOTE —The dimensions of keys for shafting, as adopted by Wm Sellers & Company, may also be derived from this table, if the values for t are increased by $\frac{1}{16}$ inch. In this table, the key seats in the hub and the shaft are of equal depth, some manufacturers prefer to make the depth of the key seat in the shaft equal to $\frac{9}{16}t$. In computing, the length of the key seat is one and one-half times the nominal diameter of the shaft.

TABLE III

TAPER PINS

Number	Diameter at Large End Inch	Approximate Fractional Sizes Inch	Number	Diameter at Large End Inch	Approximate Fractional Sizes Inch
0	.156	$\frac{5}{32}$	6	.341	$\frac{11}{32}$
1	.172	$\frac{11}{64}$	7	.409	$\frac{13}{32}$
2	.193	$\frac{3}{16}$	8	.492	$\frac{1}{2}$
3	.219	$\frac{7}{32}$	9	.591	$\frac{19}{32}$
4	.250	$\frac{1}{4}$	10	.706	$\frac{23}{32}$
5	.289	$\frac{19}{64}$			

TABLE IV

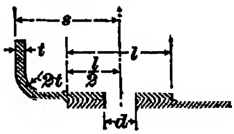
PITCH AND EFFICIENCY OF RIVETED JOINTS

Thickness of Plate Inch t	Diameter of Rivet Inch d'	Diameter of Hole Inch d	Pitch Inches p		Efficiency of Joint y	
			Single	Double	Single	Double
$\frac{1}{4}$	$\frac{5}{8}$	$\frac{11}{16}$	2	3	.66	.77
$\frac{5}{16}$	$\frac{11}{16}$	$\frac{3}{4}$	$2\frac{1}{16}$	$3\frac{1}{8}$	.64	.76
$\frac{3}{8}$	$\frac{3}{4}$	$\frac{13}{16}$	$2\frac{1}{8}$	$3\frac{1}{4}$	.62	.75
$\frac{7}{16}$	$\frac{13}{16}$	$\frac{7}{8}$	$2\frac{3}{16}$	$3\frac{3}{8}$	.60	.74
$\frac{1}{2}$	$\frac{7}{8}$	$\frac{15}{16}$	$2\frac{1}{4}$	$3\frac{1}{2}$	.58	.73

TABLE V

STANDARD SINGLE RIVETING FOR TANKS, STACKS, ETC.

(*Newport News Shipbuilding and Dry Dock Company*)



Thickness of Plate Inch t	Diameter of Rivet Inches d	Pitch Inches: Oil-Tight Joint	Pitch Inches: Water-Tight Joint	Pitch Inches: Non-Water-Tight Joint	Length of Lap Inches l	Distance of Rivet From Side of Tank Inches s
$\frac{1}{16}$	$\frac{1}{4}$	$\frac{13}{16}$	1	2	$\frac{3}{4}$	$\frac{11}{16}$
$\frac{1}{8}$	$\frac{3}{8}$	$1\frac{1}{8}$	$1\frac{1}{4}$	$2\frac{1}{2}$	$\frac{15}{16}$	1
$\frac{3}{16}$	$\frac{3}{8}$	$1\frac{1}{4}$	$1\frac{1}{2}$	3	$1\frac{1}{8}$	$1\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{3}{4}$	4	$1\frac{1}{2}$	$1\frac{5}{8}$
$\frac{5}{16}$	$\frac{5}{8}$	$1\frac{3}{4}$	2	4	$1\frac{1}{2}$	$1\frac{13}{16}$
$\frac{3}{8}$	$\frac{5}{8}$	2	$2\frac{1}{4}$	5	$1\frac{7}{8}$	$2\frac{3}{16}$
$\frac{7}{16}$	$\frac{3}{4}$	$2\frac{1}{8}$	$2\frac{1}{2}$	6	$2\frac{1}{4}$	$2\frac{9}{16}$
$\frac{1}{2}$	$\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{3}{4}$	6	$2\frac{1}{4}$	$2\frac{3}{4}$
$\frac{9}{16}$	$\frac{7}{8}$	$2\frac{1}{2}$	3	7	$2\frac{5}{8}$	3
$\frac{5}{8}$	$\frac{7}{8}$	$2\frac{5}{8}$	$3\frac{1}{8}$	7	$2\frac{5}{8}$	$3\frac{5}{16}$
$\frac{11}{16}$	$\frac{7}{8}$	$2\frac{3}{4}$	$3\frac{1}{4}$	7	$2\frac{5}{8}$	$3\frac{1}{2}$
$\frac{3}{4}$	1	3	$3\frac{1}{2}$	8	3	$3\frac{7}{8}$
$\frac{13}{16}$	1	$3\frac{1}{8}$	$3\frac{3}{4}$	8	3	$4\frac{1}{16}$
$\frac{7}{8}$	1	$3\frac{1}{4}$	$3\frac{7}{8}$	8	3	$4\frac{1}{4}$
$\frac{15}{16}$	$1\frac{1}{8}$	$3\frac{3}{8}$	$4\frac{1}{8}$	9	$3\frac{3}{8}$	$4\frac{5}{8}$
1	$1\frac{1}{8}$	$3\frac{5}{8}$	$4\frac{1}{4}$	9	$3\frac{3}{8}$	$4\frac{13}{16}$

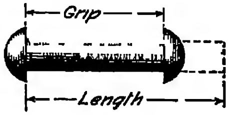
TABLE VI

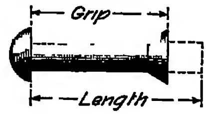
MAXIMUM SIZE OF RIVETS IN BEAMS, CHANNELS, AND ANGLES

I Beams						Channels			Angles			
Depth of Beam Inches	Weight per Foot Pounds	Size of Rivet Inch	Depth of Beam Inches	Weight per Foot Pounds	Size of Rivet Inches	Depth of Channel Inches	Weight per Foot Pounds	Size of Rivet Inch	Length of Leg Inches	Size of Rivet Inch	Length of Leg Inches	Size of Rivet Inches
3	5 50	$\frac{3}{8}$	15	42 00	$\frac{3}{4}$	3	4 00	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{4}$	$2\frac{1}{2}$	[illegible]
4	7 50	$\frac{1}{2}$	15	60 00	$\frac{3}{4}$	4	5 25	$\frac{1}{2}$	1	$\frac{3}{8}$	$2\frac{3}{4}$	$\frac{3}{4}$
5	9 75	$\frac{1}{2}$	15	80 00	$\frac{7}{8}$	5	6 50	$\frac{1}{2}$	$1\frac{1}{4}$	$\frac{1}{2}$	3	$\frac{7}{8}$
6	12 25	$\frac{5}{8}$	18	55 00	$\frac{7}{8}$	6	8 00	$\frac{5}{8}$	$1\frac{5}{16}$	$\frac{1}{2}$	$3\frac{1}{2}$	1
7	15 00	$\frac{5}{8}$	20	65 00	1	7	9 75	$\frac{5}{8}$	$1\frac{3}{8}$	$\frac{1}{2}$	4	1
8	17 75	$\frac{3}{4}$	20	80 00	1	8	11 25	$\frac{3}{4}$	$1\frac{1}{2}$	$\frac{1}{2}$	$4\frac{1}{2}$	1
9	21 00	$\frac{3}{4}$	24	80 00	1	9	13 25	$\frac{3}{4}$	$1\frac{3}{4}$	$\frac{5}{8}$	5	1
10	25 00	$\frac{3}{4}$				10	15 00	$\frac{3}{4}$	2	$\frac{5}{8}$	6	1
12	31 50	$\frac{3}{4}$				12	20 50	$\frac{3}{4}$	$2\frac{1}{4}$	$\frac{3}{4}$	7	1
12	40 00	$\frac{3}{4}$				15	33 00	$\frac{3}{4}$	$2\frac{5}{16}$	$\frac{3}{4}$		

TABLE VII

LENGTH OF RIVET SHANK REQUIRED FOR UPSETTING AND TO FORM ONE RIVET HEAD





Grip of Rivet Inches	Diameter of Rivet Inches 1/4	3/8	1/2	5/8	3/4	7/8	1	1 1/8
1/2	1	1 1/4	1 1/2	1 3/4	1 7/8	2	2 1/8	2 1/4
5/8	1 1/8	1 3/8	1 5/8	1 7/8	2	2 1/8	2 1/4	2 3/8
3/4	1 1/4	1 1/2	1 3/4	2	2 1/8	2 1/4	2 3/8	2 1/2
7/8	1 3/8	1 5/8	1 7/8	2 1/8	2 1/4	2 3/8	2 1/2	2 5/8
1	1 1/2	1 3/4	2	2 1/4	2 3/8	2 1/2	2 5/8	2 3/4
1 1/8	1 5/8	1 7/8	2 1/8	2 3/8	2 1/2	2 5/8	2 3/4	2 7/8
1 1/4	1 3/4	2	2 1/4	2 1/2	2 5/8	2 3/4	2 7/8	3
1 3/8	1 7/8	2 1/8	2 3/8	2 5/8	2 7/8	3	3	3 1/8
1 1/2	2	2 1/4	2 1/2	2 3/4	3	3 1/8	3 1/8	3 1/4
1 5/8	2 1/8	2 3/8	2 5/8	2 7/8	3 1/8	3 1/4	3 1/4	3 1/2
1 3/4	2 1/4	2 1/2	2 3/4	3	3 1/4	3 3/8	3 1/2	3 5/8
1 7/8	2 3/8	2 5/8	2 7/8	3 1/4	3 3/8	3 1/2	3 5/8	3 3/4
2	2 1/2	2 3/4	3 1/8	3 3/8	3 1/2	3 5/8	3 3/4	3 7/8
2 1/8	2 5/8	2 7/8	3 1/4	3 1/2	3 5/8	3 3/4	3 7/8	4
2 1/4	2 3/4	3	3 3/8	3 5/8	3 3/4	3 7/8	4	4 1/8
2 3/8	2 7/8	3 1/8	3 1/2	3 3/4	3 7/8	4	4 1/8	4 1/4
2 1/2	3	3 1/4	3 5/8	3 7/8	4	4 1/8	4 1/4	4 3/8
2 5/8	3 1/8	3 1/2	3 3/4	4	4 1/8	4 1/4	4 3/8	4 1/2
2 3/4	3 1/4	3 5/8	3 7/8	4 1/8	4 1/4	4 3/8	4 1/2	4 5/8
2 7/8	3 3/8	3 3/4	4	4 1/4	4 3/8	4 1/2	4 5/8	4 3/4
3	3 1/2	3 7/8	4 1/8	4 3/8	4 1/2	4 5/8	4 3/4	4 7/8
3 1/8	3 5/8	4	4 1/4	4 1/2	4 3/4	4 3/4	5	5
3 1/4	3 3/4	4 1/8	4 3/8	4 3/4	4 7/8	5	5 1/8	5 1/4

Grip of Rivet Inches	Diameter of Rivet Inches 1/4	3/8	1/2	5/8	3/4	7/8	1	1 1/8
3 3/8	3 7/8	4 1/4	4 1/2	4 7/8	5	5 1/8	5 1/4	5 3/8
3 1/2	4	4 3/8	4 5/8	5	5 1/8	5 1/4	5 3/8	5 1/2
3 5/8	4 1/8	4 1/2	4 3/4	5 1/8	5 1/4	5 3/8	5 1/2	5 5/8
3 3/4	4 1/4	4 5/8	4 7/8	5 1/4	5 3/8	5 1/2	5 5/8	5 3/4
3 7/8	4 3/8	4 3/4	5	5 3/8	5 1/2	5 5/8	5 3/4	5 7/8
4	4 1/2	4 7/8	5 1/8	5 1/2	5 5/8	5 3/4	5 7/8	6
4 1/8	4 5/8	5	5 1/4	5 5/8	5 3/4	5 7/8	6	6 1/8
4 1/4	4 3/4	5 1/8	5 1/2	5 3/4	5 7/8	6	6 1/8	6 1/4
4 3/8	4 7/8	5 1/4	5 5/8	5 7/8	6	6 1/8	6 1/4	6 3/8
4 1/2	5	5 3/8	5 3/4	6	6 1/8	6 1/4	6 3/8	6 1/2
4 5/8	5 1/8	5 1/2	5 7/8	6 1/8	6 1/4	6 3/8	6 1/2	6 5/8
4 3/4	5 1/4	5 5/8	6	6 1/4	6 1/2	6 5/8	6 3/4	6 3/4
4 7/8	5 3/8	5 3/4	6 1/8	6 1/2	6 5/8	6 3/4	6 7/8	6 7/8
5	5 1/2	5 7/8	6 1/4	6 5/8	6 3/4	6 7/8	7	7
5 1/8	5 5/8	6	6 3/8	6 3/4	6 7/8	7	7 1/8	7 1/8
5 1/4	5 3/4	6 1/8	6 1/2	6 7/8	7	7 1/8	7 1/4	7 1/4
5 3/8	5 7/8	6 1/4	6 5/8	7	7 1/8	7 1/4	7 3/8	7 3/8
5 1/2	6	6 3/8	6 3/4	7 1/8	7 1/4	7 3/8	7 1/2	7 1/2
5 5/8	6 1/8	6 1/2	6 7/8	7 1/4	7 3/8	7 1/2	7 5/8	7 5/8
5 3/4	6 1/4	6 3/4	7	7 3/8	7 5/8	7 5/8	7 3/4	7 3/4
5 7/8	6 3/8	6 7/8	7 1/8	7 1/2	7 3/4	7 3/4	7 7/8	7 7/8
6	6 1/2	7	7 1/4	7 5/8	7 7/8	7 7/8	8	8 1/8

Amount, in Inches, to Be Subtracted From Above Lengths for Countersunk Heads

1/8	1/4	1/2	1/2	5/8	3/4	7/8	7/8	1/8	1/4	1/2	1/2	5/8	3/4	7/8	7/8

TABLE VIII

RIVET SPACING

Size of Rivet Inch	Minimum Pitch Inches	Maximum Pitch at Ends of Compression Members Inches	Maximum Pitch in Flanges of Chords and Girders Inches	Distance From End of Piece to Center of Rivet Inches	
				Minimum	Usual
$\frac{1}{4}$	$\frac{3}{4}$				
$\frac{3}{8}$	$1\frac{1}{8}$				
$\frac{1}{2}$	$1\frac{1}{2}$				
$\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{1}{2}$	4	$\frac{15}{16}$	$1\frac{1}{4}$
$\frac{3}{4}$	$2\frac{1}{4}$	3	4	$1\frac{1}{8}$	$1\frac{1}{2}$
$\frac{7}{8}$	$2\frac{5}{8}$	$3\frac{1}{2}$	4	$1\frac{5}{16}$	$1\frac{3}{4}$
1	3	4	4	$1\frac{1}{2}$	2

MACHINE DESIGN

Serial 997C (PART 3) Edition 1

DESIGN OF JOURNALS AND BEARINGS

JOURNALS

END JOURNALS

1. Journals are the cylindrical portions of rotating pieces that turn within bearings and form the supports. Journals situated at or near the ends of a shaft, axle, or other rotating piece are termed *end journals*. Any journal situated between two end journals is called a *neck journal*.

The ordinary form of an **end journal** is shown in Fig. 1, and consists simply of a turned down part of the shaft, shoulders

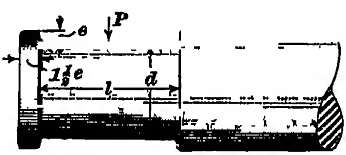


FIG. 1

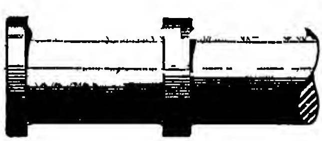


FIG. 2

being produced in this manner at each end of the journal, to prevent end play in the bearing. Sometimes, the same result is obtained without reducing the diameter of the shaft by forging collars at each end of the journal, as shown in Fig. 2.

The length of the box, or brass, on which the journal rests, however, is often made slightly shorter than the journal, permitting a slight motion lengthwise, and thereby securing uniform wear.

SIZE AND PROPORTIONS OF END JOURNALS

2. The chief element in the design of a journal moving slowly or intermittently is strength. When journals run constantly at considerable velocity, strength is not so important a consideration as durability and freedom from liability to heat.

The dimensions of an end journal to give sufficient strength may be calculated by considering the journal as a cantilever uniformly loaded.

Let l = length of journal, in inches;
d = diameter of journal, in inches;
P = total load on journals, in pounds;
S_4 = *safe* stress of material in flexure.

Then, from *Strength of Materials*, the bending moment is $\frac{w\,l^2}{2} = \frac{P\,l}{2}$, and the resisting moment is $\frac{S_4 I}{c}$, which for a circular section is

$$S_4 \frac{\frac{\pi d^4}{64}}{\frac{1}{2} d} = S_4 \frac{\pi d^3}{32}$$

Hence,
$$\frac{P\,l}{2} = S_4 \frac{\pi d^3}{32} \qquad (1)$$

or,
$$d = \sqrt{\frac{16\,P}{\pi\,S_4} \times \frac{l}{d}} = 2.26\sqrt{\frac{P}{S_4} \times \frac{l}{d}} \qquad (2)$$

Formula **2** gives the diameter of the journal when the ratio $\frac{l}{d}$ is assumed.

EXAMPLE.—Find the diameter and length of a wrought-iron journal on which there is a load of 1,200 pounds. Assume S_4 = 8,500 pounds and $\frac{l}{d} = 1.4$.

SOLUTION.—Using formula **2**,

$$d = 2\,26\sqrt{\frac{1{,}200}{8{,}500}} \times 1.4 = 1 \text{ in , nearly. Ans.}$$
$$l = 1\,4\,d = 1.4, \text{ say } 1\tfrac{7}{16}, \text{ in. Ans.}$$

3. **The bearing surface, or projected area,** of a journal is the length multiplied by the diameter; that is, it is the area of the projection of the journal on a plane

perpendicular to the direction of the load P. The total load on the journal divided by the projected area gives the pressure per square inch of projected area—a quantity that will be denoted by p.

Hence, $$p = \frac{P}{l\,d}, \text{ or } P = p\,l\,d \qquad (1)$$

In order that the journal may not heat, the pressure p must not exceed a certain limit determined by experience. When this pressure is too great, the oil used to lubricate the journal is squeezed out, and the journal heats rapidly.

From formula **1**, Art. **2**,

$$P = \frac{\pi S_4 d^3}{16\,l}$$

From formula **1**, $$P = p\,l\,d$$

Hence, $$\frac{\pi S_4 d^3}{16\,l} = p\,l\,d, \text{ or } \pi S_4 d^2 = 16 p\,l^2$$

and $$\frac{l}{d} = \sqrt{\frac{\pi S_4}{16 p}} \qquad (2)$$

Substituting this value of $\frac{l}{d}$ in formula **2**, Art. **2**, and reducing,

$$d = \sqrt{\frac{4P}{\sqrt{\pi S_4 p}}} = 1.5\sqrt{\frac{P}{\sqrt{S_4 p}}} \qquad (3)$$

From formula **1**,

$$l = \frac{P}{dp} \qquad (4)$$

Formula **3** may be used to compute the diameter of a journal when the pressure p per square inch of projected area is fixed. The length may then be obtained from formula **4**.

EXAMPLE—Compute the length and diameter of a steel journal sustaining a load of 12,000 pounds The safe stress S_4 is 14,000 pounds, and the pressure per square inch of projected area is not to exceed 750 pounds.

SOLUTION.—Using formula **3**,

$$d = 1.5\sqrt{\frac{12{,}000}{\sqrt{14{,}000 \times 750}}} = 2.89, \text{ say } 2\tfrac{7}{8}, \text{ in. Ans.}$$

Hence, $$l = \frac{P}{dp} = \frac{12{,}000}{2\frac{7}{8} \times 750} = 5\tfrac{9}{16} \text{ in. Ans.}$$

TABLE I

PRESSURE ON BEARINGS AND SLIDES

Kind of Journal Bearing	Pressure p per Square Inch of Projected Area Pounds
Bearings subject to intermittent load at slow speeds, such as crankpins of shearing machines	3,000
Crosshead neck journals; motion oscillating	800 to 1,400
Crosshead neck journals in locomotives	1,500 to 2,000
Crankpins (slow engines)	800 to 900
Crankpins (fast engines)	500 to 800
Crankpins (marine engines)	400 to 500
Crankpins (locomotives)	1,200 to 1,800
Crankpins (small land engines)	150 to 200
Main crank-shaft bearings (slow)	300 to 450
Main crank-shaft bearings (fast)	200 to 300
Locomotive driving-axle journal	180 to 350
Flywheel-shaft bearings	150 to 250
Eccentric straps	80 to 100
Railway-car journals	250 to 300
Locomotive and tender axle bearings	200 to 250
Line shafting on cast-iron steps (Sellers)	50
Pivots	700
Collar-thrust bearings (slow)	70 to 80
Collar-thrust bearings (fast)	50 to 70
Slides, cast iron on Babbitt metal	200 to 300
Slides, cast iron on cast iron (slow)	60 to 100
Slides, cast iron on cast iron (fast)	40
Steel or iron shaft on lignum vitæ; water lubrication	350

4. The pressure p per square inch of projected area may be taken at from 400 to 800 pounds when the journal runs constantly at a speed under 150 revolutions per minute. For journals that run slowly or intermittently, p may be much greater, while for journals that run faster than 150 revolutions per minute, the pressure p should vary inversely as the number of revolutions. That is, letting N = number of revolutions per minute, $p = \frac{a}{N}$, where a is a constant.

Another consideration affecting the allowable pressure p is the direction of the load. In some journals the load acts only in one direction, generally downwards; in others, as, for example, crankpins and crosshead pins, the direction of the load changes at every revolution. In the latter case, the pressure p may be twice as great as where the load acts in one direction only, because the change in the direction of the load permits a more perfect lubrication of the bearing. When, however, the direction of the load is variable, the safe stress S_4 must be taken smaller than when the direction is constant.

5. The values of p for different kinds of journals, according to Unwin, are given in Table I.

For journals not given in Table I, the value of p must be determined by the judgment of the designer. The value adopted should seldom exceed 750 pounds. For journals running faster than about 150 revolutions per minute, p

TABLE II

SAFE WORKING STRESSES ON END JOURNALS

Direction of Load Constant		Direction of Load Variable	
Material	Safe Working Stress S_4	Material	Safe Working Stress S_4
Steel	14,000	Steel	12,000
Wrought iron	8,500	Wrought iron	7,000
Cast iron	4,000	Cast iron	3,000

should vary inversely as the number of revolutions. For example, if a given journal is allowed a bearing pressure of 300 pounds at 150 revolutions, it should be allowed a bearing pressure of only $\frac{300 \times 150}{250} = 180$ pounds if it is required to run at 250 revolutions.

6. The permissible working stress S_4 may be taken as given in Table II.

NECK JOURNALS

7. A **neck journal** similar to an engine crosshead pin may be considered as a beam supported at both ends and uniformly loaded. Consequently, from *Strength of Materials*, the bending moment is $\frac{w\,l^2}{8} = \frac{P\,l}{8}$.

Then,

$$\frac{P\,l}{8} = \frac{S_4 \pi d^3}{32} \text{ (see formula 1, Art. 2)}$$

and

$$d = \sqrt{\frac{4\,P}{\pi\,S_4} \times \frac{l}{d}} = 1.13\sqrt{\frac{P}{S_4} \times \frac{l}{d}} \qquad (1)$$

Formula **1** may be used to calculate a neck journal when $\frac{l}{d}$ is given or assumed.

But, $P = p\,d\,l$ (see formula **1**, Art. **3**)

Hence,

$$\frac{P\,l}{8} = \frac{p\,d\,l^2}{8} = \frac{S_4 \pi d^3}{32}, \text{ or } \frac{l}{d} = \sqrt{\frac{\pi\,S_4}{4\,p}} = .886\sqrt{\frac{S_4}{p}} \qquad (2)$$

Substituting this value of $\frac{l}{d}$ in formula **1**,

$$d = 1.06\sqrt{\frac{P}{\sqrt{p\,S_4}}} \qquad (3)$$

$$l = \frac{P}{p\,d} \qquad (4)$$

From $P = p\,d\,l$,

The same values for S_4 and p may be taken as for end journals.

By comparing formula **3** with formula **3**, Art. **3**, it will be seen that for the same load the neck journal need be only

about two-thirds the diameter of the end journal. A comparison of formula 2 with formula 2, Art. 3, shows that the ratio $\frac{l}{d}$ in a neck journal is double what it would be in an end journal.

NOTE —In using formula 3, Art 3, and formula 3, Art. 7, it will be found most convenient to use logarithms. Thus, applying logarithms to formula 3, Art 3, it becomes

$$\log d = \log 1.5 + \tfrac{1}{2}[\log P - \tfrac{1}{2}(\log p + \log S_4)]$$

EXAMPLE —Find the length and diameter of a wrought-iron neck journal, the load being 9,600 pounds and variable in direction. Allow a bearing pressure of 600 pounds per square inch.

SOLUTION —Using formula 3,

$$d = 1.06\sqrt{\frac{9{,}600}{\sqrt{600 \times 7{,}000}}}$$

or,

$$\log d = \log 1.06 + \tfrac{1}{2}[\log P - \tfrac{1}{2}(\log p + \log S_4)]$$
$$= .02531 + \tfrac{1}{2}[3.98227 - \tfrac{1}{2}(2.77815 + 3.84510)] = .36063$$

Whence $d = 2.294 = 2\frac{19}{64}$, or $2\frac{5}{16}$, in., nearly. Ans.

$$l = \frac{P}{p\,d} = \frac{9{,}600}{600 \times 2\frac{5}{16}} = 7 \text{ in., nearly. Ans.}$$

8. Sometimes, simply as a matter of taste, it may be desirable to make the length of a journal greater than the calculated value. To give the journal the same strength, its diameter must be increased in the proportion given by the following formula:

$$\frac{l_1}{l} = \left(\frac{d_1}{d}\right)^3$$

where l and d are the original and l_1 and d_1 the new lengths and diameters, respectively, of the journal.

For example, suppose that a wrought-iron end journal is subjected to a load of 350 pounds. Assuming $\frac{l}{d} = 1.4$, the dimensions of the journal will be

$$d = 2.26\sqrt{\frac{P}{S_4} \times \frac{l}{d}} = 2.26\sqrt{\frac{350}{8{,}500}} \times 1.4 = .5426 \text{ inch;}$$
$$l = 1.4\,d = 1.4 \times .5426 = .7596 \text{ inch}$$

Hence, call the diameter $\frac{9}{16}$ inch and the length $\frac{3}{4}$ inch.

Though a journal of these dimensions would be sufficiently stiff and durable for the load, it is rather small to

look well. Suppose the length is made double the calculated value; that is, $l_1 = 2\,l$. Then, from the formula just given,

$$d_1 = d\sqrt[3]{\frac{l_1}{l}} = .5426\sqrt[3]{2} = .6837, \text{ or } \tfrac{11}{16}, \text{ inch nearly;}$$

$$l_1 = 2\,l = 2 \times \tfrac{3}{4} = 1\tfrac{1}{2} \text{ inches}$$

FRICTION OF JOURNALS

9. When relative motion takes place between two surfaces in contact with each other, and subjected to more or less heavy pressure, as in a bearing and a journal, a certain amount of energy is expended in overcoming the frictional resistance to motion. This energy is continually transformed into heat; the temperature of the journal and bearing will therefore increase until a point is reached where the heat is dissipated as rapidly as it is produced. Thus, in designing a journal, the question is not alone that of giving it sufficient strength, but also that of so proportioning it that it may freely radiate the heat developed.

10. The work expended in overcoming the friction of a journal is

$$W = .26\,f\,P\,N\,d \text{ foot-pounds per minute,}$$

where f is the coefficient of friction, and N the revolutions per minute.

It is therefore apparent that with the same load and speed, the work expended against the resistance of friction is directly proportional to the diameter of the journal.

In order to illustrate how the dimensions of a journal affect its heat-radiating qualities, a comparison will be made between two journals of equal lengths, but having different diameters, one being 2 and the other 4 inches. The load P is the same for both journals, but the unit pressure p on the larger journal will necessarily be one-half that on the smaller. The frictional resistance of both journals will be the same, as the total pressures P on the journals are equal; but the amount of work lost in friction and converted into heat will not be the same. If both journals make the same number

of revolutions per minute, the circumferential speed of one is twice that of the other, and as the amount of work performed is the product of the resistance and the distance through which the resistance is overcome, it follows that the energy lost in revolving the larger journal is twice that expended on the smaller one. The temperature attained by both journals will nevertheless be the same, because the 4-inch journal has a radiating surface twice as great as that of the 2-inch one. As should be clearly seen, not only is no advantage derived from an increase in diameter, but, on the contrary, by reason of the enlarged surface, a greater amount of energy is lost in overcoming the frictional resistance. If, on the other hand, the length of the 2-inch journal were doubled, the peripheral speed would remain the same, and likewise the energy lost in friction, but the radiating surface would be doubled.

As another example, take two journals, one 2 inches in diameter and 6 inches long, and the other 4 inches in diameter and 3 inches long. They each have the same projected area; that is, 12 inches. The latter journal, however, requires double the work to overcome friction that the former requires, and, besides, contains twice as much material. Hence, the $2'' \times 6''$ journal is preferable for a steady load. However, the $4'' \times 3''$ journal is $\frac{4^3 \times 6}{2^3 \times 3} = 16$ times as strong as the other, and would be preferred in situations where the load is variable; also, the journal is liable to shocks, as in the case of crankpins of high-speed engines.

11. It must not be inferred from the preceding that an increase in the length of a journal will always overcome its heating tendency, and that in all cases it would be useless to increase the diameter of the journal. A journal running slowly under heavy pressure might heat less by increasing its diameter and thereby reducing the unit bearing pressure. On the other hand, the advantage derived from increasing the length of a journal may be lost by a tendency of the journal to bend, whereby an excess of pressure is produced

at one place, followed by a rise in temperature. By reason of the various points to be taken into consideration, it is difficult to give uniform rules regarding the relation between length and diameter of a journal. In general, the diameter is made as small as the requirements for strength will allow, while the length is made sufficient to keep the bearing pressure within the limits found by experience to be safe.

12. The height of the journal collars, Fig. 1, may be

$$e = \frac{d}{10} + \tfrac{1}{8} \text{ inch}$$

The width of the outer collar is $1\frac{1}{2}\,e$. It is good practice to turn the journal with a fillet in the corner, as the shaft or axle is more liable to crack and fracture if the shoulder is turned with a square corner. The fillet may be outlined by a circular arc drawn with a radius equal to $\frac{1}{2}e$.

PIVOTS AND COLLAR JOURNALS

13. Pivot Journals.—A **pivot journal** is shown in Fig. 3. This type differs from an ordinary journal in that the direction of pressure is parallel with the axis of the shaft instead of perpendicular to it. Thus, the bearing area is the area of the end of the pivot; that is, $.7854\,d^2$.

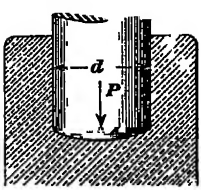


FIG. 3

The diameter of a flat pivot may therefore be found at once by assuming a value for the pressure per square inch of projected area and solving for the area. The formulas in Table III give good results for ordinary cases:

Let P = load on pivot;
d = diameter of pivot;
N = revolutions per minute of pivot.

EXAMPLE.—What should be the diameter of a steel pivot on gun-metal bearings, the load being 200 pounds, and the number of revolutions 320 per minute?

SOLUTION.—

$$d = .004\sqrt{PN} = .004\sqrt{200 \times 320} = 1 \text{ in., nearly} \quad \text{Ans.}$$

The difficulty met with in pivot journals is the unequal wear of the rubbing surfaces. This is caused by the difference in velocity between parts near the axis of rotation and other parts near the periphery of the journal, the velocity of any particular part being proportional to its distance from the axis of rotation. As the resulting wear of the rubbing surfaces is directly proportional to their respective velocities, it follows that the parts farthest away from the axis of rotation will wear away more quickly. The result is that the

TABLE III

FORMULAS FOR PIVOT JOURNALS

Speed	Wrought-Iron or Steel Pivot on Gun-Metal Bearing	Cast-Iron Pivot on Gun-Metal Bearing	Iron or Steel on Lignum Vitæ Bearing, Moistened With Water	
CASE I—Pivot turning very slowly or intermittently.	$d = 035\sqrt{P}$	$d = 05\sqrt{P}$	. . . . .	(1)
CASE II—Revolutions per minute less than 150 . .	$d = 05\sqrt{P}$	$d = .07\sqrt{P}$	$d = .035\sqrt{P}$	(2)
CASE III—Revolutions per minute more than 150 . .	$d = 004\sqrt{PN}$	. . . . .	$d = 035\sqrt{P}$	(3)

central part of the journal will be gradually exposed to an excess of pressure, followed by abrasion and, possibly, crushing.

14. Collar Journals.—To reduce the extreme differences in velocity between the portions of the bearing surface nearest to and farthest from the centers, it is customary to make use of only the outer part of the rubbing surfaces and to make them ring-shaped. This is the theory on which **collar journals** are based, the aim being to make the collar shallow, thereby avoiding great differences in velocity. In journals exposed to very heavy pressure, the bearing surface is made great enough by increasing the number of collars.

A journal with one collar is shown in Fig. 4, and one with several collars in Fig. 5.

Let d_1 = diameter of collars;

d = diameter of shaft;

b = number of collars;

p = pressure per square inch of projected area;

P = total load, or thrust.

Then,
$$.7854\,(d_1^2 - d^2)\,b p = P$$

The value of p is usually taken at 60 pounds per square

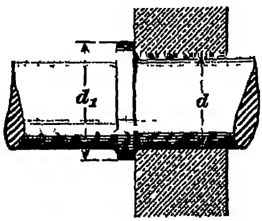


FIG. 4

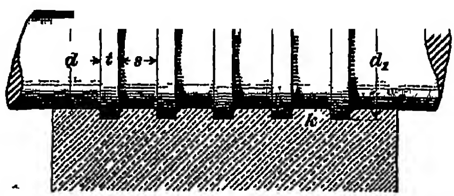


FIG 5

inch for the thrust bearings of propeller shafts, and this value should not be much exceeded in any case.

Then,
$$d_1^2 - d^2 = \frac{P}{47\,b}$$

or,
$$d_1 = \sqrt{d^2 + \frac{P}{47\,b}}$$

The number of collars b depends on the judgment of the designer. The larger the number, the smaller is the diameter, and the less will be the wear and work of friction. However, when many collars are used, there is danger of bringing all the thrust on one or two.

In Fig. 5, the dimension $e = \frac{1}{2}(d_1 - d)$. Usually the thickness t of the collars is $.8\,e$, and the width s of the space is equal to the thickness t, unless the annular encircling rings k are hollow for water circulation or unless they are lined with white metal. In the latter case, $s = 2$ to $2\frac{1}{2}\,t$.

EXAMPLES FOR PRACTICE

1. Find the proportions of a cast-iron end journal that turns slowly under a steady load of 15,000 pounds, assuming the length equal to the diameter. Ans. $4\frac{3}{8}$ in. $\times$ $4\frac{3}{8}$ in.

2. Find the proportions of a wrought-iron end journal that turns under a load of 8,600 pounds, assuming the safe pressure per square inch of projected area to be 650 pounds. The direction of the load is variable. Ans 3 in $\times$ $4\frac{7}{16}$ in.

3. Find the dimensions of a steel neck journal, working under a load of 12,000 pounds, direction variable, the allowable bearing pressure to be 1,200 pounds per square inch. Ans $1\frac{7}{8}$ in $\times$ $5\frac{5}{16}$ in.

4. Find the minimum dimensions of a wrought-iron end journal, the required conditions being that the bearing pressure shall not exceed 750 pounds and that the length shall be one and one-fourth times the diameter The load on the journal is 9,600 pounds, and variable in direction. Ans $3\frac{1}{4}$ in $\times$ 4 in.

NOTE —By using formula 3, Art 3, for solving example 4, the diameter is found to be $3\frac{1}{16}$ inches, nearly By substituting the value $\frac{l}{1\ 25}$ for d in formula 4, Art 3, l is found to be equal to 4 inches, but as the quotient $\frac{P}{d\,l}$ in this instance is greater than 750, which is the value of p, the values of d or l must be adjusted to give the desired pressure p Making $l = 4$ inches, the required value of d may be found by transforming formula 4, Art 3, into the form $d = \frac{P}{l\,p}$, when $d = 3\ 2$ inches, or $3\frac{1}{4}$ inches, nearly Ans

5. Find the diameter of a wrought-iron pivot running in a gun-metal bearing at a speed of 80 revolutions per minute and bearing a load of 800 pounds. Ans. $1\frac{7}{16}$ in.

6. Assuming that five thrust collars on a shaft 10 inches in diameter are subjected to an end thrust of 20,000 pounds, find the diameter and thickness of collars. Ans $\begin{cases} d_1 = 13.6 \text{ in.} \\ t = 1\frac{7}{16} \text{ in.} \end{cases}$

15. Conical Pivot Journals.—When the bearing surface of a pivot journal is conical in shape, as in Fig. 6, it is termed a **conical pivot journal.** Examples of these bearings are found on lathes and similar machines where the end thrust of the journal is not excessive.

The angle u, Fig. 6, of the pivot point does not affect the unit pressure p; this pressure is the same as that of a journal having a flat end whose area is equal to the projection of the conical surface on a plane perpendicular to the axis of

the journal. This can be proved in the following manner: In Fig. 6, the load P may be resolved into the two components S, S acting at right angles to the bearing surface, it being supposed that the load is acting in these two directions only. Then,

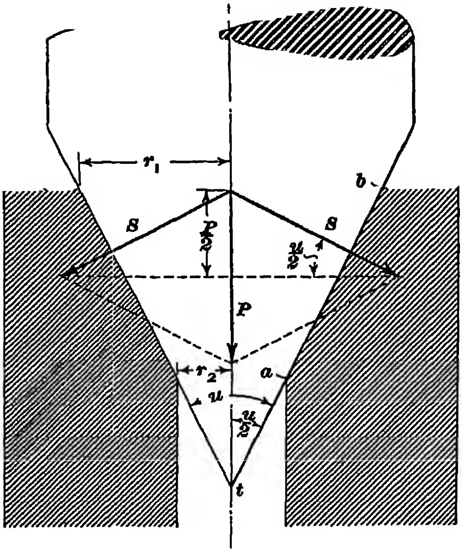


FIG. 6

$$S = \frac{\frac{P}{2}}{\sin\frac{u}{2}}$$

and

$$2S = \frac{P}{\sin\frac{u}{2}} \qquad (1)$$

If A is equal to the surface area of the conical point, then

$$p = \frac{2S}{A}$$

As the active part of the point is the frustum of a cone, the area

$$A = \tfrac{1}{2}(2\pi r_1 + 2\pi r_2)(bt - at)$$

Expressing the lengths bt and at in terms of r_1, r_2, and $\frac{u}{2}$, the formula will be

$$A = \tfrac{1}{2}(2\pi r_1 + 2\pi r_2)\left(\frac{r_1}{\sin\frac{u}{2}} - \frac{r_2}{\sin\frac{u}{2}}\right)$$

or,

$$A = \frac{\pi}{\sin\frac{u}{2}}(r_1^2 - r_2^2) \qquad (2)$$

Inserting this value of A in the formula $p = \frac{2S}{A}$,

$$p = \frac{2S}{\frac{\pi}{\sin\frac{u}{2}}(r_1^2 - r_2^2)}$$

According to formula **1**,

$$2S = \frac{P}{\sin \frac{u}{2}}$$

Therefore, $$p = \frac{P}{\pi (r_1^2 - r_2^2)}$$

It is seen that the angle u is eliminated from this formula and that it therefore does not affect the unit pressure. The value $\pi (r_1^2 - r_2^2)$ represents the annular surface produced by projecting the conical contact area on a plan perpendicular to the pivot axis.

BEARINGS WITH SLIDING FRICTION

16. Solid Journal Bearings.—The simplest form of bearing for a journal is merely a hole in the frame that supports the rotating piece. Such a bearing is shown in Fig. 7.

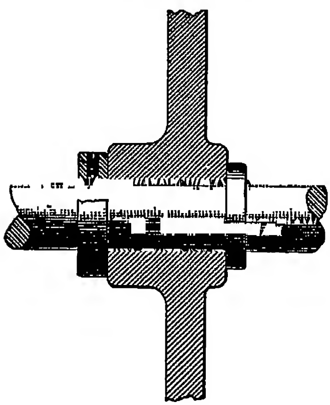


FIG 7

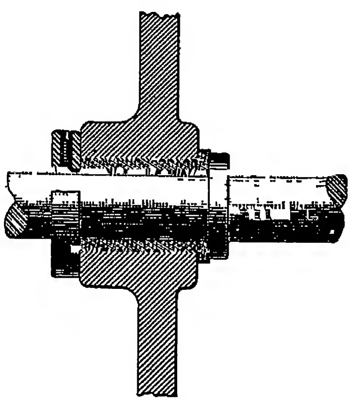


FIG 8

Motion endwise is prevented by two collars, one of which may be forged on the shaft, and the other made separately and held in place by a setscrew or pin. A boss is cast on the frame, as shown in the figure, in order to give the journal the necessary length and bearing area.

Such a bearing has no means of adjustment to take up the wear; for this reason it is better to use the form of solid

bearing shown in Fig. 8. In this bearing the hole is bored out larger than the journal and is lined with a bushing made of brass or other metal. The wear thus comes on the bushing, which can easily be replaced.

17. Divided Bearings.—In many cases, solid bearings are undesirable, and in others it will be impossible to use them. The bearing is then divided, and the parts held together by bolts; when this is done, the parts of the bushing are called *boxes*, or *brasses*. The division of the bearing permits its adjustment for wear.

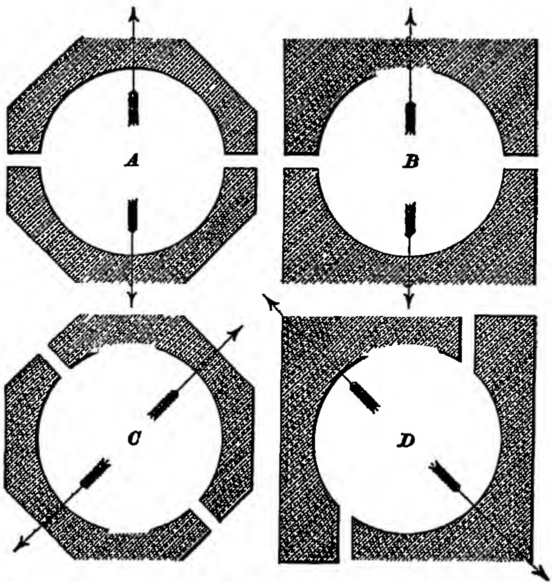


FIG 9

The load on a bearing usually acts constantly in one direction, and the bearing should be divided so that the line of division is perpendicular, or nearly so, to the direction of the load. Fig. 9 shows various methods of dividing the boxes. At *A* and *B* the direction of the load is vertical; consequently, the boxes are divided horizontally.

Naturally, the wear will come on the top and bottom of the boxes, and the hole will become oval, with the long

diameter vertical. The boxes are then screwed closer together until the hole regains its original circular form. The necessity of making the division perpendicular to the direction of the load is thus apparent. At C the direction of the load is oblique, and the adjustment perpendicular to it; at D the direction of the load is oblique also, but the boxes are divided to allow a horizontal adjustment.

In Fig. 10 is shown a simple form of divided bearing; it is made of cast iron, with no separate boxes, but with means of adjustment. The bearing forms part of the frame of the machine. This type of bearing is used only in cheap work. Sometimes, however, the bearing has a recess, or groove,

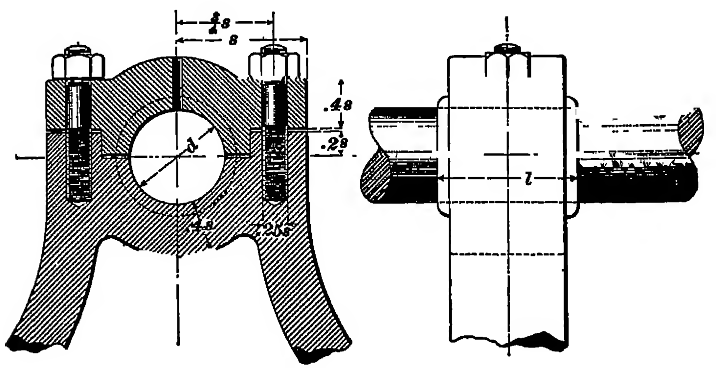


FIG 10

cast in it, which is filled with Babbitt metal, on which the journal rests. The proportional unit for this bearing is

$$s = 1.15\,d + .4$$

The dimensions l and d of the bearing are the same as those for the journal; the other proportions are given in terms of the unit s, and are shown in the figure.

18. Boxes, or Brasses.—Carefully made bearings are always lined with so-called **boxes**, or **brasses**, which are made of brass, gun metal, phosphor-bronze, white metal, or Babbitt metal, although other alloys are also used. The boxes when worn out may be easily replaced, and being made of softer material than the journal, the latter wears but very little.

An ordinary form of box is shown in Fig. 11. A part of the bearing at each end is made octagonal in cross-section. This part is fitted into an octagonal recess in the *pillow-block*, or *pedestal*, which holds it, and the boxes are thus prevented from turning. Sometimes, the octagonal parts are dispensed with, and the boxes are turned in a lathe, the pillow-blocks, or pedestals, being bored to receive them. It is then necessary to provide the boxes with lugs, or pins, to prevent them from turning.

These boxes may be made according to the following pro-

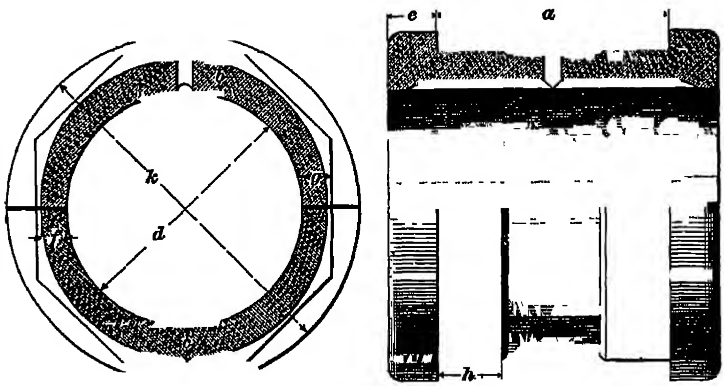


Fig. 11

portions, in which d, the diameter of the journal, is the proportional unit:

$b = \frac{1}{10}d + \frac{3}{16}$ in.; $g = \frac{1}{8}d$;
$c = \frac{1}{8}d + \frac{3}{16}$ in.; $h = \frac{1}{4}d$;
$e = \frac{1}{8}d + \frac{1}{8}$ in.; $k = 1\frac{1}{2}d$;
$f = \frac{1}{10}d$; d = diameter of journal

The dimension a should be made to fit the pillow-block for which the boxes are intended.

Boxes should be well supplied with grooves and channels, so that oil may be conducted to every part of the journal. It is a good plan to place a groove parallel with the axis of the journal in the upper half of the bearing, or top box, so as to be in direct communication with the oil holes; then, the advancing side of the journal will always carry a thin film of

oil along with it. The grooves may be from $\frac{1}{8}$ to $\frac{1}{2}$ inch wide, according to the size of the journal.

19. Lining for Boxes.—Boxes for large bearings are often lined with Babbitt, or some other antifriction, metal. Experience has proved that a bearing will run cooler when so lined, probably because the antifriction metal is softer and thus accommodates itself to the journal more readily than the more rigid metal, mostly brass, of which the boxes are made.

Some of the common methods of lining the boxes are shown in Fig. 12. At (*a*), the Babbitt metal is shown cast

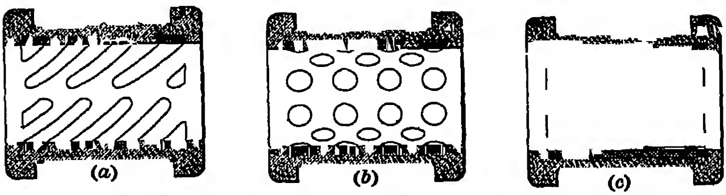


FIG 12

into shallow helical grooves; at (*b*), into a series of round holes; and at (*c*), into shallow rectangular grooves. Consequently, the journal rests partly on the brass and partly on the Babbitt metal.

In cheap work, the bearing is frequently simply lined with Babbitt metal. A mandrel, the exact size of the journal, is placed inside the bearing, and the melted Babbitt metal is poured around it. In better work, a smaller mandrel is used. After the metal hardens, it is hammered in; the bearing is then bored out to the exact size of the journal.

PEDESTALS

20. The names **pedestal, pillow-block, bearing,** and **journal-box** are applied indiscriminately to one form of bearing, and mean a support for a rotating piece.

21. A form of bearing frequently used for small shafts is shown in Fig. 13. It consists of two parts: the seat

that supports the journal, and the cap that is screwed down to the seat. This bearing is lined with Babbitt metal, or, as it is commonly expressed, the bearing is babbitted. The cap is held in place by capscrews—an invariable method in small pedestals.

22. The proportioning of a pedestal is largely a matter of experience. Few or none of the parts are calculated for

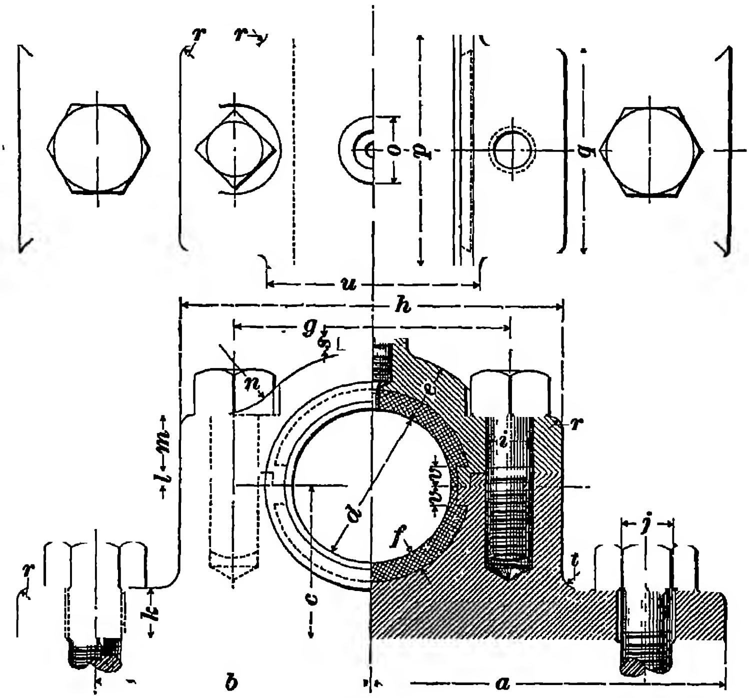


FIG 13

strength. All the proportions of the pedestals that follow are based on the diameter d of the journal as the unit; the length of the bearing is the same as that of the journal.

For the bearing shown in Fig. 13, the following proportions may be used for sizes of journals from $\frac{1}{2}$ to 2 inches in diameter, inclusive. The diameter d of the shaft is the unit.

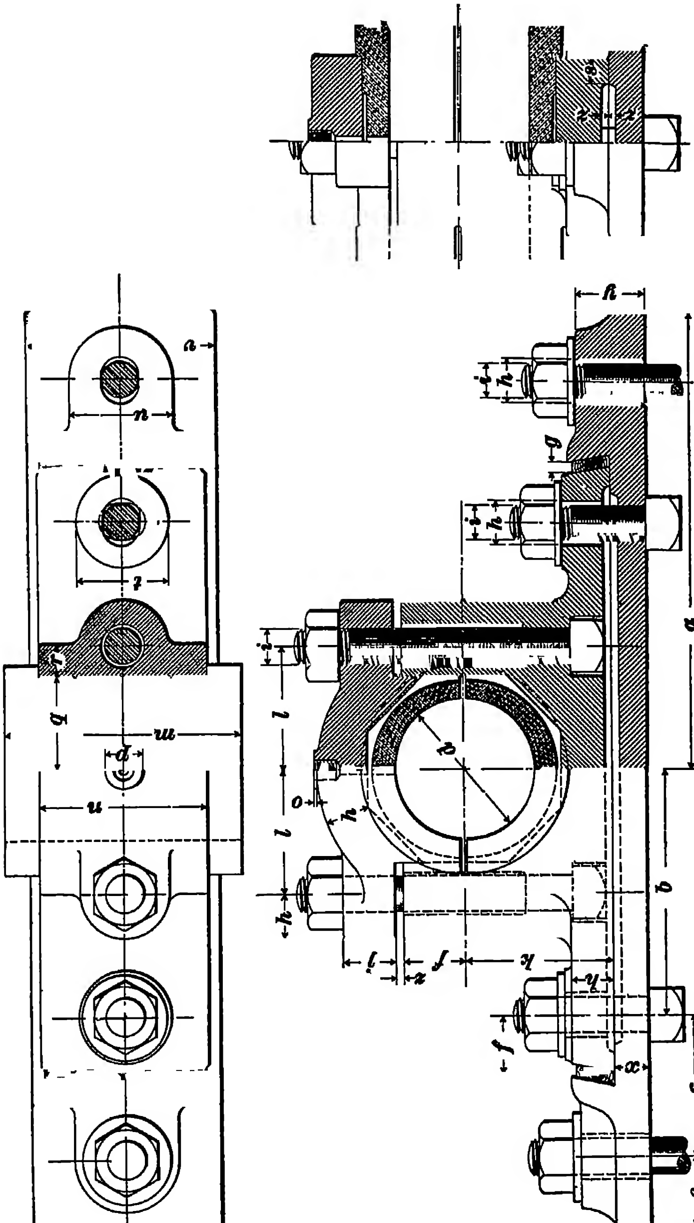


Fig. 14

$a = 2.25\,d$;
$b = 1.75\,d$;
$c = d$;
$e = .375\,d$;
$f = .08\,d + .0625$ in.;
$g = 1.75\,d$;
$h = 2.45\,d$;
$i = .3\,d$;
$j = .33\,d$;
$k = .25\,d + .125$ in.;
$l = .08\,d$;
$m = .25\,d + .1875$ in.;
$n = .5\,d$;
$o = .625$ in. (constant);
$p = 1.5\,d$;
$q = 1.333\,d$;
$r = .08\,d$;
$s = .125$ in. (constant);
$t = .16\,d$;
$u = 1.333\,d$;
$v = .125\,d$.

23. In Fig. 14 is shown a common form of pedestal used for somewhat larger journals than the one shown in Fig. 13. This pedestal consists of: (1) a foundation plate that is bolted to the foundation on which the pedestal rests; the plate is essential when the pedestal rests on the brickwork or masonry, but may be dispensed with when the pedestal rests on the frame of the machine; (2) the block that carries the brasses and supports the journal; and (3) the cap that is screwed down over the brasses. The bolt holes in both the foundation plate and the block are oblong, so that the pedestal may be readily adjusted.

The following proportions may be used for this kind of pedestal, with journals from 2 to 6 inches in diameter, inclusive. An oil cup having a $\frac{1}{4}$-inch pipe tap shank may be used on pedestals for journals having diameters from 3 to 4 inches, and $\frac{3}{8}$-inch pipe tap shank for larger sizes up to 6 inches in diameter. The diameter d of the journal is the unit.

NOTE.—The shanks of oil cups and grease cups in the market are made with $\frac{1}{8}$-, $\frac{1}{4}$-, $\frac{3}{8}$-, and $\frac{1}{2}$-inch pipe threads. The amount of oil or grease the cup holds when filled is usually expressed in ounces.

$a = 3.25\,d$;
$b = 1.75\,d$;
$c = d$;
$e = .5\,d$;
$f = .4375\,d$;
$g = .09\,d$;
$h = .3125\,d$;
$i = .25\,d$;
$j = .375\,d$;
$k = 1.0625\,d$;

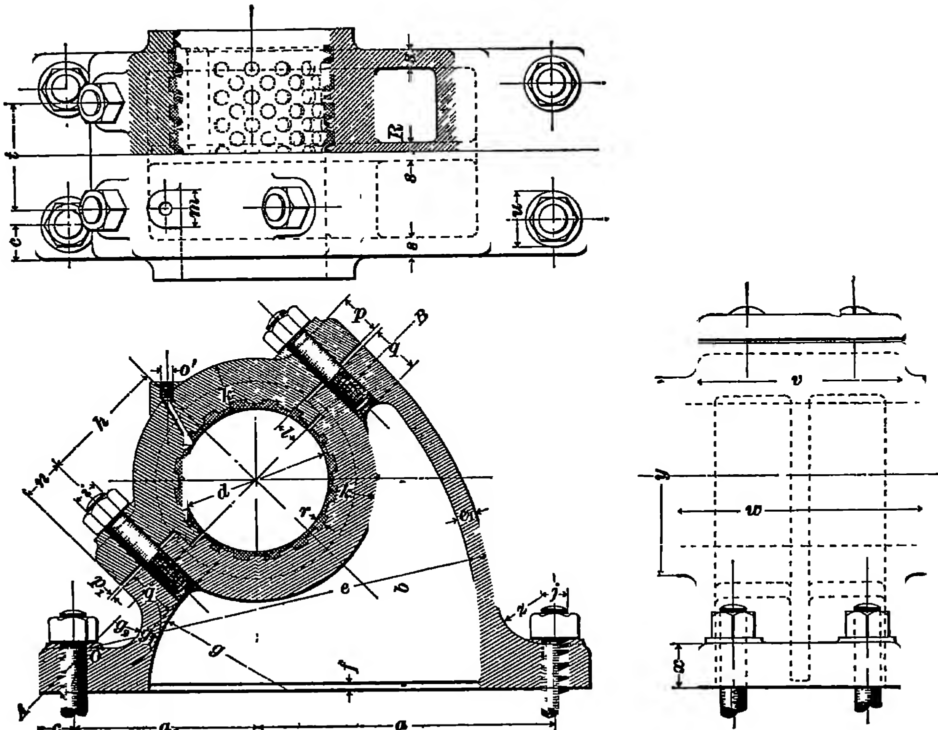


FIG. 1b

$l = .875\,d;$	$s = .1875\,d;$
$m = 1.75\,d;$	$t = .65\,d;$
$n = 1.25\,d;$	$u = .75\,d;$
$o = .125$ in. (constant);	$v = 1.375\,d;$
$p = .875$ in. (constant);	$x = .25\,d;$
$q = .625\,d;$	$y = .5\,d;$
$r = .25\,d;$	$z = .0625\,d.$

24. Crank-Shaft Pedestals.—The load on a crank-shaft bearing is due partly to the weight of the shaft and flywheel, and partly to the alternate push and pull of the connecting-rod. The direction of the resultant of these two forces is therefore more or less oblique; consequently, the pedestal is often divided obliquely.

25. A pedestal of the kind just mentioned is shown in Fig. 15, and may be proportioned by taking the diameter d of the journal as the unit and using the following proportions:

$a = 2\,d + 1$ in.;	$n = .3\,d;$
$a_1 = 1.2\,d + 1$ in.;	$p = .25\,d + .25$ in.;
$b = 1.5\,d;$	$p_1 = .04\,d;$
$c = .22\,d + .5$ in.;	$q = .35\,d;$
$e_1 = .1\,d + .25$ in.;	$r = .02\,d + .3125$ in.;
$f = .5$ in. (constant);	$s = .1\,d + .25$ in.;
$g_1 = .1\,d + .25$ in.;	$t = .75\,d;$
$h = .9\,d;$	$u = .27\,d + 1.125$ in.;
$i = .15\,d + .25$ in.;	$v = 1.45\,d;$
$j = .15\,d + .375$ in.;	$w = 1.75\,d;$
$k = .3\,d + .625$ in.;	$x = .25\,d + .625$ in.;
$l = .1\,d + .375$ in.;	$y = 1.3\,d + 1$ in.;
$m = .25\,d;$	$z = .25\,d + 1$ in.

The line AB is at an angle of 45° to the base line. This pedestal, as shown, is babbitted. The Babbitt is held in place and prevented from turning with the journal by providing the surfaces with which the Babbitt comes in contact with round projections, as shown. The projections are about 1 inch in diameter for the larger sizes of pedestals.

To find the radius e, draw a line parallel with, and at a distance x above, the base line. The point of intersection O

of this line with the line AB is the center of the arc having the radius e. The radius g_2 is found by trial. The center of the arc must lie on the line AB. With a radius $g_2 + g_1$, describe an arc from the same center. Draw a line parallel

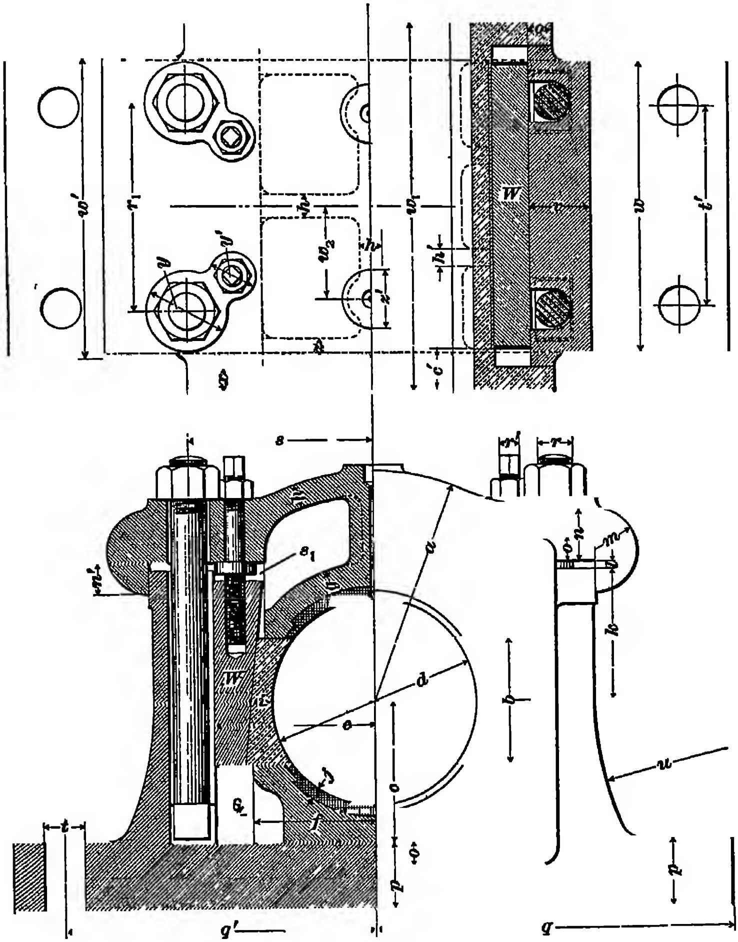


FIG 16

with the face of the bearing at a distance from it determined by q. The radius g is to be found by trial, the center being on the base line, and the arc tangent to the line determined

by q and to the arc determined by $g_2 + g_1$. The rib R may be used on all pedestals in which the diameter of the journal is above 6 inches. For 3- to 4-inch journals, o' may be tapped with a $\frac{1}{4}$-inch pipe tap; up to 6-inch journals, a $\frac{3}{8}$-inch pipe tap; up to 9-inch, a $\frac{1}{2}$-inch pipe tap; and all sizes above 9 inches, a $\frac{3}{4}$-inch pipe tap.

Two oil cups may be used on all bearings in which the diameter of the journal is above 8 inches. This style of pedestal may be used for journals from 3 inches up to 15 inches in diameter, inclusive.

26. Fig. 16 shows a pedestal suitable for the crank-shaft of a horizontal engine with journals from 8 to 20 inches in diameter. The block may be complete in itself, as shown in the figure, but more often it forms part of the engine bed.

The box is in three parts, the cap, the cheek boxes, or cheeks, and the bottom box. The cheeks may be adjusted horizontally by means of wedges W, and the bottom box may be raised by placing packing pieces under it. To obtain the dimensions of this pedestal, use the following proportions, which are based on the unit d, or the diameter of the crank-shaft journal:

$a = d + 1$ in.;
$b = .5\,d + 1$ in.;
$c = .66\,d$;
$c' = .2\,d$;
$e = .825\,d - .25$ in.;
$f = .6\,d$;
$g = .1\,d + .5625$ in.;
$h = .1\,d + .25$ in.;
$h' = .08\,d$;
$i = .11\,d$;
$j = .625$ in. (constant);
$k = .5\,d + 1.25$ in.;
$l = .375$ in. (constant);
$m = .175\,d + .3125$ in.;
$n = .25\,d + .25$ in.;
$n' = .1\,d + .375$ in.;
$o = 1$ in. (constant);

$p = .25\,d + .625$ in.;
$q = 1.75\,d$;
$q' = 1.5\,d$;
$r = .15\,d$;
$r' = .1\,d$;
$r_1 = d$;
$s = .9\,d$;
$t = .15\,d + .375$ in.;
$t' = .9\,d$;
$u = 1.5\,d$;
$v = .25\,d + .375$ in.;
$w = 1.45\,d$;
$w' = 1.47\,d$;
$w_1 = 1.75\,d$;
$w_2 = \frac{w_1}{4}$;
$w_3 = w_1 - 2\,x$;

$w_4 = w_1 - 2\,c' + \frac{1}{8}$ in.; $y' = .2\,d + .5$ in.;
$x = .1\,d$; $z = .09\,d$;
$y = .3\,d + .75$ in.; $z' = 2.5$ in. (constant).

Taper of adjusting wedge = $1\frac{1}{4}$ inches per foot.

The form and location of the wedge W are shown in the plan view, Fig. 16, and the perspective views, Fig. 17 (*b*) and (*c*). It will be noticed that the upper part of the wedge extends across the whole width of the pedestal, until it reaches the flanges of the cap and the cheeks which have corresponding recesses s_1, s_1, Fig. 17 (*a*) and (*b*). As shown

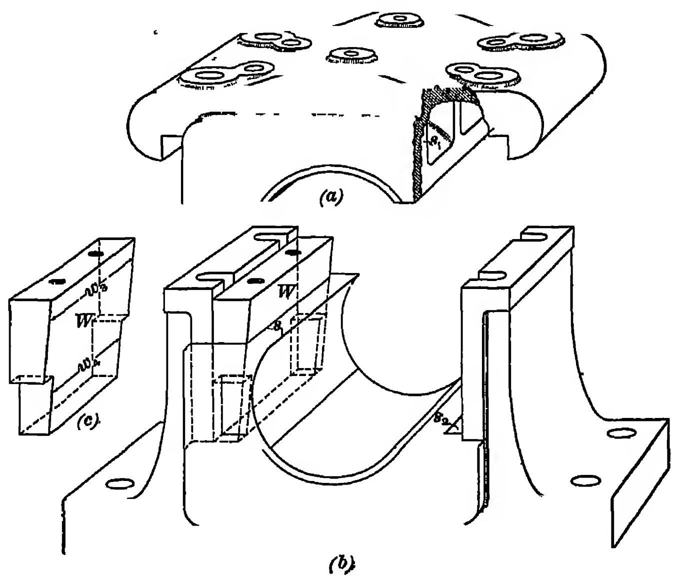


Fig. 17

in Fig. 17 (*b*), the bottom box has shoulders that make the recess s_2 narrower than those in the cheeks and cap. For this reason, the lower part of the wedge is made narrower, so as to enter the recess s_2. The perspective views will assist in locating the various parts and in understanding the peculiar shape of the cap, the view of the latter being partly broken away in order to show the lower central part.

The dimensions of the wedge may be found in the following manner: Lay off the lengths i and e, Fig. 16, along the horizontal center line of the bearing, the difference between the two dimensions being equal to the thickness of the wedge along this line. Through the left-hand end of dimension i draw a vertical line equal in length to b, the lower end of the line beginning at the lower side of the cheek. As the taper of the wedge is $1\frac{1}{4}$ inches to the foot, calculate the amount of taper for a length equal to b. Draw horizontal lines through the upper and lower ends of this vertical line, and along the upper line set off one-half of the taper to the right of the vertical line, and along the lower one, one-half to the left. By connecting these two points, the taper side of the wedge is determined; this side may now be extended upwards to any point desired, the bottom of the wedge being made flush with the lower side of the cheek.

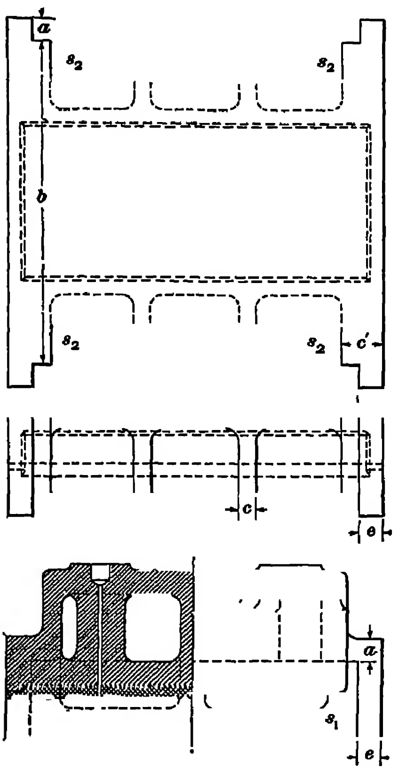


FIG 18

The center of the circle of diameter y', shown in the plan view, Fig. 16, should be approximately over the center of the adjusting wedge and far enough from the center of the large bolt to permit both nuts to be turned freely.

Further details of the bottom box and the cap are shown in Fig. 18, in which the unit is the same as in Fig. 16, and the proportions are as follows:

$a = 1$ in. (constant); $\quad c' = .2\,d$;

$b = 1.65\,d - .5$ in.; $\quad e = .1\,d$.

$c = .08\,d$;

The outlines of the dovetailed projections that hold the Babbitt lining in place are indicated by dotted lines.

27. The foundation, or bed casting, is shown in Fig 19 (*a*), and has dimensions to suit the pedestal shown in Fig. 16. The

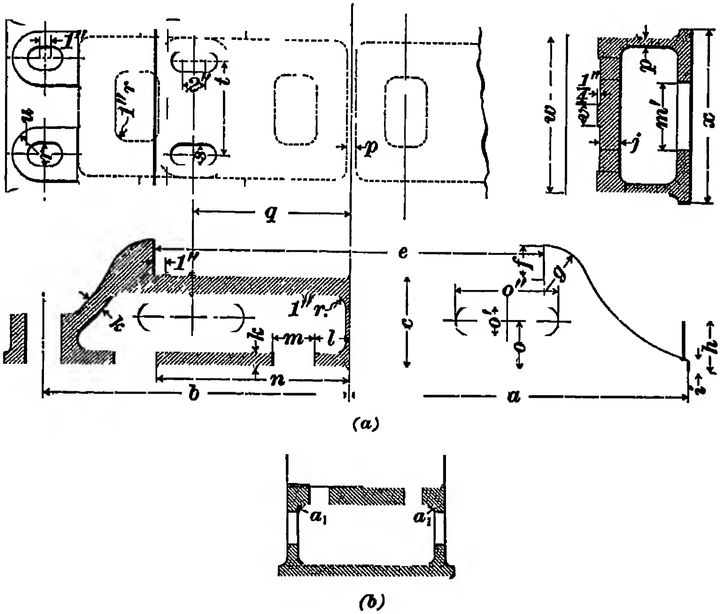


FIG. 19

proportions for this foundation are as follows, the diameter d of the crank-shaft journal being taken as the unit.

$a = 2.45\,d + 7.25$ in.;	$i = .05\,d + .5$ in.;
$b = 2.3\,d + 5\ 25$ in ;	$j = .05\,d + 1.125$ in.;
$c = .5\,d + 3\ 5$ in.;	$k = .05\,d + .75$ in.;
$e = 3.5\,d + 2$ in.;	$l = .25\,d + .75$ in.;
$f = .25\,d + .5$ in.;	$m = .4\,d$;
$g = .25\,d + 1\ 75$ in.;	$m' = .6\,d$;
$h = .25\,d + 2\ 25$ in.;	$n = 1.55\,d + 2.5$ in.;

$o = .25\,d + 2$ in.;	$s = .15\,d + .375$ in.;
$o' = .25\,d + .5$ in.;	$t = .9\,d$;
$o'' = .5\,d + 4.5$ in.;	$u = .15\,d + .875$ in.;
$p = .08\,d$;	$v = .2\,d$;
$q = 1.5\,d$;	$w = 1.5\,d$;
$r = .15\,d + .375$ in.;	$x = 1.65\,d$.

Fig. 19 (b) shows a cross-section taken through two of the holes for the pedestal bolts, and explains the presence

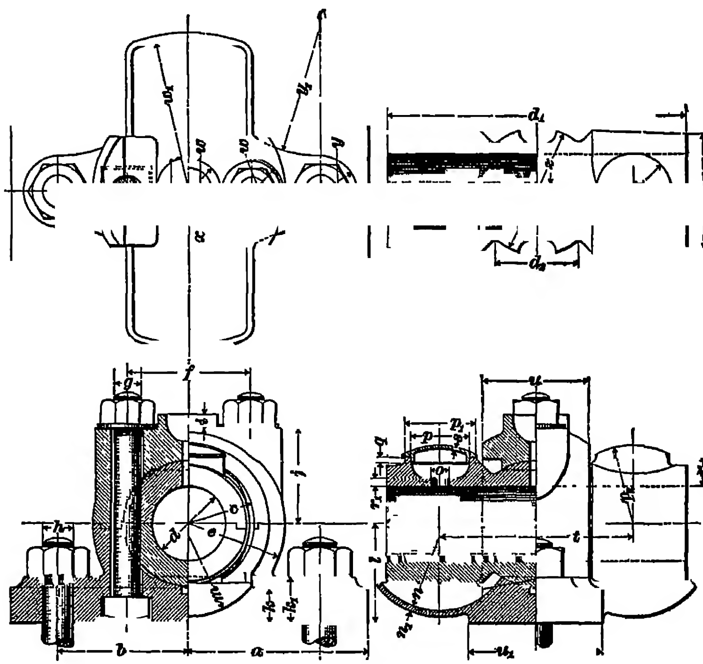


FIG. 20

of the dotted curve at the outside of the holes in the plan view. As shown at a_1, the fillet at the inside of the top has been milled away near the bolt hole, in order to make room for the pedestal bolt head and give the latter a flat bearing surface.

28. **Ball-and-Socket Bearings.**—The **ball-and-socket bearing** now largely used in the United States was

first introduced by William Sellers & Co., of Philadelphia. This bearing has very long boxes made of cast iron bored to fit the journal. In some cases, however, the boxes are cast with a recess in which a Babbitt lining is poured to form a bearing surface for the journal. The boxes have a spherical enlargement at the center, which fits into corresponding hollows in the block and cap, thus making a ball-and-socket joint, which leaves the box free to move slightly in any direction to conform to an inequality or want of alinement in the shaft. When rigid boxes are used, they must necessarily be short on account of unavoidable deflections of the shaft due to belt pull, thrust of gearing, etc. Pivoted boxes, on the contrary, by reason of their flexibility, may be made long, thereby giving a large wearing surface and increased durability.

29. Fig. 20 shows a pedestal with ball-and-socket bearing. The boxes are made of cast iron and have a length equal to four times the diameter of the journal. Ordinarily, the journal is lubricated through the oil hole in the center of the cap, but the top box also has two cups, which are filled with a mixture of tallow and oil that melts at about 100° F. If the bearing becomes heated, this mixture melts, and thus helps in the lubrication.

The dimensions of this bearing are obtained from the following proportions, which are based on the diameter d of the journal as the unit:

$a = 2.375\,d$;	$j = 1.27\,d$;
$b = 1.75\,d$;	$k = .3\,d + .25$ in.;
$c = .875\,d$;	$k_1 = .4\,d + .375$ in.;
$d_1 = 4\,d$;	$l = 1.33\,d$;
$d_2 = 1.1\,d$;	$m = d + .125$ in.;
$d_3 = 1.25\,d + .5$ in.;	$n = d$;
$e = 1.3\,d$;	$n_1 = .125$ in. (constant);
$f = 1.65\,d$;	$o = .25\,d$;
$g = .2\,d + .25$ in.;	$p = .8\,d$;
$h = .2\,d + .375$ in.;	$p_1 = .95\,d$;
$i = .1875\,d$;	$p_2 = d$;

$p_1 = d$;
$q = .07\,d$;
$r = .4\,d$;
$r_1 = .1\,d$;
$s = .0625$ in. (constant);
$t = 2.5626\,d$;
$u = 1.45\,d$;
$u_1 = 1.8\,d$;
$v = .875\,d$;
$w = .2\,d + .375$ in.;
$w_1 = 2.06\,d$;
$x = .18\,d + 2.5$ in.;
$x_1 = .1875\,d$;
$y = .25\,d + .375$ in.;
y_1 to be found by trial;
$z = 1.5\,d + .3125$ in.

WALL BRACKETS AND HANGERS

30. Wall Brackets.—A shaft must sometimes be supported by a bearing fixed to a wall or pillar. In such a case, the bearing is generally supported by a **wall bracket.** It will readily be seen that the bracket is a cantilever with practically a uniform load, due to its own weight, and a concentrated load, due to the weight of the shaft; hence, the bending moment may be determined by the principles of strength of materials.

In order that a cantilever shall be equally strong at all sections, the resisting moment $\frac{SI}{c}$ of that section must be proportional to the bending moment at that section. Suppose the bracket in question to be a plate of a constant thickness b and a varying height h. Then, at any section, the bending moment

$$M = \frac{SI}{c} = \frac{S\frac{1}{12}b\,h^3}{\frac{1}{2}h} = \frac{1}{6}S\,b\,h^2$$

Since $\frac{1}{6}S\,b$ remains constant for all sections, h^2 is proportional to the bending moment; or, in other words, the height of a cantilever at any section should be proportional to the square root of the bending moment at that section. It can be shown mathematically that a cantilever of constant width should have a parabolic form when the load is concentrated at the end, and a triangular form when the load is uniformly distributed. In practice, a bracket or other machine part of the cantilever class is given a form that is often neither exactly parabolic nor triangular, but approximates closely

to one or the other, according to the taste and judgment of the designer. The bearing of the wall bracket may be made the same as in the hanger shown in Fig. 21.

31. Hangers.—A **hanger** is used when a shaft bearing is to be suspended from the ceiling. Fig. 21 shows a form of a hanger frequently employed.

The frame of the hanger is divided, and the parts are connected by bolts. With such a form the shaft may be more easily removed than when the hanger frame is a solid piece.

The units for determining the leading dimensions of a shaft hanger are the diameter d of the shaft and the drop D of the hanger. The following proportions are suitable for shafts ranging from $1\frac{1}{2}$ inches up to $4\frac{1}{2}$ inches in diameter:

$A = 6\,d + .45\,D$;
$A_1 = 2\,d + .03\,D$;
$B = 4\,d + .35\,D$;
$C = 2\,d + .3\,D$;
$E = 2\,d + .25\,D$;
$F = .5\,d + .01\,D$;
$F_1 = 1.5\,d + .05\,D$;
$G = 1.25\,d$;
$H = 2\,d$;
$I = .4\,d$;
$J = .125\,d + .01\,D$;
$K = .5\,d + .5$ in.;
$L = .25\,d + .5$ in.;
$M = .75\,d + .6875$ in.;
$N = .25\,d + .375$ in.;
$O = 1\ 25\,d$;
$O_1 = .094\,d + .002\,D$;
$P = .375\,d + .008\,D$;
$Q = .375\,d + .008\,D$;
R and R_1 = (see note);
$S = .25\,d + .005\,D$;
$S_1 = .125\,d + .003\,D$;
$T = .125\,d + .01\,D$;
T_1 = (see note);
$U = 2\,d$;
$V = .5\,d$;
$W = .75\,d$;
$X = .375\,d$;
$Y = .25\,d + .125$ in.;
$Z = .625\,d$;
$a = .15\,d + .375$ in.;
$a_1 = 2.4\,d + .3125$ in.;
$b = .08\,d$;
$c = .125\,d + .0625$ in.;
$e = .2\,d$;
$e_1 = .4\,d$;
$e_2 = .2\,d$;
$f = .375\,d + 1$ in.;
$f_1 = .09\,d + .25$ in.;
$g = .75\,d$;
$g_1 = 1.3125\,d + .125$ in.;
$h = 1.25\,d + .1875$ in.;
$i = .1\,d$;
$j = 25\,d + .25$ in.;
$j_1 = .125\,d + .0625$ in.:
$k = 2\ 2\,d$;
$l = 4\,d$;
$m = 1.4\,d + .375$ in.;

$n = d$;
$o = .25\,d$;
$o_1 = .0625\,d$;
$p = d$;
$p_1 = .0625\,d$;
$q = .4\,d$;
$q_1 = .15\,d$;
$r = 2.125\,d$;
$s = 1.5\,d$;
$s_1 = .125\,d$;
$t = 2\,d$;
$t_1 = .5\,d$;
$t_2 = d$;
$t_3 = .25\,d$;
$u = .95\,d$;
$u_1 = .85\,d$;
$v = .25\,d + .125$ in.;
$v_1 = .5\,d$;
$w = d$;
$w_1 = .125$ in. (constant);
$x = .25\,d$;
$x_1 = d$;
$x_2 = 4\,d + 2$ in.;
$y = 1.25\,d$;
$y_1 = .75\,d + .0625$ in.,
$y_2 = .4\,d + .0625$ in.;
$z = .06\,d + .25$ in.;
$z_1 = .12\,d + .75$ in.;
$z_2 = .3125$ in. (constant).

Thread of plugs = .5-inch pitch for all sizes.

NOTE.—To find R_1, draw the arc J, also, draw the arc Q tangent to P, then, draw a straight line tangent to these arcs, and R_1 will be the distance along the center line, determined by B included between this tangent and the upper face of the hanger. Having found R_1, make R equal to it.

The radius T_1 is made equal to three-eighths the thickness at the middle

The boxes of the ball-and-socket bearings shown in Figs. 20 and 21 are made of cast iron, and are bored to fit the journal without lining. The ball, and the recesses in the ends of the plugs into which the ball is fitted, should be finished. The screw threads on the plugs, Fig. 21, may be either cast on the plugs or turned, the latter method being preferable.

The spherical enlargement at the center of the boxes is held between the ends of two hollow cast-iron plugs, which are threaded at their outer ends and screw into bosses cast on the bracket. The boxes may be raised or lowered by screwing the plugs up or down. In order to turn the plugs, the hole at the end is made square to receive a key. When the plugs are in the desired position they are locked by setscrews.

32. Pivot, or Footstep, Bearings.—Bearings that are used to support the ends of vertical shafts are known as **pivot, or footstep, bearings.** An ordinary pivot bearing

is shown in Fig. 22. The end of the pivot rotates on a disk *A*, which may be made of steel, brass, or bronze. The brass bushing *B* prevents the pivot from moving sidewise. The end of the pivot should be made of steel, and it may be flat on the end or slightly convex. The proportions are

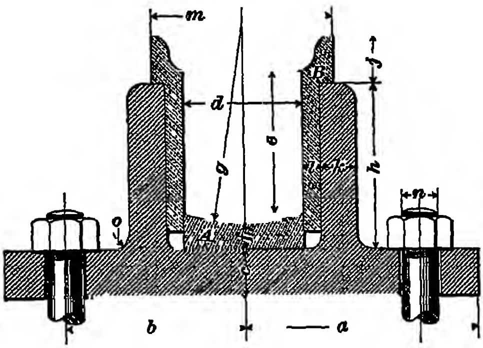


FIG 22

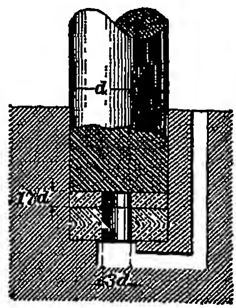


FIG 23

given in terms of the diameter *d* of the pivot as a unit, and are as follows:

$a = 2\,d$; $\quad j = .4\,d$;
$b = 1.5\,d$; $\quad k = .3\,d$;
$c = .25\,d + .375$ in.; $\quad l = .15\,d$;
$e = 1.25\,d$; $\quad m = 1.5\,d$;
$f = .2\,d + .125$ in.; $\quad n = .2\,d + .25$ in.;
$g = 1.75\,d$; $\quad o = .15\,d$.
$h = 1.4\,d$;

33. Pivots With Loose Disks.—When pivots are required to run at great speed, the relative speed of the surfaces in contact may be reduced by placing a number of loose disks between the pivot and the footstep, shown in Fig. 23. These disks are sometimes made slightly convex.

If *a* is the number of disks and *n* the revolutions per minute of the pivot, then the relative speed of any of the surfaces in contact is $\frac{n}{a+1}$ revolutions per minute. Suppose, for example, a pivot runs at 3,000 revolutions per minute, and it is desired to reduce the speed between adjacent surfaces to

750 revolutions per minute; then, $750 = \frac{3{,}000}{a+1}$, or $a = 3$, the number of disks required. The pivot and disks may be lubricated by a channel, as shown in the figure. The disks may be made of gun metal or phosphor-bronze. Proportions are given in the figure.

ROLLER AND BALL BEARINGS

ROLLER BEARINGS

34. Rolling Friction.—In Fig. 24, C illustrates a cylinder at the center of which the force P is acting, compelling the cylinder to roll over the surface D and to revolve in the direction shown by the arrow v. As indicated, the surface D undergoes a certain amount of deformation, which may be permanent, if the material is of a plastic nature, or only temporary, if the material is elastic and the compressive stress does not exceed the elastic limit. In the position shown, the point e is lower than b, and in order that the motion may continue, the cylinder must revolve about b as an instantaneous axis of rotation. The moment causing rotation is $P \times ab$, and the resisting moment is $W \times bc$, W being the load supported by the cylinder. These moments must be equal; hence,

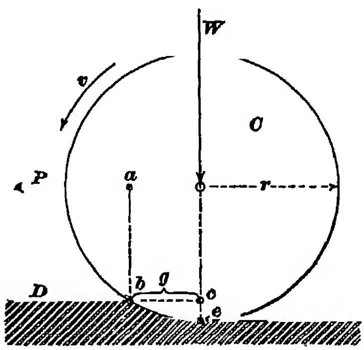


FIG 24

$$P \times ab = W \times bc, \text{ and } P = \frac{W \times bc}{ab}$$

If $bc = g$ and the cylinder radius is r, then

$$P = \frac{Wg}{r - ce}$$

For hard materials, as used in bearings, the length ce is very small compared with r, and may be left out of consideration. The formula then becomes

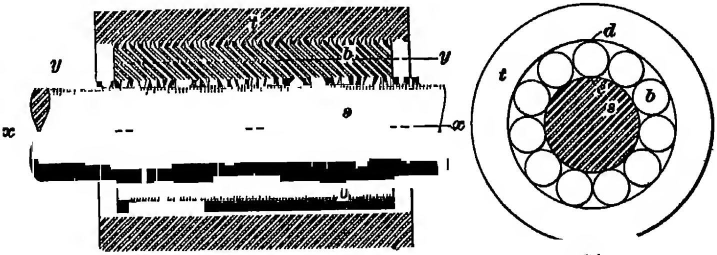
$$P = \frac{Wg}{r}, \text{ approximately}$$

This formula shows that the rolling resistance decreases with an increase of r.

Very few experiments have been made to determine the value of g. Some tests give g a value of from .007 to .015 inch for steel wheels rolling on steel rails.

As the rolling resistance between two polished metal surfaces is very low, the advantage to be derived from this combination has been utilized in the construction of roller and ball bearings. In all bearings of this class, it is important that the rollers or balls, as well as the paths provided for them, be of the greatest hardness and smoothness. For this reason, if the whole of the parts in contact cannot be made of the required hardness, it is customary to provide them with linings that possess the requisite qualities.

35. Cylindrical Rollers.—The principal parts of a roller bearing, with the accessory parts omitted, are shown in Fig. 25, (*a*) being a longitudinal section and (*b*) an end



(a) (b)

FIG 25

view. The part s may represent part of a shaft or journal surrounded by a series of rollers b, b, which are held in position by means of the box t. In order that pure rolling may take place, the casing, rollers, and journal must be perfectly cylindrical; also, the axes yy of the rollers and the axis xx

of the journal must be parallel, and endwise motion of the rollers must be prevented. For the purpose of holding the rollers parallel to one another and to the journal, various methods are in use, as slotted cylinders, or cages, rings with pins, and rods guiding and separating the individual rollers.

Rollers may be conical as well as cylindrical. In order that true rolling may be insured with conical rollers, it is necessary that all the axes of rotation of the rollers shall intersect the axis of the journal at the same point, the position of which remains constant throughout a revolution of the journal.

The application of this rule may be more clearly shown by means of Fig. 26, where the rollers b, as well as the journal s, are conical. The axes $y\,y$ of the rollers, produced, intersect

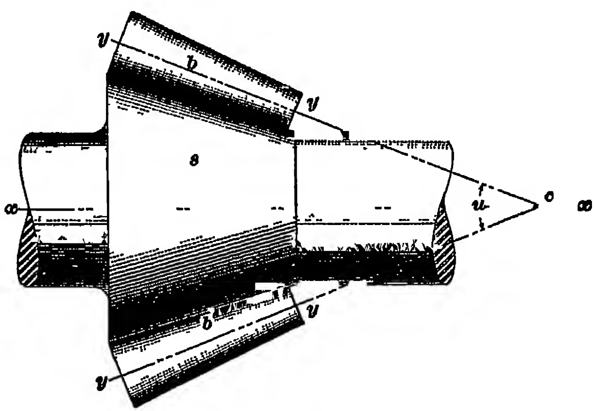


FIG. 26

the axis $x\,x$ at a point c. If the position of this point is unaltered while the journal completes a revolution, true rolling takes place.

36. According to the preceding rule, there are cases where true rolling should occur, but where, nevertheless, sliding friction is present. An instance of this kind is shown in Fig. 27, which may be considered as a variation of Fig. 26, the angle u being increased to 180° so that the axes y_1y_1 and y_2y_2 are at right angles to $x\,x$. In Fig. 27, b_1 and b_2 are

cylindrical rollers resting on the bearing plate t and supporting the circular end of the shaft s. If the rollers are guided by suitable means, such as a perforated disk, the intersection of the axes $y_1 y_1$ and $y_2 y_2$ with the axis $x x$ of the shaft will be

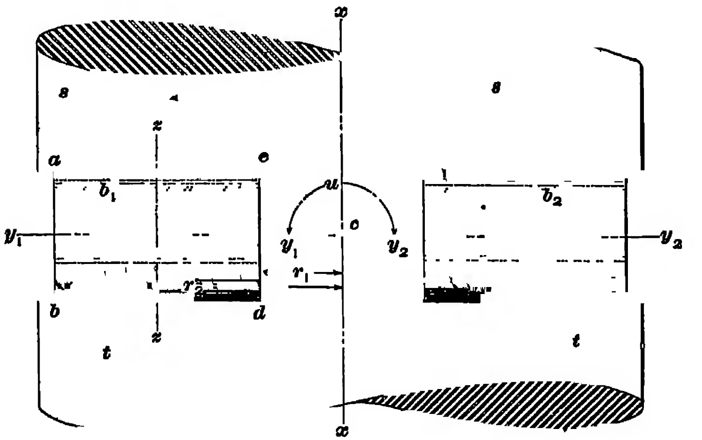


FIG 27

at the point c, and the position of this point will be unaltered throughout a revolution of the shaft.

Owing to the difference in peripheral speed between the points a and e at the end of shaft s, it is evident that the end $a b$ of the roller b_1 tends to rotate at a greater speed than the end $d e$. As this is impossible, it follows that there must be relative motion between one end or both ends and the contact surfaces, which will result in friction. As the rollers are caged, they are compelled to revolve around the axis $x x$ and, simultaneously, to revolve around a vertical axis located anywhere between a and e, as at $z z$. This latter motion is termed *spinning*, and is one that must be eliminated as far as possible if the bearing is to be efficient and durable.

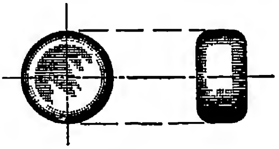


FIG. 28

Attempts have been made to obviate the injurious effect of spinning in thrust bearings, as in Fig. 27, by using a great

number of very short rollers having rounded edges, as shown in Fig. 28. The difference between the circumferential speeds at the two ends of the roller is then decreased to such an extent as to be of little consequence.

Fig. 29 shows such rollers inserted in a *cage*, or guiding plate *e*, of a thrust bearing. This cage is provided with rectangular openings and several series of rollers, as a_1, a_2, etc., and b_1, b_2, etc., every alternate series being so arranged relatively to the others that the paths of any two series do not coincide; consequently, the wear is more evenly distributed. The axes of all the series converge toward the common center *c* located on the axis of the shaft. The cage *e* rests on a raised edge, or flange, *d*.

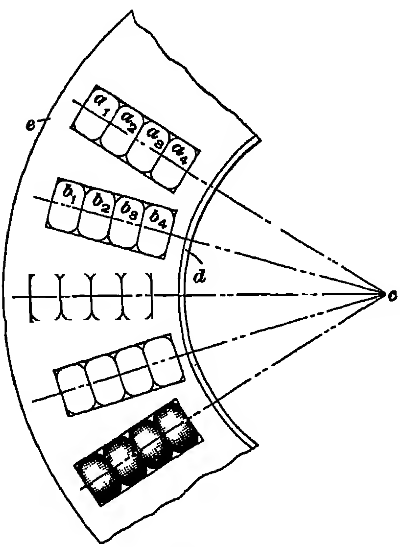


FIG. 29

Under certain conditions, thrust bearings of this kind have been very successful and have sustained very high pressures; both the rollers and the contact plates were made of hardened tool steel. Great care should be taken to see that the diameters of the rollers are perfectly uniform, and that the surfaces are absolutely plane and highly polished.

37. Conical Thrust Rollers.—If rollers having their axes at right angles to the axis of the shaft are to comply with the rules for true rolling, they must be made conical, and be proportioned according to the principles illustrated in Fig. 30 (*a*). That is, any roller constituting the frustum of a cone will have a true rolling motion if its apex coincides with the apexes of the conical surfaces on which it rolls.

The apex angles u_1, u_2 of these surfaces do not have to be equal; they may be varied until one of them becomes 180°, as at u_2, Fig. 30 (*b*), and may be increased beyond 180° if necessary.

It can be proved that if the roller b_1 is to have a true roll-

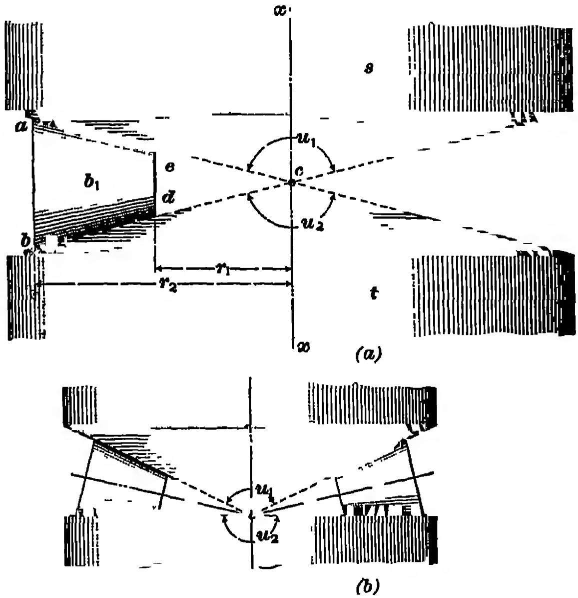


FIG 30

ing motion the following proportion must be complied with:

$$ab : de = r_2 : r_1$$

38. Axial Pressure of Conical Rollers.—In Fig. 31 (*a*), the shaft *s* is revolving in the direction of the arrow, and is supporting a load *P* that is transmitted to the bearing plate *t* by a series of rollers, two of which, *b*, b_1, are shown. Two forces P_1 and P_2 will therefore be acting normally to the sides of each roller. A resultant of these forces will act along the axis *ce*, tending to force the roller outwards; that is, away from the center *c*. Thus, a force P_3 will have to be applied at *e*, so as to prevent a longitudinal motion of the roller.

The magnitude of these forces may be ascertained as indicated in Fig. 31 (*b*). Draw the line *fg* parallel to *P*. Let P_0 represent the portion of the total load carried by each roller, and to some given scale, make *fg* of a length representing the magnitude of P_0. Draw *fh* at right angles to *fg*, and from point *g* line *gh* parallel to P_1. Then, the point of intersection *h* will determine the length of $gh = P_1$. As the action of P_1 is opposed by the reactive force P_2, draw

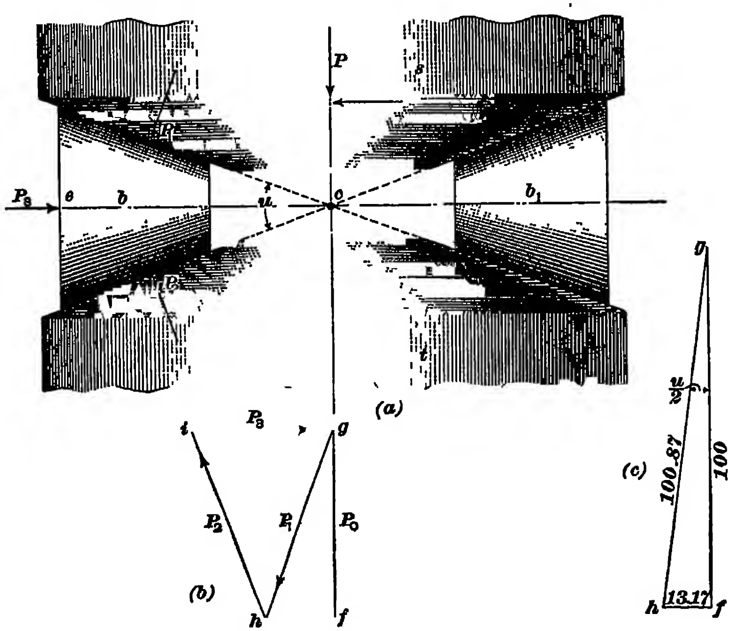


FIG. 31

the line *hi* parallel to the latter, and from *g*, the line *gi* parallel to the axis *ce*. Then, *gi* will indicate the magnitude of the reaction P_3 required to hold the roller in place. It is evident that this force P_3 should be as small as possible, and that, therefore, the angle *u* should be as small as the conditions will allow. This angle should not exceed 15°.

EXAMPLE.—In a roller thrust bearing similar to Fig. 31 (*a*), the pressure P_0 on each roller is 100 pounds. If the vertex angle *u* of the roller is 15°, what will be the pressures P_1 and P_3?

SOLUTION.—In solving the problem graphically, it is sufficient to draw the portion corresponding to fgh, Fig. 31 (*b*). Draw a line fg, Fig. 31 (*c*), representing the pressure of 100 pounds to any convenient scale, as, for instance, 1 in = 10 lb Therefore, $fg = 10$ in. Lay off the angle $fgh = \frac{1}{2} u = 7° 30'$, and draw gh, then, draw fh perpendicular to fg By measuring the lines according to the chosen scale, it is found that the line $fh = \frac{1}{2} P_s = 13.17$ lb, and, therefore, $P_s = 26.34$ lb. The normal pressure $P_1 = gh = 100.86$ lb. Ans.

An example of a roller thrust bearing is shown in Fig. 32, which illustrates a full-sized transverse section through one side of the bearing. The bearing plate *g* revolves with the

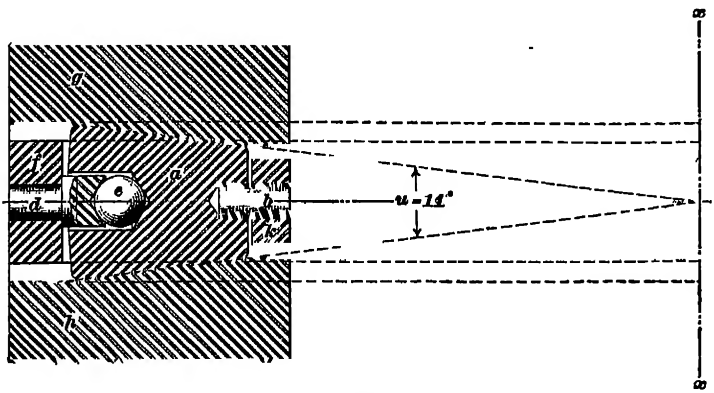


FIG. 32

shaft, and the plate *h* is stationary. One of the conical rollers is shown at *a*, and is held in position by the pins *b* and *d*. A ball *e* is inserted in a recess in the roller and rests against the concave end of the pin *d*, its purpose being to reduce the frictional resistance between the roller and the pin *d*. The inside pins are supported by the ring *k*, and the outside ones by the ring *f*, both rings revolving around the axis *xx*. In this instance, the apex angle *u* is 14°.

39. Most of the roller bearings manufactured come under one of the classes just described. On account of the rigidity of the parts, all bearings must be accurately shaped, and the material used must be extremely hard. In one form of roller bearing, the rollers are not solid, but are made

of steel bands wound spirally into the shape of a roller. Rollers constructed in this manner have a certain amount of flexibility, and thus require less accuracy in their manufacture; likewise, the surfaces against which the rollers revolve need not be of such extreme hardness.

40. Construction of Roller Bearings.—The strength of a roller bearing cannot be safely determined except by actual test; it depends to a great extent on the material used for the rollers and bearing surfaces. In some cases, bearings that are identical in size and construction will show great differences in durability owing to insufficient hardening of some of the parts. Calculations based on the strength of the material and the form of the rollers are not safe guides, as the actual strength is often found to be far below the estimated value. Extensive tests have been made by Striebeck, the results of which are embodied in the following formulas:

Let P = total load on bearing, in pounds;
P_1 = safe load for one roller;
l = length of roller, in inches;
d = diameter of roller, if cylindrical, and average diameter, if conical;
k = a coefficient which is 1,000 for hardened steel;
n = number of rollers in bearing.

Then, for a thrust bearing in which the load is supposed to be evenly distributed among the rollers,

$$P_1 = \frac{P}{n} \qquad (1)$$

For a cylindrical bearing in which the load acts in a direction normal to the journal axis, some of the rollers will carry the load while the rest are idle. Under these conditions, it is generally found that

$$P_1 = \frac{5P}{n} \qquad (2)$$

The safe load for one roller in either of these bearings is

$$P_1 = k\,d\,l \qquad (3)$$

41. Before constructing a roller bearing, it is necessary to ascertain the number of rollers required to support a given load. Having found this number, the diameters of the surfaces along which the rollers revolve must be calculated. Referring to Fig. 33, let

D_1 = diameter of circle on which the roller centers are located;

D_2 = diameter of circle tangential to the inner side of the rollers;

d = diameter of rollers;

C = diameter of circle circumscribing the rollers.

Then, $$C = D_1 + d \qquad (1)$$

and

$$D_2 = D_1 - d \qquad (2)$$

From the center o, draw radii $o\,l$ and $o\,m$ to the centers of two adjoining rollers. Connect these centers with the chord $l\,m$, and draw a line $o\,n$ to the point of tangency between the two rollers. Then, the angle $l\,o\,m = \frac{360}{n} = 2u$, and the angle $m\,o\,n$ is

$$u = \frac{180}{n} \qquad (3)$$

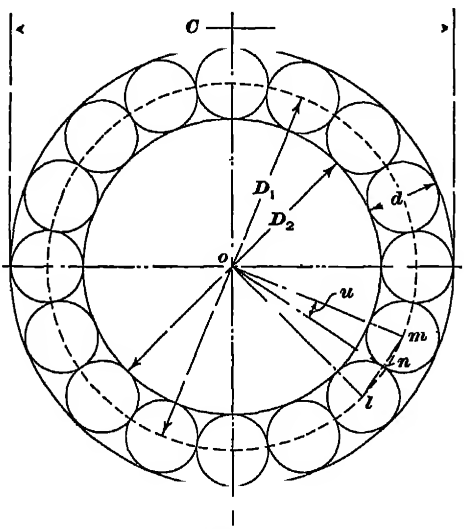


FIG 33

In the right triangle $m\,n\,o$, the side $m\,n = m\,o \times \sin u$; and as $m\,o = \frac{1}{2}D_1$ and $m\,n = \frac{1}{2}d$, it follows that $\frac{1}{2}d = \frac{1}{2}D_1 \sin u$, or $d = D_1 \sin u$. Hence,

$$D_1 = \frac{d}{\sin u} \qquad (4)$$

Having found the value of D_1 by means of formula **4**, the values of C and D_2 may be found from formulas **1** and **2**.

These formulas are based on the assumption that all the

rollers are in contact with one another. In practice, the cage in which the rollers are inserted leaves a certain amount of space between each pair of rollers, and this space must be considered in using formula **4**.

Let i = space between two adjoining rollers;

d_1 = roller diameter plus one-half the space on both sides (which is equal to the space between two adjoining rollers), or $d + i$.

Then, $\frac{1}{2} d_1 = \frac{1}{2} D_1 \sin u$; $d_1 = D_1 \sin u$

and $$D_1 = \frac{d_1}{\sin u} \qquad (5)$$

The value of the angle u, as found from formula **3**, is not affected by the addition of the space i. The values of C and D_2 for the new value of D_1 are still found by means of formulas **1** and **2**.

EXAMPLE.—A roller bearing is to contain twelve cylindrical rollers, each 1 inch in diameter. Assuming that there is to be a space of $\frac{1}{4}$ inch between each pair of rollers, find the diameter of the circle passing through the roller centers of the journals, and of the outside casing.

SOLUTION.—From formula **3**,

$$u = \frac{180}{12} = 15°, \text{ and } d_1 = d + i = 1 + .25 = 1.25 \text{ in.}$$

From formula **5**,

$$D_1 = \frac{1.25}{\sin 15°} = \frac{1.25}{.2588} = 4.83 \text{ in.}$$

From formula **1**,

$$C = D_1 + d = 4.83 + 1 = 5.83 \text{ in.}$$

From formula **2**,

$$D_2 = D_1 - d = 4.83 - 1 = 3.83 \text{ in. Ans.}$$

BALL BEARINGS

42. Distinctive Features.—Fig. 34 (*a*) illustrates a cylindrical roller *a b c d*, having hemispherical ends. If the length of this roller is reduced until the lines *a b* and *c d* coincide, as in (*b*), a ball is produced. Hence, **ball bearings** may be considered as a special form of roller bearings in which the length of the linear contact is reduced to a point. In reality, owing to the elasticity of the material, the contact of a roller with its support does not constitute a line, but

rather a rectangular surface. Similarly, the contact between a ball and its support is not a point, but a circular or an elliptical surface, depending on the shape of the support. In all theoretical considerations, however, it is assumed that there is either line or point contact.

In reducing the length of a roller to that of a ball, certain advantages are gained. By reason of the line contact of the rollers in a roller bearing, they suffer from a certain lack of adjustability and from stringent requirements as regards the exactness of the engaging surfaces and the parallelism of their axes. If the journal should show signs of deflection under load, it would materially affect the efficiency and durability of the bearing. In a ball bearing, the length of the line contact is reduced to a minimum; hence, the adjustability of the rolling parts is a maximum. If, in roller bearings, it is important that true rolling take place, and that material and finish be of the best, it is still more so in ball bearings, where the supporting area is more limited and therefore more sensitive to irregularities in relative motion and character of the contact surfaces.

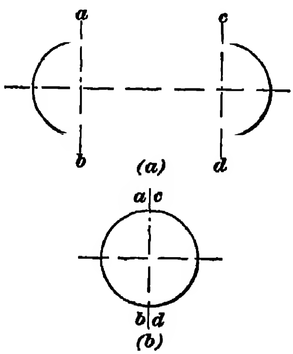


FIG. 84

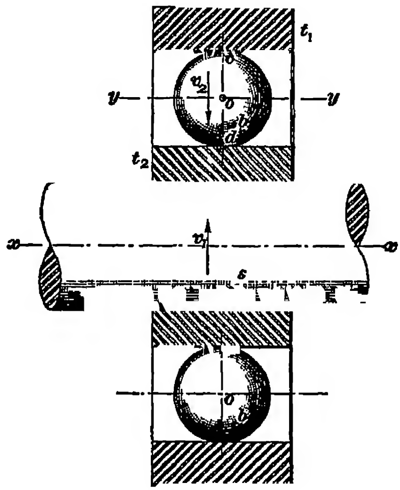


FIG. 85

43. Radial Bearings.—The fundamental form of a ball bearing is one in which the axes of rotation of the balls

are parallel with that of the journal, as shown in Fig. 35, in which s is the journal, t_1 the *outer race*, t_2 the *inner race*, and b, b the balls. The axis $x x$ of the journal is parallel with the axis of rotation $y y$ of any one ball. This bearing corresponds in principle to the roller bearing shown in Fig. 25, and is termed a **radial ball bearing**, because the two points of contact c and d lie on a line radial to the journal axis.

44. As a bearing of the form shown in Fig. 35 would not, without the use of supplementary means, prevent the balls from leaving their races, the latter are provided with grooves c_1 and c_2, as shown in Fig. 36. In cross-section, these grooves appear as circular arcs of radii r_1 and r_2, which are somewhat larger than the radius of a ball. Authorities differ as to the most suitable ratio between these radii, but Striebeck, from extensive experiments, recommends $r_1 = r_2 = \frac{2}{3} d$, where d is the diameter of a ball. These proportions are given the grooves shown in Fig. 36.

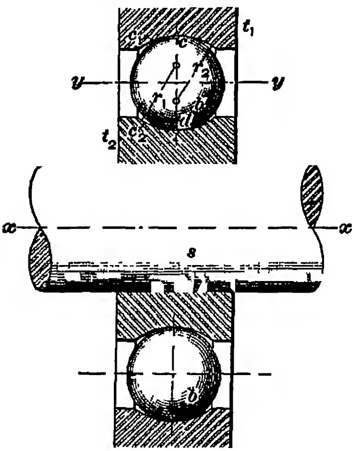


FIG 36

In a bearing of this form, the journal is still allowed a certain amount of axial play by reason of the flatness of the curves in the grooves. This form possesses practically the same quality of pure rolling inherent in the form shown in Fig. 35, but has in addition the advantage of a greater surface contact, because the curvature of the supporting surface conforms more nearly to that of ball. Ball bearings of this class have proved very efficient, even under heavy pressures. If the curvature of the ball race is made as shown in Fig. 36, the end thrust of the journal may be as high as one-third of the load, normal to the journal axis, without requiring any provisions for preventing end motion.

45. Another variation of Fig. 35 is shown in Fig. 37 (*a*), in which one of the rolling surfaces, instead of being cylindrical, consists of two conical surfaces joined along the line *fg*. This variation is termed a **three-point bearing,** there being three contact points *c*, *d*, and *e*, which are points of tangency between the ball and the ball races. In this example, the balls partake of another motion besides pure rolling. This motion may be seen more clearly in Fig. 37 (*b*) and (*c*), where the angle between the sides of the groove is made more acute. If while rolling along the race t_1 the ball *b* turns in the direction of the arrow v_1, then, at any moment, the ball revolves around a line *z z*, passing through the points *c* and *d*, as an instantaneous axis of rotation. Theoretically, these places of contact should be merely points, but, actually, they are two circular surfaces with *c* and *d*, respectively, as centers. By looking along the line *io*, the ball will appear as in Fig. 37 (*c*), revolving around the center *c* in the direction of the arrow v_2. The contact area at *c* may appear somewhat like the shaded circular surface, having a sliding friction similar to that found at the end of a revolving shaft resting on a plane surface.

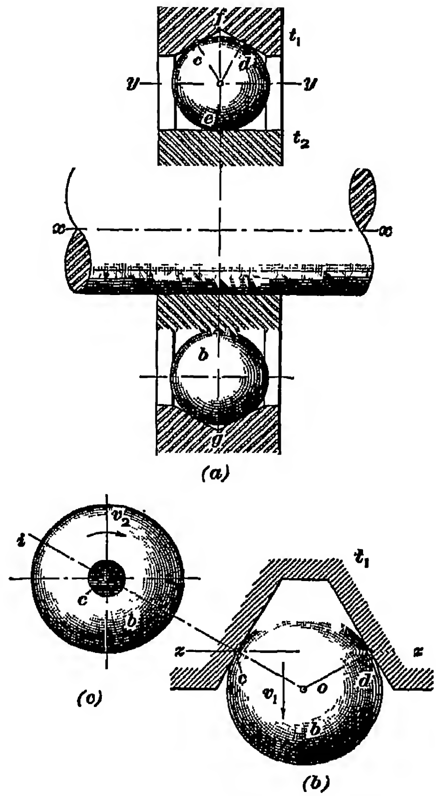


FIG 37

This supplementary motion of the ball around a point of contact, termed spinning, should be absent from any bearing where high efficiency is required. The spinning motion of a ball is somewhat similar to that of a top, which revolves around its axis while it advances over the supporting surface.

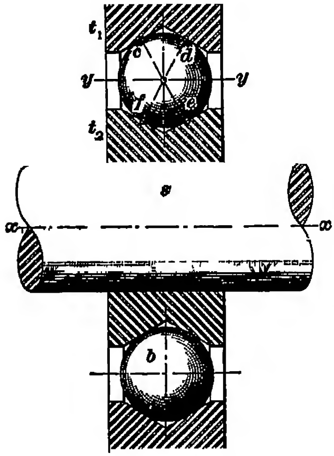


FIG 38

46. If the race t_1 also is provided with a groove, as in Fig. 38, then a **four-point bearing** is produced, having contact point c, d, e, and f, at each of which spinning takes place.

47. Thrust Bearings.—In **thrust bearings**, the axis of rotation yy of the ball is normal to the axis xx of the journal, as shown in Fig. 39. This bearing corresponds to the roller bearing illustrated in Fig. 27. In this form, the balls will not retain their positions unaided, and a cage, or perforated plate, must therefore be used for the purpose. The balls while following a circular path with radius R will revolve, or spin, once around the axis zz for each revolution around the axis xx.

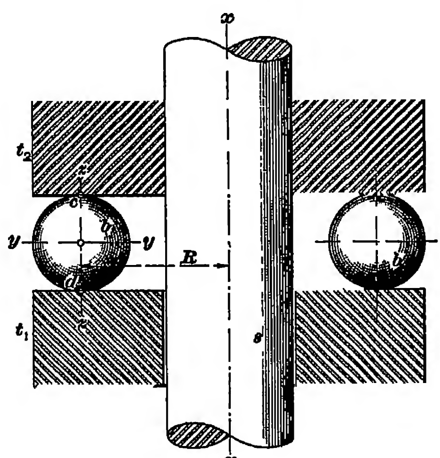


FIG 39

48. To prevent the balls from running off tangentially to their circular path, one of the races, as shown in Fig. 40, is provided with a groove

having sloping sides. As here shown, the ball will not have a true rolling motion for the following reasons: It may be considered part of two separate cones, one of which has its vertex at z_1 and rolls along the plane cz_1, while the other has its vertex at z_2 and rolls along the plane dez_2. Evidently it is impossible for two cones revolving around different axes to be rigidly connected through the ball b; hence, there must be a resultant of the two motions, represented by a common axis of rotation, having one point at o and another point somewhere between

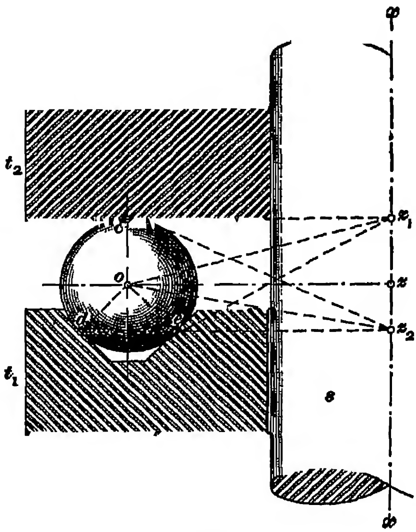


FIG 40

z_1 and z_2, as at z. There will be spinning at all three points of contact. If the race t_1 causes an excess of resistance to relative motion between it and the ball, then the spinning at c will be increased and the point z will be nearer z_2.

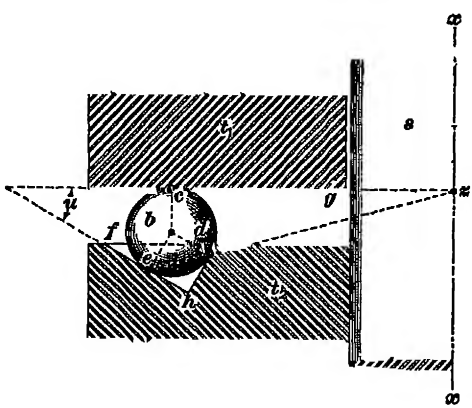


FIG. 41

49. Fig. 41 illustrates a rearrangement of Fig. 40, in compliance with the requirements for true rolling. The method of construction is as follows: If cg is to be one of the bearing surfaces, produce the line cg until it intersects

the axis $x x$ at z, which point will be the vertex of an imaginary cone. From z draw a line intersecting the ball b at two points, as d and e, which are to act as contact points between the ball and the race t_2. The sides $f h$ and $d h$ of the groove should be tangent to the ball at these points, as shown. While a certain freedom prevails in the choice of the contact points by varying the angle $c z e$, there is a position below which the point e must not fall. Experiments have shown that if the angle between any two adjoining surfaces, such as the angle u between the planes $e f$ and $c g$, are below a certain value, the ball tends to wedge itself between the surfaces. The angle should be at least 25°.

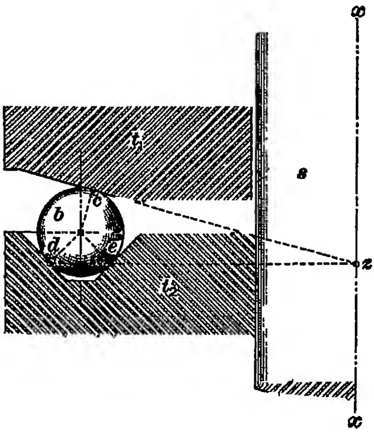


FIG 42

On the other hand, the angle u should not be too large, as in that case the angle $f h d$ would become too acute. This would give the ball a tendency to wedge itself in the groove at h, thereby greatly increasing the pressure at the points d and e where spinning takes place.

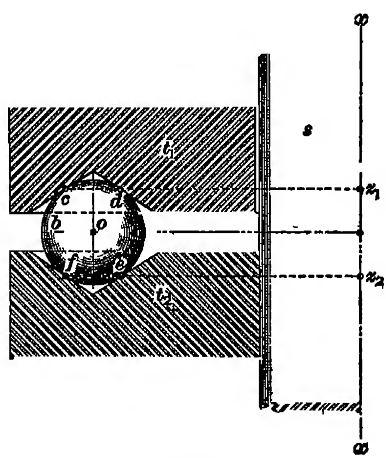


FIG. 43

50. The position of the vertex z may be changed without affecting the action of the ball. In Fig. 42, for instance, the point z has been lowered so as to make the lower side of the imaginary cone normal to the axes $x x$. If

the construction is made as explained in Art. **49,** the conditions for practically true rolling are also here fulfilled.

51. Fig. 43 shows a form of ball bearing that has been extensively used in various machines, although it is very inefficient. In this instance, the lines $c z_1$ and $f z_2$, which intersect the ball at the points c, d, e, and f, are parallel. From the foregoing, it is clear that if the ball is supposed to be part of a cone, its sides should roll simultaneously along the surfaces $c z_1$ and $f z_2$: but as it cannot do so, there must be some sliding at one or more points, which should be avoided.

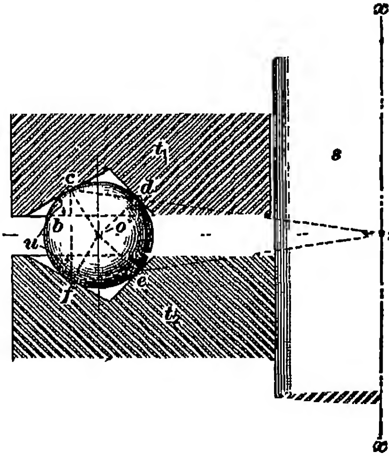


FIG. 44

52. In order to insure true rolling, the bearing should be constructed as shown in Fig. 44. When produced, the lines $c d$ and $e f$ passing through the four contact points should have a common point of intersection z located on the axis $x x$. Of course, the axis $o z$ need not be normal to $x x$; the point z can be moved to some extent above or below its present position, but care must be taken that neither of the angles u, u is too small. In the diagram, each angle is 30°.

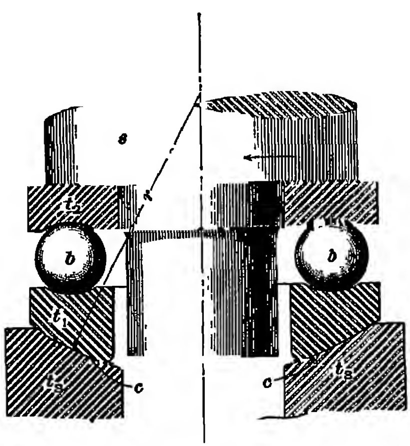


FIG 45

53. Fig. 45 shows a thrust bearing with ball races of the form shown in Fig. 36. The surface of plate t_1 is curved at *c* to the radius *r*, thus providing a certain amount of adjustability relative to the main bearing t_2. This is necessary if the load is to be evenly distributed among all the balls *b*. The plate t_2 is fastened to the shaft *s*.

54. Hub Bearings.—There is still another class of ball bearings, known as **hub bearings**, which may be considered as a combination of a radial and a thrust bearing. This type differs from the regular thrust bearing in that the end thrust does not proceed from without the bearing, but

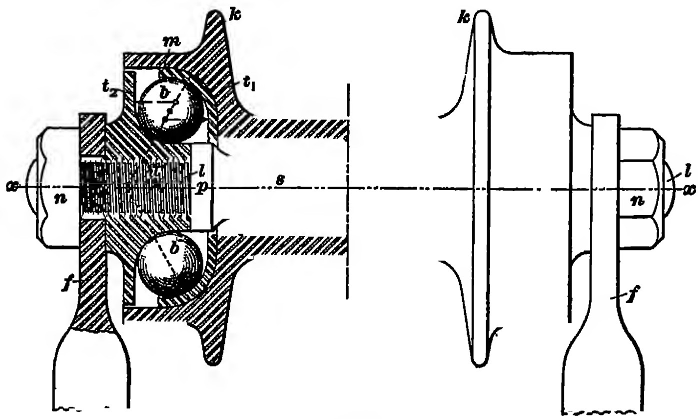


FIG. 46

from within, as a result of the arrangement of the ball races. These bearings are always used in pairs, so that the end thrust produced by one is neutralized by that produced by the other. The evolution of the bicycle served to bring this class of bearing into prominence.

In order that its action may be better understood, the type of hub bearing used in a bicycle wheel is shown in Fig. 46. The stationary spindle, or axle, *s* of a rear wheel has threaded ends *l*, *l*, which engage with nuts *n*, *n* and cones t_2 on each end. The cone shown is permanently screwed against the shoulder *p*; at the other end of *s* there is no shoulder, thus permitting the other cone to be moved longitudinally on *l*

in order that the bearings may be adjusted. By screwing the nuts *n* tightly against the rear forks *f*, *f* of the bicycle frame, they are forced against the cones, preventing them from turning and at the same time fastening the spindle to the fork. The balls *b*, *b* transmit the load supported by the spindle to the cups or ball races *m* inserted in each end of the hub t_1. The hub is provided with flanges *k*, *k*, to which are fastened the inner ends of the spokes, not shown, which connect the hub and the wheel rim.

55. Fig. 47 shows, diagrammatically, one end of a **two-point hub bearing,** t_1 being the hub, t_2 the cone, *s* the spindle, or axle, *b* the ball, and *c* and *d* the two contact points

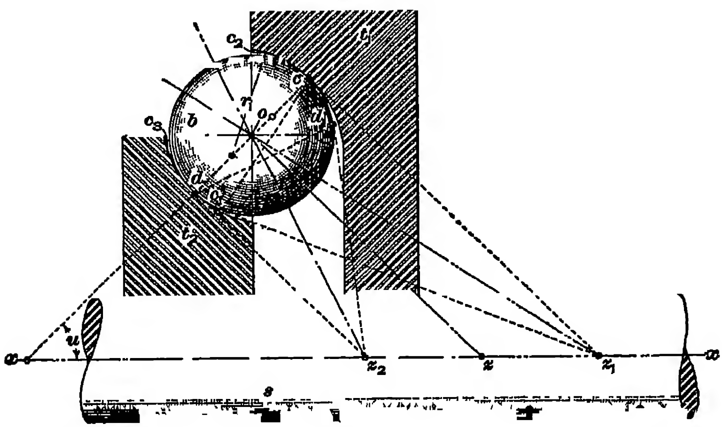


FIG 47

with the races c_2 and c_3. The principles on which the action of this bearing is based are similar to those explained in Art. **48.** Drawing the tangents $c z_1$ and $d z_2$, and considering the ball as part of two cones with vertexes z_1 and z_2, respectively, it is again evident that true rolling cannot take place at both planes $c z_1$ and $d z_2$. In the present instance, the ball will revolve around an axis situated somewhere between $o z_1$ and $o z_2$, as, for example, at $o z$. Spinning will therefore occur at both *c* and *d*. In a bearing having the relative proportions shown in Fig. 47, it is found that for each revolution

of the hub t_1 around the axis $x x$, the ball will make about four-fifths revolution around the axis $c d$. The amount of this rotary motion will increase with an increase of the angle u.

The radius r_1 varies between nine-sixteenths to three-fourths of the ball diameter. On account of this curvature the bearing cannot be so closely adjusted that the rolling paths of the ball will be strictly defined. For this reason, the hub has a certain amount of longitudinal play. This play will increase with an increase of the angle u, which, in a bicycle, may be as much as 60°, because there the side pressure is of secondary importance.

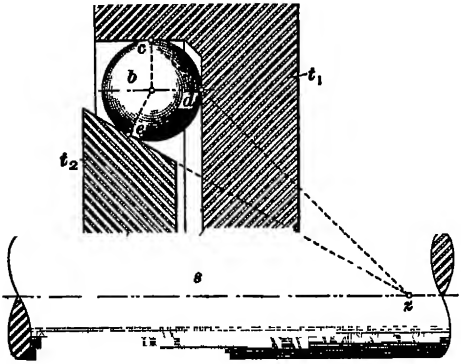


FIG 48

56. Figs. 48 and 49 are diagrams illustrating **three-point** and **four-point bearings**, respectively. In Fig. 48, the contact points lie on the sides of the imaginary cone $c d z e$, and in Fig. 49, on the cone $c d z e f$. The method of construction is similar to that shown in Figs. 42 and 44.

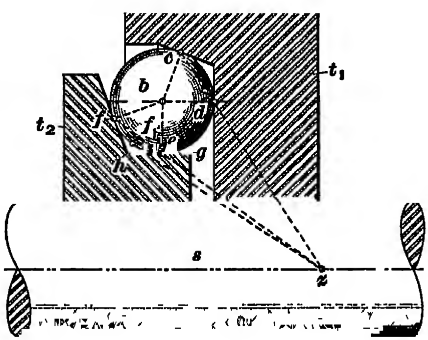


FIG 49

57. By referring to the four-point bearings shown in Figs. 38, 44, and 49, it should be noted that these bearings require great accuracy in both their construction and adjustment in order to operate correctly. The reason for this may be seen, for instance, from Fig. 49. It is evident that

if the cylindrical surface gh should have a radius somewhat smaller than shown, the ball would fail to make contact at e. The ball would still retain its position, but would make a three-point contact, the point f assuming the functions of the points e and f, but at a greatly increased pressure. It will be further observed that under these circumstances the position of point f is incorrect; it should occupy a position somewhere between points e and f, as at f_1, the point of tangency between a line gf_1 and the ball.

58. By comparing the two-, three-, and four-point bearings, it will be seen that in the last the balls are confined to a definite path, but that the bearing demands a very careful construction. In a two-point bearing, the path may vary on either bearing surface; hence, the bearing possesses a maximum of adjustability. A three-point bearing occupies an intermediate position, the rolling path being definitely determined for one surface, while that on the other has a certain amount of freedom.

59. Diameter of Ball Circle.—The diameter of the circle passing through all the centers o, Fig. 35, of the balls in a bearing is found, as described in Art. **41**, by means of formula **4**. This is under the supposition that the balls touch one another; but the conditions of easy running demand that a certain amount of clearance be left between the balls. The space left between each pair of balls varies with the total number of balls in the bearing. This space may be given a trial value of .005 inch, which is satisfactory if the sum of all the spaces does not exceed one-fourth of the ball diameter, otherwise the value should be reduced accordingly. The diameter of the ball circle is then found by means of formula **5**, Art. **41**.

EXAMPLE.—A ball bearing contains fourteen balls, each $\frac{1}{4}$ inch in diameter. Calculate the diameter of the ball circle if the balls are given suitable clearance.

SOLUTION.—If a clearance of .005 in. is given each pair of balls, the total clearance is $14 \times .005 = .07$ in.; as the value $\frac{d}{4} = \frac{.25}{4} = .0625$ in.,

the total clearance should be reduced to this amount Therefore, the clearance for each pair of balls should be .0625 ÷ 14 = .0045 in.

From formula **5**, Art. **41**, $D_1 = \frac{d_1}{\sin u}$. Here, d_1 = .25 + .0045 = .2545 in. From formula **3**, Art. **41**, $u = \frac{180}{n} = \frac{180}{14} = 12° 51'$. Hence,

$$D_1 = \frac{.2545}{\sin 12° 51'} = \frac{.2545}{.22240} = 1.1443 \text{ in. Ans.}$$

60. Safe Load on Ball Bearings.—In thrust bearings of the type shown in Fig. 45, all the balls are supposed to take equal portions of the load, it being assumed that they are uniform in size. In bearings where the load acts in a direction normal to the axis of the journal, as in Figs. 36 and 46, the balls will be idle during one part of their travel around the journal and the load will be supported by two, three, or four balls, depending on the total number.

Let P = total load on bearing, in pounds;
P_1 = safe load for one ball;
n = number of balls in bearing, varying from ten to twenty;
d = diameter of the balls, in inches;
k = a coefficient that varies with the form of the bearing and the number of contact points. For hardened steel it has the following values: 400 for a straight-line race, as in Fig. 35; 700 for three- and four-point bearings; 1,400 for a two-point radial bearing, as in Fig. 36.

Then,
$$P_1 = k d^2 \qquad (1)$$

For a thrust bearing,
$$P = n P_1 \qquad (2)$$

For a bearing in which the load acts normal to its axis,
$$P_1 = \frac{5P}{n} \qquad (3)$$

For the radial type of bearings, as in Fig. 36, the load is independent of the speed if the latter is uniform and below 3,000 revolutions per minute. If sudden changes of load and speed take place, then the coefficient k should be lowered.

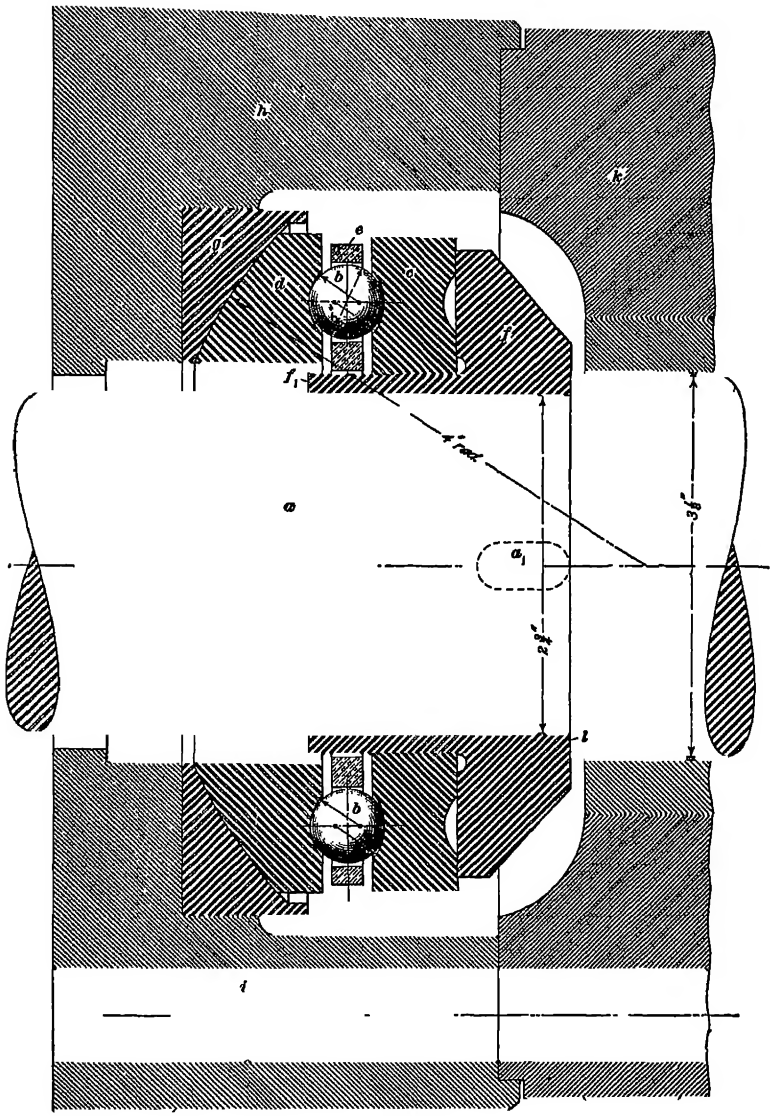


FIG. 50

EXAMPLE.—In a two-point ball bearing, as in Fig 36, with fourteen balls each $\frac{1}{4}$ inch in diameter, the load is normal to the journal. Find the safe load.

SOLUTION.—From formula **1**,

$$P_1 = 1{,}400 \times .25^2 = 87.5 \text{ lb.}$$

From formula **3**, solving for P,

$$P = \frac{14 \times 87\ 5}{5} = 245 \text{ lb. Ans.}$$

61. Examples of Ball Bearings.—Figs. 50 and 51 illustrate two types of ball bearings. They are not given as standards of design, but simply as examples of bearings that have worked efficiently under the conditions for which they were intended. Both are two-point bearings with grooves of the form shown in Fig. 36.

Fig. 50 shows a longitudinal section of one of the thrust bearings of a mine pump that was required to be in constant operation, the bearing sustaining an end thrust of about 2,000 pounds and the shaft revolving at a speed of 890 revolutions per minute. A pump with this style of bearing ran continuously for 289 days.

In the illustration, a is the pump shaft, which is driven by an electric motor, and b, b the $\frac{5}{8}$-inch balls, of which there are sixteen, placed in holes in the cage e. The two halves of the ball bearing are lettered c and d. In its earlier form the ring c rested directly on the shaft, but it was found that the intensity of pressure at the shoulder l was too great, resulting in a partial crushing of the material and a tendency of the ring to tip—hence, the insertion of the bushing f with the extension f_1, and the key a_1. The end surface of the other ring d is made spherical to allow it a certain amount of adjustment; it rests against the concave side of the ring g. The casing h of the bearing is bolted to the main body k of the pump by bolts inserted through holes i.

62. In Fig. 51 is shown a pedestal bearing with fifteen balls, each $\frac{7}{8}$ inch in diameter, supporting a shaft $2\frac{9}{16}$ inches in diameter; (a) is a longitudinal section and (b), a side view, partly in section, the section being taken along the line $x\,x$ in (a). In this bearing, the balls b are not evenly

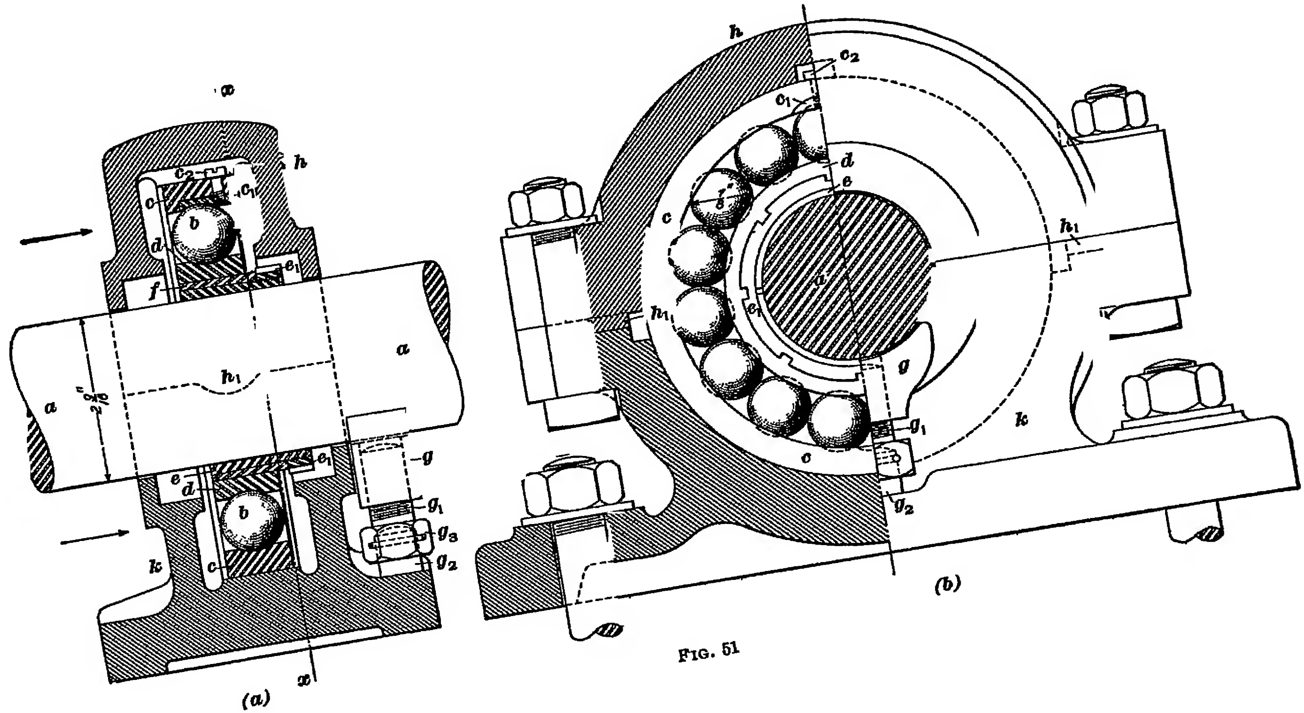


FIG. 51

loaded, being idle while passing over the upper side of the shaft. The ball races are shown at *c* and *d*, the latter resting directly on the shaft if there is little vibration present. Otherwise, this race is fastened in the manner shown in (*a*). The sleeve *f* has a conical hole into which is fitted the split conical bushing *e*, one end of which is threaded to engage with the ring e_1. By turning up ring e_1, the bushing is drawn outwards, locking the ring securely to the shaft.

As rings *c* and *d* are not split, some means must be provided for the insertion or removal of the balls. For this purpose, a recess large enough to allow a ball to pass is made in the ring *c*. Into this recess is fitted a piece c_1, which is held by the screw c_2. The recess should be located at a place where the piece c_1 will not be subjected to any pressure from the balls. The outer casing is divided into two parts *h* and *k*, and is held together by bolts as indicated, the tongues h_1 insuring the correct alinement of the two parts. Between the casing *h* and the ring *c* there is a play of .004 inch, to allow for longitudinal adjustment of the rings and for expansion in case the parts become warm.

The chair *g* serves the purpose of supporting the shaft while the bearing is being inspected. The threaded end of the bolt g_1 is screwed into the chair, while the head revolves loosely on the stud g_2, to which it is connected by a pin, as indicated. By turning the head g_2, the chair may be raised until it supports the shaft, thus permitting the removal of the ball races.

In a more recent form of this bearing, the piece c_1 is omitted and the ring *c* is made solid throughout. To permit the insertion of the balls, their number is decreased, and they are inserted while the ring *c* is placed eccentrically relative to *d*. After the ball races are arranged concentrically, elastic distance pieces are inserted between each pair of balls, thus spacing them evenly around the shaft

It is important that both roller and ball bearings be incased in such a manner as to retain a lubricant and exclude grit and dust. The lubricant will also prevent rust, but it should be free from acid.

MACHINE DESIGN

Serial 997D (PART 4) Edition 1

SHAFTS, COUPLINGS, AND SPRINGS

SHAFTS

VARIETIES OF STRESSES IN SHAFTS

1. Shafts may be divided into three classes, according to the kinds of stresses to which they are subjected:

1. Shafts subjected chiefly to torsion or twisting, as, for example, line shafting in mills and shops, and, in general, shafts used to transmit power.
2. Shafts subjected chiefly to bending action, such as the axles of gears, etc.
3. Shafts subjected to both twisting and bending, such as engine crank-shafts.

When a shaft is intended simply to transmit power from one point to another, it does so by means of its resistance to torsion. Conditions under which a shaft is subjected to torsion alone exist only when it, if horizontal, is supported by a sufficient number of bearings, so as to prevent any bending tendency; also, that no pulleys or gears are supported by the shaft subjecting it to bending stresses. If the shaft is vertical, its weight must be supported by thrust bearings along its length, to prevent a tendency to buckle and thus to prevent bending stresses.

In order that the turning force shall not be accompanied by bending forces at the place where power is transmitted to the shaft, it is necessary that the driving power be applied at least at two points diametrically opposite each other, as in Fig. 1. This illustration represents, diagrammatically, an overhanging shaft *a* projecting beyond the bearing *g*, and having at its end the pulley *b* as a driving member. P, P are the turning forces acting at the distance R from the center of the shaft. The turning moment T is therefore $P \times R + P \times R = P \times 2R$.

Turbines and electric motors are examples of machines by means of which a shaft may be driven by torsion alone; that is, without bending, as the turning forces are applied

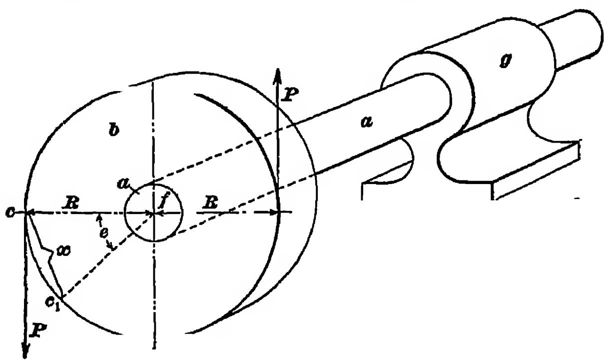


FIG. 1

at numerous points on the circumference of the revolving member.

2. When the turning force is applied at one point only —at one side of the shaft—as is the case with most machines, then the shaft is exposed not only to torsion, but also to bending. An example coming under this class is the combination of a connecting-rod and a crank-shaft. Another example, but one in which the turning and bending forces are constant for each revolution, is that of a belt and pulley, as shown in Fig. 2. Here, T_1 represents the tension of the tight and T_2 that of the slack side of the belt. The turning force P is equal to their difference, or $P = T_1 - T_2$.

In this case, the force P not only tends to twist the shaft, but the combined tensions T_1 and T_2 also tend to bend it. The twisting moment T is PR, when R is the radius of the pulley.

If the pulley shaft is overhanging, as in Fig. 1, the bending moment M varies from zero at the pulley to a maximum at the supporting bearing. If the pulley is located between two bearings, then the bending moment is a maximum at the pulley.

In Fig. 3, a gear is attached to the shaft and driven by

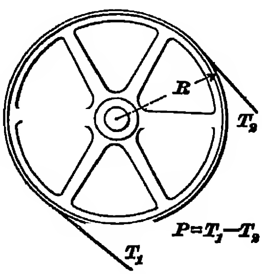


FIG. 2

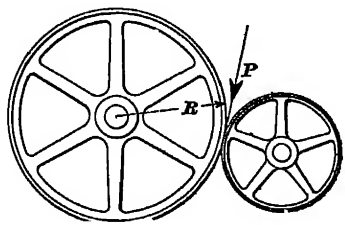


FIG 3

another gear, in which case P will act at the pitch circle with a leverage equal to the radius R.

SHAFTS SUBJECTED TO TORSION

3. Moments of Inertia.—In the case of pure torsion, the stresses produced are those of shearing, the plane of shearing being at right angles to the axis of rotation. The moment of inertia found with reference to this axis, termed the **polar moment of inertia,** is generally denoted by the letter J, and has been referred to in *Strength of Materials*, Part 2. If I is the *rectangular moment of inertia*, then for a solid circular section,

$$J = 2I = 2 \times \frac{\pi d^4}{64} = \frac{\pi d^4}{32}$$

4. Torsional Strength of a Solid Round Shaft.—The ultimate resistance of a beam to bending must be equal to

the bending moment; or, according to the formula given in *Strength of Materials*, Part 2,

$$M = S_b \frac{I}{c}$$

If $I = \frac{\pi d^4}{64}$, and S_4 is the safe fiber stress in flexure, the resisting moment of a round shaft will be

$$S_4 \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = S_4 \frac{\pi d^3}{32}$$

or,

$$M = \frac{S_4 d^3}{10.2} \qquad (1)$$

The resisting moment of a shaft to twisting is found by a formula similar to formula **1**, except that the polar moment of inertia J is substituted for the rectangular moment of inertia I.

Let T = twisting moment, in inch-pounds;
P = turning force, in pounds;
R = length of lever arm of turning force, in inches;
S_s = ultimate shearing strength of shaft, in pounds;
S_2 = safe shearing strength of shaft, in pounds;
J = polar moment of inertia of cross-section of shaft about neutral axis;
c = distance from neutral axis to outermost fiber;
f = factor of safety.

Then, the moment that resists twisting is $\frac{S_s J}{c}$, and the safe moment is $\frac{S_s J}{f c} = \frac{S_2 J}{c}$, since $S_2 = \frac{S_s}{f}$. Therefore,

$$T = \frac{S_2 J}{c}$$

Inserting the value given for J in Art. **3**, and that of $c = \frac{d}{2}$, then the resisting moment is

$$S_2 \frac{\frac{\pi d^4}{32}}{\frac{d}{2}} = S_2 \frac{\pi d^3}{16}$$

Equating the twisting and resisting moments,

$$T = PR = S_s \frac{\pi d^3}{16} = \frac{S_s d^3}{5.1} \qquad (2)$$

whence

$$d = \sqrt[3]{\frac{5.1\, PR}{S_s}} = 1.72 \sqrt[3]{\frac{PR}{S_s}} \qquad (3)$$

Instead of using the twisting moment PR, it may be more convenient at times to use the horsepower transmitted by the shaft and the number of revolutions per minute. From the formula referring to keys in *Machine Design*, Part 2, where H is the horsepower and N the number of revolutions per minute,

$$T = PR = 63{,}025 \frac{H}{N}$$

Substituting this value of PR in formula **3**,

$$d = 1.72 \sqrt[3]{\frac{PR}{S_s}} = 1.72 \sqrt[3]{\frac{63{,}025\ H}{N S_s}}$$

or,

$$d = 68.45 \sqrt[3]{\frac{H}{N S_s}} \qquad (4)$$

The values for S_s may be taken as 3,400 for cast iron, 6,800 for wrought iron, and 9,000 for steel. Ordinarily, these values are the ones that are used in good practice. Under exceptional circumstances, it may be necessary to use lower values, particularly if the shaft is subjected to shocks or if the stress changes suddenly or violently.

5. Angle of Torsion, or Twist, of a Solid Round Shaft.—As shown in *Strength of Materials*, Part 2, the angle of twist is directly proportional to the twisting moment, if the material is not strained beyond the elastic limit. Let the angle of torsion be indicated in Fig. 1 by the angle $c f c_1 = e$. If the amount of twist, or distortion, $c c_1 = x$ is produced in a length l of the shaft, it can be proved by higher mathematics that

$$\frac{x}{l} = \frac{S_s}{E_s} \qquad (1)$$

In this formula, E_s is the modulus of torsional elasticity, or of elasticity in shear. E_s may also be said to be the ratio

between the unit torsional shearing stress and the strain, or distortion, per unit length, or

$$E_s = \frac{S_s}{\frac{x}{l}} = \frac{S_s l}{x}$$

The circumference of the shaft is πd, and the angle of twist is e. The arc included by any angle e is $x = \frac{\pi d e}{360}$. Inserting this value of x in formula **1**,

$$\frac{\pi d e}{360 l} = \frac{S_s}{E_s}, \text{ hence } e = \frac{360 S_s l}{\pi d E_s}$$

From formula **2**, Art. **4**, $T = \frac{S_s d^3}{5.1}$ and $S_s = \frac{5.1 T}{d^3}$. Substituting this value in the formula $e = \frac{360 S_s l}{\pi d E_s}$,

$$e = \frac{360 \times \frac{5.1 T l}{d^3}}{\pi d E_s}$$

or,

$$e = \frac{584 T l}{E_s d^4} \qquad (2)$$

The value of E_s for steel is about 11,000,000, for cast iron 6,000,000, and for wrought iron 10,000,000. If the horsepower transmitted by the shaft and the number of revolutions per minute are given, then by substituting the value of $T = 63{,}025 \frac{H}{N}$ in formula **2**,

$$e = \frac{36{,}806{,}600 H l}{N E_s d^4} \qquad (3)$$

If the shaft is made of steel, formula **2** will be

$$e = \frac{.0000531 T l}{d^4} \qquad (4)$$

and

$$T = \frac{e d^4}{.0000531 l} \qquad (5)$$

Substituting the value $E_s = 11{,}000{,}000$ in formula **3**,

$$e = \frac{3.346 H l}{N d^4} \qquad (6)$$

Example.—A 2-inch steel shaft 20 feet long transmits 25 horsepower at a speed of 120 revolutions per minute. Find the angle of torsion of the shaft.

SOLUTION.—By formula **6**,

$$e = \frac{3\,346 \times 25 \times 240}{120 \times 16} = 10.46°. \quad \text{Ans.}$$

6. Limits of the Angle of Torsion.—The diameter found by means of formula **4**, Art. **4**, will give the shaft sufficient *strength*, but it may be deficient in torsional *stiffness*. Under these conditions, a long shaft of small diameter may act as a spring, with the result that one or both ends will revolve at a variable velocity. This is objectionable when the conditions are such that a shaft should run at a constant velocity under variable loads. It is therefore customary to add from $\frac{3}{8}$ to $\frac{5}{8}$ inch to the diameter found according to formula **4**, Art. **4**, in order to increase the stiffness of the shaft and to prevent vibration.

More accurate results may be obtained by calculating the angle of torsion, which should not exceed 1° for a length of shaft equal to twenty times the diameter. Testing the results found in the example of Art. **5** by this rule, the angle of torsion should not exceed $\frac{l}{20\,d} = \frac{240}{20 \times 2} = 6°$. As the angle is 10.46°, the diameter should be increased to give greater stiffness.

The foregoing statement may be incorporated directly into a formula by inserting the values $e = 1°$, and $l = 20\,d$ in formula **5**, Art. **5**. Then,

$$T = \frac{d^4}{.0000531 \times 20\,d} = \frac{d^3}{.00106}$$

and $$d^3 = .00106\,T$$

Whence $$d = .102\sqrt[3]{T} \qquad \textbf{(1)}$$

If the horsepower to be transmitted by the shaft is known, then the value $T = 63{,}025\,\frac{H}{N}$ may be inserted in formula **1**, and

$$d = 4.059\sqrt[3]{\frac{H}{N}} \qquad \textbf{(2)}$$

Recalculating the example given in Art. **5** by formula **2**,

$$d = 4.059\sqrt[3]{\frac{25}{120}} = 2.406$$

or, $$d = 2\tfrac{13}{32}, \text{ say } 2\tfrac{7}{16}, \text{ inches}$$

By comparing formula **2**, Art. **4**, with formula **5**, Art. **5**, it will be seen that for *strength* the diameter of a shaft is proportional to the *cube root* of the twisting moment, and that for *stiffness* the diameter of a shaft is proportional to the *fourth root* of the twisting moment.

7. Torsional Strength of a Hollow Shaft.—For a hollow round shaft, $J = \frac{\pi(d_1^4 - d_2^4)}{32}$, d_1 and d_2 being the outside and inside diameters, respectively. Inserting this value of J in the formula $T = \frac{S_s J}{c}$ and making $c = \frac{d_1}{2}$, then

$$T = PR = \frac{S_s \frac{\pi(d_1^4 - d_2^4)}{32}}{\frac{d_1}{2}}$$

or,

$$T = \frac{S_s(d_1^4 - d_2^4)}{5.1\, d_1} \qquad (1)$$

If the ratio $\frac{d_2}{d_1}$ is represented by n, then $n = \frac{d_2}{d_1}$ and $d_2 = n\, d_1$. Inserting this value of d_2 in the preceding formula,

$$T = PR = \frac{S_s[d_1^4 - (n\, d_1)^4]}{5.1\, d_1}$$

$$= \frac{S_s d_1^4(1 - n^4)}{5.1\, d_1}$$

$$= \frac{S_s d_1^3(1 - n^4)}{5.1}$$

from which

$$d_1 = 1.72\sqrt[3]{\frac{PR}{S_s(1 - n^4)}} \qquad (2)$$

If the horsepower transmitted by the shaft is given, then, by following the method given in Art. **4**,

$$d_1 = 68.45\sqrt[3]{\frac{H}{S_s N(1 - n^4)}} \qquad (3)$$

EXAMPLE —A hollow steel shaft is to transmit 2,000 horsepower at 160 revolutions per minute. If S_s = 9,000 pounds, and the ratio between the inside and outside diameters is as 1 : 2, what will be the outside diameter?

SOLUTION —Substituting these values in formula **3**,

$$d_1 = 68\ 45\sqrt[3]{\frac{2,000}{9,000 \times 160\,(1 - \ 5^4)}} = 7\ 803, \text{ or } 7\tfrac{13}{16}, \text{ in., nearly.} \quad \text{Ans.}$$

8. Angle of Torsion of a Hollow Round Shaft.—The angle of torsion of a hollow round shaft may be found by means of a formula resembling formula **2**, Art. **5**, the only difference being that the value $d_1^4 - d_2^4$ is substituted for d^4. Thus,

$$e = \frac{584\,T\,l}{E_s\,(d_1^4 - d_2^4)} \qquad (1)$$

Similarly, formula **3**, Art. **5**, changed so as to apply to hollow shafts, will be

$$e = \frac{36,806,600\,H\,l}{N\,E_s\,(d_1^4 - d_2^4)} \qquad (2)$$

9. Ratio of Strength of Solid and Hollow Shafts. A close study of formulas **2** and **3**, Art. **7**, shows that the strength of a shaft is not seriously diminished by removing the central part. For instance, if the ratio of $d_2 : d_1 = n = \frac{1}{2}$, then the factor $1 - n^4$ becomes $1 - \frac{1}{16}$, showing that the strength of the shaft has been reduced by only $\frac{1}{16}$, or that the ratio of the strength of the solid to that of the hollow shaft is 1 : .9375. Similarly, if $n = .6$, then $1 - n^4 = .8704$, indicating that the ratio is as 1 : .87.

10. In a solid shaft, the poorest material and that which is least effective in resisting torsion is usually found at the center. In a hollow shaft, however, this material is removed, thereby increasing the average strength of the material that remains. Also, by forging shafts of large diameter hollow, the material is more thoroughly worked and will consequently have greater strength. Hollow steel shafts are much used for marine engines.

11. A hollow shaft will have a strength equal to that of a solid shaft if the moduli of the two sections are equal, or if formula **2**, Art. **4**, is equal to formula **1**, Art. **7**. That is.

$$\frac{S_s\,d^3}{5.1} = \frac{S_s\,(d_1^4 - d_2^4)}{5.1\,d_1}$$

or,

$$d^3 = \frac{d_1^4 - d_2^4}{d_1} \text{ and } d_1 = \frac{d_1^4 - d_2^4}{d^3}$$

If $d_2 = n d_1$, then $d_1 = \frac{d_1^4 - (n d_1)^4}{d^3}$ and $d_1^3 = \frac{d^3}{1 - n^4}$.

Or, $$d_1 = d\sqrt[3]{\frac{1}{1 - n^4}}$$

EXAMPLE.—A solid shaft has a diameter of 9 inches and is to be replaced by a hollow shaft of equal strength in which the ratio $d_2 : d_1 = n = .4$ Find the outside diameter

SOLUTION —By substituting these values in the formula,

$$d_1 = 9\sqrt[3]{\frac{1}{.9744}} = 9.078, \text{ or } 9\tfrac{1}{8}, \text{ in. Ans.}$$

12. Line Shafting.—**Line shafting** is a term applied to the long and continuous lines of shafting used in mills, factories, and shops for the distribution of power. The shaft is principally strained by torsion, but, in addition, there is always a bending action due to the weight of the shaft itself and the pulleys carried by it, and to the tension of belting or the thrust of gearing.

In calculating the diameter of a shaft for a given twisting moment, two things must be considered: *strength* and *stiffness*. Very large shafts or short shafts need be calculated for strength only; but in long lines of shafting of small diameter attention must be paid to stiffness and rigidity.

13. Speed of Line Shafting.—The **speed of a shaft** is fixed largely by the speed of the driving belt or by the diameters of the pulleys on it. In general, machine-shop shafts run from about 120 to 150 revolutions per minute, shafts driving woodworking machinery from about 200 to 250 revolutions per minute, while in cotton mills, the practice is to make the shaft diameter smaller and to run at a higher speed. Line shafts should generally not be less than $1\frac{3}{4}$ inches in diameter.

14. Distance Between Bearings.—The **distance between bearings** should not be great enough to permit a deflection of more than $\frac{1}{100}$ inch per foot of length. Hence, when the shaft is heavily loaded with pulleys, the bearings must be closer than when it carries only a few pulleys. Pulleys that give out a large amount of power should be placed as near to hangers as possible.

In Table I are given the maximum distances between the bearings of different sizes of continuous shafts used for the transmission of power.

TABLE I

MAXIMUM DISTANCE BETWEEN BEARINGS OF CONTINUOUS SHAFTS

Diameter of Shaft Inches	Distance Between Bearings Feet		Diameter of Shaft Inches	Distance Between Bearings Feet	
	Wrought-Iron Shaft	Steel Shaft		Wrought-Iron Shaft	Steel Shaft
2	11	11.5	6	19	20
3	13	13.75	7	21	22.25
4	15	15.75	8	23	24
5	17	18.25	9	25	26

SHAFTS SUBJECTED TO BENDING

15. The axles of large waterwheels, gear-wheels, etc. are loaded transversely and are not generally subjected to torsional stresses. They may therefore be treated as beams transversely loaded, and the bending stresses in the same may be ascertained analytically or by a combination of analytical and graphical methods.

When calculating bending moments for shafts and axles, it is customary to consider the center of the bearing as the center of moments, on the supposition that the fit between the journal and its bearings is of such nature as not to restrain a possible deflection of the journal. The strength of the shaft may therefore be calculated as if the shaft simply rested on two supports at the centers of the bearings. While this is not entirely correct, the supposition is on the safe side. In cases where the shaft supports a pulley, gear, etc., the latter being driven tight on the shaft, it is assumed that the bending stresses do not go much beyond the edge of the hub. To allow for a sufficient margin of safety in this case, it is also supposed that the center of the bending

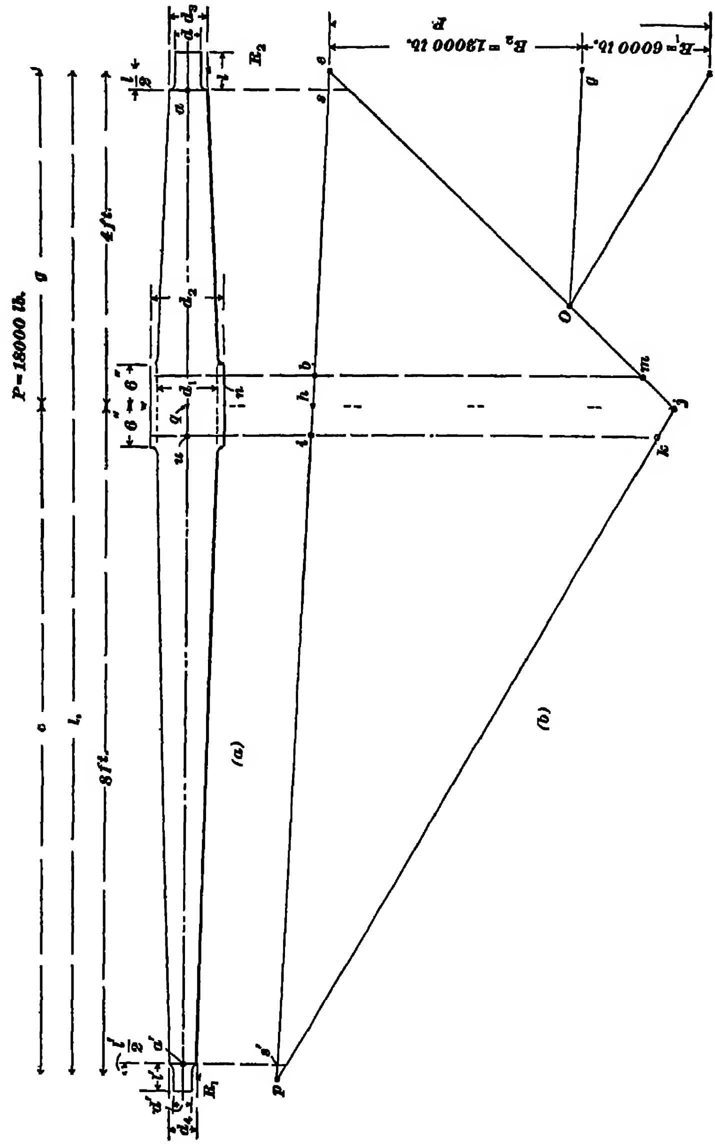


FIG 4

moment is located at the center of the hub, though some designers prefer to locate it nearer the end of the hub.

16. Axles Supporting One Load.—The methods used in finding the reactions and bending moments for axles supporting one load are best shown by an example.

EXAMPLE—An axle 12 feet long between centers of bearings carries a wheel weighing 9 tons. The wheel is 4 feet from one end of the shaft and the length of its hub is 12 inches. The axle is of wrought iron Required, the dimensions.

NOTE—Two solutions of this problem will be given, the first wholly analytical and the second partly graphical and partly analytical.

SOLUTION 1—Let Fig. 4 (*a*) represent, temporarily, the axle in question, P being the load of 9 tons at the distance indicated from either bearing, and R_1 and R_2 the reactions. To find the value of the reactions, proceed as follows:

Take, for instance, the center of the right-hand bearing as the center of moments and find the reaction at the center of the other bearing Then,

$$P \times g = R_1 \times l_1 \text{ and } R_1 = \frac{Pg}{l_1} = \frac{18{,}000 \times 48}{144} = 6{,}000 \text{ lb.}$$

$$R_2 = P - R_1 = 18{,}000 - 6{,}000 = 12{,}000 \text{ lb.}$$

The resisting moment of a beam is $S_4 \frac{I}{c}$, and since the safe stress S_4 remains constant for the same beam, the resisting moment is proportional to $\frac{I}{c}$. For a beam of circular section,

$$\frac{I}{c} = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^3}{32}$$

Hence, since $\frac{\pi}{32}$ is a constant quantity, the resisting moment at any section of a circular shaft is proportional to the cube of the diameter at that section. Conversely, the diameter of the shaft at any section must be proportional to the cube root of the resisting moment at that section, and to have the shaft of equal strength throughout, the diameter at each section should be proportional to the cube root of the *bending moment* at that section.

In finding the diameter d_1, the bending moment is taken at the point q, and is $R_1 c = 6{,}000 \times 96$. For ascertaining the diameter at this point, use formula **1**, Art. **4**,

$$M = R_1 c = \frac{S_4 d_1^3}{10.2}$$

from which

$$d_1 = \sqrt[3]{\frac{10.2 \times R_1 c}{S_4}}$$

Assuming a safe fiber stress $S_4 = 8{,}500$,

$$d_1 = \sqrt[3]{\frac{10.2 \times 6{,}000 \times 96}{8{,}500}} = 8.842, \text{ or } 8\tfrac{7}{8}, \text{ in., nearly}$$

The shaft should be enlarged at this place to allow a keyway to be cut without weakening the shaft. According to the formula given in *Machine Design*, Part 2, the depth of the sunk key is $\frac{1}{6} d_1 = 1\frac{1}{2}$ in. The depth of the keyway is, therefore, $\frac{3}{4}$ in., and the diameter d_3 of the enlarged portion should be at least $8\frac{7}{8} + 1\frac{1}{2} = 10\frac{3}{8}$ in.

The lengths and diameters of the journals may now be calculated. For the journal nearest the load, from the formula given in *Machine Design*, Part 3,

$$d = 1.5\sqrt{\frac{R_2}{\sqrt{S_4}\,p}}$$

Assuming that $p = 700$,

$$d = 1.5\sqrt{\frac{12{,}000}{\sqrt{8{,}500} \times 700}} = 3\tfrac{3}{8} \text{ in., nearly}$$

$$l = \frac{R_2}{p\,d} = \frac{12{,}000}{700 \times 3\frac{3}{8}} = 5\tfrac{1}{8} \text{ in., nearly}$$

For the other journal,

$$d' = 1.5\sqrt{\frac{R_1}{\sqrt{S_4}\,p}} = 1.5\sqrt{\frac{6{,}000}{\sqrt{8{,}500} \times 700}} = 2\tfrac{3}{8} \text{ in.}$$

$$l' = \frac{6{,}000}{700 \times 2\frac{3}{8}} = 3\tfrac{5}{8} \text{ in.}$$

$d_4 = d' + 2e$, where $e = \frac{1}{10} d' + \frac{1}{8} = .3625$, or $\frac{3}{8}$, in., nearly. Hence, $d_4 = 2\frac{3}{8} + \frac{3}{4} = 3\frac{1}{8}$ in. Likewise, $d_5 = d + 2\left(\frac{1}{10} d + \frac{1}{8}\right) = 4\frac{5}{16}$ in.

To draw the taper outline of the axle, lay off half the diameter d_1 on each side of the horizontal center line of the enlarged portion n, and on either side of the vertical center line draw vertical lines at the distances indicated, assuming that the total width of this part is 12 in. Then construct the journals with their shoulders, and connect each shoulder with the adjacent diameter d_1. Finally, lay off diameter d_3 and draw the enlarged portion n. All corners should be filleted.

Solution 2.—The graphical part of this solution is shown mainly in Fig. 4 (*b*), and partly in Fig. 4 (*a*). To ascertain graphically the values of the bending moments, proceed as follows.

Produce downwards the line that indicates the position of R_2, and to any convenient scale mark off from e to f, the value of P in pounds. Choose a pole O and connect it with points e and f. Extend line $O e$ until it intersects at j the perpendicular representing the position of load P. From j draw a line parallel with $O f$, intersecting line R_1 at p. Connect p and e, and from O draw a line parallel with $p e$, intersecting $e f$ at g. The lines $f g$ and $g e$ will then represent the values of R_1 and R_2, respectively, read to the scale at which $e f = P$ was laid off.

The next step is to ascertain the bending moment at the enlarged portion *n*, as in the preceding solution. On perpendiculars dropped from any point on the axle across the polygon *p e j*, the lengths of the intercepted portions will be proportional to the bending moments at these points. The line *i k*, for instance, is proportional to the bending moment at the point *u*. To ascertain the value in inch-pounds that this line represents select an intercept whose value can easily be determined. For instance, the intercept *h j* represents the bending moment $R_1 c = 6{,}000 \times 96 = 576{,}000$ in.-lb. If the length of *h j* is found to be 3.2 in., then each inch of any intercept represents $576{,}000 - 3.2 = 180{,}000$ in.-lb. It should be observed that the larger the scale to which this polygon is drawn, the more accurate will be the readings of the intercepts.

Assuming that the strengthening action of the wheel hub may be ignored, the center of the moment is taken at *q* instead of at points nearer the ends of part *n*, which would give intercepts *b m* and *i k*. The bending moment at *q* being 576,000 in.-lb., the diameter required may be found from formula **1**, Art. **4**. Thus,

$$M = \frac{S_4 d_1^3}{10.2} = 576{,}000$$

Hence,

$$d_1 = \sqrt[3]{\frac{10.2 \times 576{,}000}{8{,}500}} = 8.84, \text{ or } 8\tfrac{7}{8}, \text{ in., nearly}$$

The remainder of the solution is wholly analytical and is identical with the corresponding part of the first solution. The dimensions *d* and *d'* having been determined by formulas based on a unit bearing pressure *p* of 700 lb., it may be well to check these values by calculating the diameters required to sustain the bending moments at *a* and *a'*. This may be done by measuring the intercepts at *s* and *s'*, but as they are very small in the diagram, it may be more accurate to calculate the bending moments by formula **1**, Art. **4**, from which

$$d = \sqrt[3]{\frac{10.2 \times R_2 \times \frac{l}{2}}{S_4}} = \sqrt[3]{\frac{10.2 \times 12{,}000 \times 2.5625}{8{,}500}}$$
$$= 3.329, \text{ or } 3\tfrac{21}{64}, \text{ in., nearly}$$

Using the same formula,

$$d' = \sqrt[3]{\frac{10.2 \times R_1 \times \frac{l'}{2}}{S_4}} = \sqrt[3]{\frac{10.2 \times 6{,}000 \times 1.8125}{8{,}500}}$$
$$= 2.354, \text{ or } 2\tfrac{23}{64}, \text{ in., nearly}$$

As both of these values are smaller than those found by the other formula, the axle is sufficiently strong at these points to resist the bending moments. The axle may or may not be tapered as shown in Fig. 4; if it is not tapered, its diameter must be calculated from the

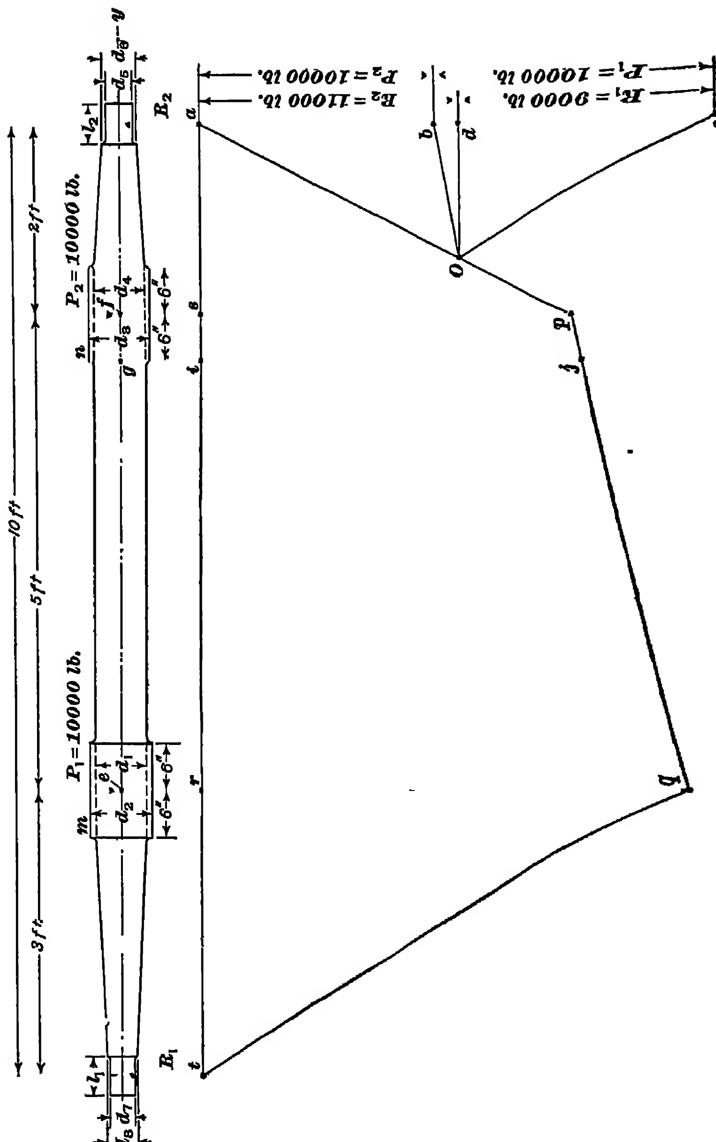

10 ft
3 ft
5 ft
2 ft
$P_1 = 10000$ lb.
$P_2 = 10000$ lb.
$R_1 = 9000$ lb.
$P_1 = 10000$ lb.
$R_2 = 11000$ lb.
$P_2 = 10000$ lb.
B_1
B_2
6"
6"
6"
6"
l_1
l_2
d_1
d_2
d_3
d_4
d_5
d_6
d_7
d_8
m
n
e
f
g
a
b
c
d
O
p
q
r
s
t

FIG. 5

maximum bending moment (hj in this case) A straight shaft is easier to construct, but the end journals, of course, must be larger, and there is considerable loss by friction on that account.

17. Axles Supporting More Than One Load.—The dimensions of axles supporting more than one load may be found graphically by the method used in Art. **16.** To assist in applying the principles, however, the following example is given:

EXAMPLE —An axle 10 feet long between centers of bearings carries two wheels, each supporting a load of 5 tons. The distances of these loads from the ends of the axle are indicated in Fig 5 The axle is made of steel and is not subjected to torsion. Find its dimensions.

SOLUTION.—Draw a line xy representing the axis of the axle and lay off on it, to scale, its length and the places where the loads P_1 and P_2 are applied; also the lengths of the enlarged parts m and n and the locations and directions of the reactions R_1 and R_2. On the line R_2, produced, and to any convenient scale, mark off from a to b the value in pounds of P_2, and from b to c that of P_1. Select a pole O, and connect it with points a, b, and c Produce $a\,O$ until it intersects the produced perpendicular P_2 at p, then, from this point draw a line parallel to $O\,b$, intersecting the produced perpendicular P_1 at q. Parallel to $O\,c$, draw a line from q until it intersects R_1 at t. A line from t to a completes the polygon, and a line from O parallel to $t\,a$, intersecting $a\,c$ at d, divides the latter line into the parts $c\,d$ and $d\,a$, thus indicating the values of the reactions R_1 and R_2. By reading to the same scale as the loads P_1 and P_2, it is found that the values of R_1 and R_2 are 9,000 lb and 11,000 lb., respectively

Having found the reactions, ascertain the values of the intercepts in the moment diagram $a\,p\,q\,t$ by finding, for instance, the value of the intercept $q\,r$, which represents the moment $R_1 \times 3$ ft $= 9{,}000$ lb. $\times\, 36$ in. $= 324{,}000$ in.-lb. The value of the unit length of the intercepts in inch-pounds may now be found, as explained in Art. **16.**

Disregarding the supporting action of the wheel hub at m, the diameter of this part is calculated from the bending moment at e. Assuming a safe fiber stress $S_4 = 9{,}000$ lb and using formula **1,** Art. **4,**

$$d_1 = \sqrt[3]{\frac{10.2\,M}{S_4}} = \sqrt[3]{\frac{10.2 \times 324{,}000}{9{,}000}} = 7.161, \text{ or } 7\tfrac{3}{16}, \text{ in., nearly}$$

The depth of the sunk key is $\frac{1}{6}\,d_1 = 1\frac{1}{4}$ in , nearly The depth of the keyway is therefore $\frac{5}{8}$ in , and the diameter $d_2 = 7\frac{3}{16} + 1\frac{1}{4} = 8\frac{7}{16}$, or $8\frac{1}{2}$, in , nearly

Proceeding to the part n, it is found that the intercept $p\,s$ at the point f is smaller than the intercept $i\,j$ at g The latter should therefore be used as a basis for calculating the diameter at this place.

Measuring the intercept ij to the same scale as gr, it is found to represent a bending moment $M = 270{,}000$ in.-lb. From formula **1**, Art. **4**,

$$d_8 = \sqrt[3]{\frac{10.2\,M}{S_4}} = \sqrt[3]{\frac{10.2 \times 270{,}000}{9{,}000}} = 6.74, \text{ or } 6\tfrac{3}{4}, \text{ in., nearly}$$

The increase in diameter required for the key is $\frac{1}{6}\,d_8 = 1\frac{1}{8}$ in., making $d_4 = 6\frac{3}{4} + 1\frac{1}{8} = 7\frac{7}{8}$ in., at least. The length and diameter of each journal is found again according to formulas given in Art. **15**. Assuming that $p = 700$,

$$d_5 = 1.5\sqrt{\frac{R_2}{\sqrt{S_4}\,p}} = 1.5\sqrt{\frac{11{,}000}{\sqrt{9{,}000} \times 700}} = 3.14, \text{ or } 3\tfrac{3}{16}, \text{ in., nearly}$$

and the length is

$$l_2 = \frac{R_2}{p\,d_5} = \frac{11{,}000}{700 \times 3\frac{3}{16}} = 4.93, \text{ or } 5, \text{ in., nearly}$$

The height of the shoulder at this journal is

$$e = \frac{d_5}{10} + \tfrac{1}{8} = .444, \text{ or } \tfrac{1}{2}, \text{ in., nearly}$$

Hence, $d_6 = 3\frac{3}{16} + 1 = 4\frac{3}{16}$ in. Proceeding in the same manner, it is found that

$$d_7 = 1.5\sqrt{\frac{9{,}000}{\sqrt{9{,}000} \times 700}} = 2.84, \text{ or } 2\tfrac{7}{8}, \text{ in., nearly}$$

The height of the shoulder at this end is

$$e = \frac{2\frac{7}{8}}{10} + \tfrac{1}{8} = .412, \text{ or } \tfrac{7}{16}, \text{ in., nearly}$$

The diameter d_8 is then $2\frac{7}{8} + \frac{7}{8} = 3\frac{3}{4}$ in., and the length is

$$l_1 = \frac{R_1}{p\,d_7} = \frac{9{,}000}{700 \times 2\frac{7}{8}} = 4.47, \text{ or } 4\tfrac{1}{2}, \text{ in., nearly}$$

The axle is drawn in the same manner as described in Art. **16**.

SHAFTS SUBJECTED TO TORSION AND BENDING

18. Equivalent Twisting Moment.—When a shaft is subjected to combined bending and twisting moments, the two moments may, by means of a formula derived from higher mathematics, be combined in such a manner as to give an increased bending moment, which in its effect is equivalent to the combined effects of the two separate moments; this is termed the **equivalent bending moment.** Or, another formula may be used in which the two moments are so combined as to give an increased twisting moment, equal in its effect to the two separate moments; this is called the **equivalent twisting moment.** The latter method is the

more general and is the one mentioned in *Strength of Materials*, Part 2.

Let M = bending moment for any section;

T = twisting moment for the same section;

T_1 = equivalent twisting moment.

Then, $$T_1 = M + \sqrt{M^2 + T^2}$$

19. Graphical Determination of the Equivalent Twisting Moment.—It is evident that the quantity $\sqrt{M^2 + T^2}$ found in the formula of Art. **18** may represent the length of the hypotenuse in a right triangle with sides M and T. Using this length as a basis, the whole value of the formula can easily be found graphically in the following manner:

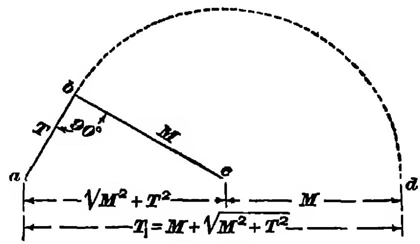


FIG 6

Lay off the values of the moments T and M in inch-pounds, as sides $a\,b$ and $b\,c$ in the right triangle $a\,b\,c$, Fig. 6, which may be drawn to any convenient scale. The hypotenuse $a\,c$ will represent the value $\sqrt{M^2 + T^2}$, read to the same scale as the separate moments represented by the sides $a\,b$ and $b\,c$. Produce the side $a\,c$, and, in the manner indicated, lay off a length $c\,d$ equal to $b\,c$. The line $a\,d$ will then be equal to $M + \sqrt{M^2 + T^2}$, and will thus represent the total value of T_1 in inch-pounds.

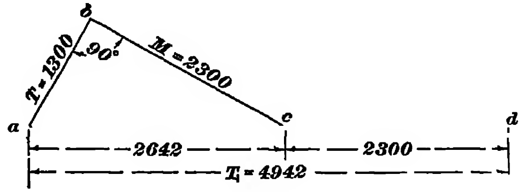


FIG 7

EXAMPLE 1.—A shaft is subjected to a bending moment of 2,300 inch-pounds and a twisting moment of 1,300 inch-pounds Find the equivalent twisting moment.

SOLUTION.—Lay off $a\,b$ and $b\,c$, Fig. 7, at right angles to each other and to any scale, representing 1,300 and 2,300 in -lb , respectively. Draw $a\,c$ and produce it until $c\,d$ is equal to $b\,c$. T_1 is then equal to 2,642 + 2,300 = 4,942 in.-lb. Ans.

NOTE —It is not absolutely necessary to extend the line $a\,c$, as the sum of the values represented by the lines $b\,c$ and $a\,c$ can be found by ordinary addition.

EXAMPLE 2.—Calculate the diameter of a steel crank-shaft for an engine from the following data. The length of stroke is 10 feet, the maximum tangential force acting on the crank-pin is 25 tons; and the shaft is 10 feet long between centers of bearings, and carries midway between bearings a flywheel weighing 55 tons. Assume the safe shearing stress to be 9,000 pounds = $4\frac{1}{2}$ tons.

SOLUTION —The twisting moment T = maximum tangential force × length of crank = 25 T × 5 ft. = 125 ft -T. = 1,500 in.-T. The bending moment is

$$\tfrac{1}{4}Pl = \frac{55 \times 10}{4} = 137.5 \text{ ft -T.} = 1{,}650 \text{ in.-T.}$$

The value of T_1 may be determined graphically in the manner indicated in Fig. 8, but care should be taken to read off the value of T_1 in *inch-tons*, as this term is used to express the other moments Thus, the value of T_1 is found to be 3,880 in -T. The bending moment has therefore the same effect as if the tangential force at the crank were increased to

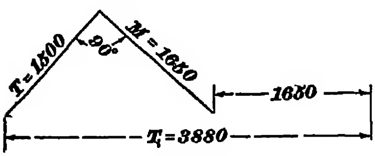


FIG 8

$$\frac{3{,}880}{5 \times 12} = 64.7 \text{ T.}$$

If T_1 is to be found by calculation, then from the formula of Art. **18**,

$$T_1 = M + \sqrt{M^2 + T^2}$$
$$= 1{,}650 + \sqrt{1{,}650^2 + 1{,}500^2} = 3{,}880 \text{ in.-T.}$$

Now, using formula **3**, Art. **4**,

$$d = 1.72\sqrt[3]{\frac{PR}{S_s}} = 1.72\sqrt[3]{\frac{T_1}{4.5}} = 1.72\sqrt[3]{\frac{3{,}880}{4.5}} = 16\tfrac{3}{8} \text{ in., nearly. Ans.}$$

NOTE —In the foregoing solution it will be noticed that the units selected are inches and tons Generally, when the bending and twisting moments are large, it is more convenient to use tons than pounds Care must be taken, however, to make the units consistent, for example, it would be wrong to take the moments in *inch-tons*, and the safe stress in *pounds*.

EXAMPLE 3 —A wrought-iron shaft transmits 150 horsepower at 125 revolutions per minute, and at the same time is subjected to a bending moment of 60,000 inch-pounds. Calculate the diameter of the shaft.

SOLUTION.—From the formula of Art 4, $PR = 63{,}025\,\frac{H}{N}$; the twisting moment $T = 63{,}025 \times \frac{150}{125} = 75{,}630$ in.-lb. M being 60,000

in.-lb., T_1 is found graphically in the manner described, its value being 156,540 in.-lb. Using formula **3**, Art. **4**, viz., $d = 1.72\sqrt[3]{\frac{PR}{S_s}}$, and making $S_s = 6{,}800$,

$$d = 1.72\sqrt[3]{\frac{156{,}540}{6{,}800}} = 4\tfrac{15}{16} \text{ in., nearly. Ans.}$$

20. Equivalent Torsional Moment of a Hollow Shaft.—The methods described in Arts. **18** and **19** for finding the equivalent torsional moment for solid shafts apply also to hollow shafts; but, after the moment T_1 has been determined, the diameters of the shaft must of course be found by means of formula **2**, Art. **7**.

EXAMPLE.—A hollow steel shaft transmitting 200 horsepower at 120 revolutions per minute is subjected to a bending stress of 50,000 inch-pounds. Calculate the outside and inside diameters d_1 and d_2, respectively, when the ratio $\frac{d_2}{d_1} = n = .5$.

SOLUTION.—The twisting moment T is found from the formula of Art. **4**. Thus,

$$PR = 63{,}025\,\frac{H}{N} = 63{,}025 \times \frac{200}{120} = 105{,}042 \text{ in.-lb.}$$

To find the equivalent twisting moment T_1 by calculation, use the formula of Art. **18**, from which $T_1 = M + \sqrt{M^2 + T^2} = 50{,}000 + \sqrt{50{,}000^2 + 105{,}042^2} = 166{,}335$ in.-lb. Inserting this value in formula **2**, Art. **7**,

$$d_1 = 1.72\sqrt[3]{\frac{PR}{S_s\,(1 - n^4)}} = 1.72\sqrt[3]{\frac{166{,}335}{9{,}000\,(1 - .5^4)}} = 4.646,$$

or $4\tfrac{11}{16}$, in., nearly

The inside diameter d_2 is $n \times d_1 = .5 \times 4.646 = 2.323$, or $2\tfrac{5}{16}$, in., nearly. Ans.

SHAFT COUPLINGS

21. Couplings are used to connect the ends of shafts, and are of three kinds: (1) *Fast*, or *permanent*, *couplings;* (2) *loose couplings*, or *clutches*, by means of which shafts may be connected or disconnected at pleasure; and (3) *friction clutches*, which are loose couplings that hold by friction.

PERMANENT COUPLINGS

22. Sleeve, or Muff, Couplings.—The *sleeve*, or *muff, coupling*, as shown in Fig. 9, consists of a cast-iron cylinder fitting over the ends of the shafts. The two ends are prevented from moving relatively to each other by a sunk key, the keyway being cut half into the box and half into the shaft ends. Quite commonly, the ends of the shafts are

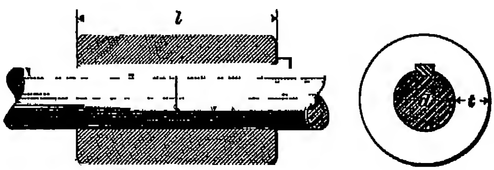


FIG 9

enlarged so as to allow the keyway to be cut without weakening the shafts.

The key may be proportioned by the formulas already given in *Machine Design*, Part 2. For the other dimensions take

$$l = 2\tfrac{1}{2}\,d + 2 \text{ in.}$$
$$t = .4\ d + .5 \text{ in.}$$

EXAMPLE.—Find the dimensions of a muff coupling for a shaft $2\frac{1}{2}$ inches in diameter.

SOLUTION.—For the key, apply the formula given in *Machine Design*, Part 2.

$$\left.\begin{array}{l} b = \frac{1}{4}\,d = \frac{1}{4} \times 2\frac{1}{2} = \frac{5}{8} \text{ in} \\ t = \frac{1}{6}\,d = \frac{1}{6} \times 2\frac{1}{2} = \frac{7}{16} \text{ in} \end{array}\right\}\text{Ans.}$$

For the muff, use the foregoing formula.

$$\left.\begin{array}{l} l = 2\frac{1}{2}\,d + 2 = 2\frac{1}{2} \times 2\frac{1}{2} + 2 = 8\frac{1}{4} \text{ in.} \\ t = .4\ d + .5 = .4 \times 2\frac{1}{2} + .5 = 1\frac{1}{2} \text{ in.} \end{array}\right\}\text{Ans.}$$

23. Clamp Couplings.—A **clamp coupling** is shown in Fig. 10. The faces for the joint are first planed off, the holes are drilled, and then the two halves are bolted together with pieces of paper between them, after which the coupling is bored out to the exact size of the shaft. The pieces of paper on being removed leave a slight space between the halves, and the coupling when bolted to the shaft grips

it firmly. This form of coupling is very easily put on, or removed, and has no projecting parts. The key is straight, and fits only at the sides.

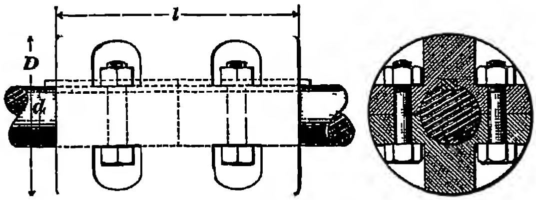


FIG 10

The following are the proportions used in practice:

d = diameter of shaft;
D = diameter of coupling $= 2\frac{1}{2}d + \frac{1}{2}$ in.;
l = length of coupling $= 4d$.

The diameter of the bolts may be $\frac{5}{8}$ inch for shafts under $2\frac{1}{2}$ inches in diameter; $\frac{3}{4}$ inch for $2\frac{1}{2}$-inch shafts, and $\frac{7}{8}$ inch for larger shafts. For shafts up to 3 inches in diameter, use four bolts; for larger shafts use six bolts.

24. Flange Couplings.—A **flange coupling** is shown in Fig. 11. Cast-iron flanges are keyed to the ends of the

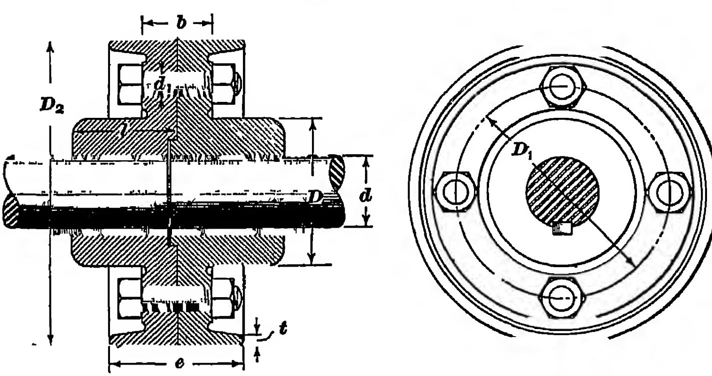


FIG. 11

shafts. To insure a perfect joint, the flange is usually faced in a lathe after being keyed to the shaft. The two flanges are then brought face to face and bolted together.

Sometimes, the ends of the shafts are enlarged to allow for the keyway. To prevent the possibility of the shafts getting out of line, one flange may be made with a projection that fits into a recess in the other, as shown.

The following proportions may be used for the form of flange coupling shown in Fig. 11:

d = diameter of shaft;

$D = 1\frac{3}{4}d + 1$ in.;

D_1 = diameter of bolt circle $= 2\frac{1}{2}d + 2$ in.;

$l = 1\frac{1}{8}d + 1$ in.;

n = number of bolts $= 3 + \frac{d}{2}$;

d_1 = diameter of bolts $= \frac{d}{n} + \frac{1}{4}$ in.;

D_2 = diameter of coupling $= 1.4\,D_1$;

$b = \frac{1}{2}d + \frac{5}{8}$ in.;

$e = 2b$;

$t = \frac{1}{8}d$.

As most designers prefer an even number of bolts, select for n the nearest even number.

The key may be proportioned by the formulas already given for keys. The dimensions for l and D_2, as found by the preceding formulas, should not be used for shafts exceeding 4 inches in diameter. For larger shafts, the dimensions given in Table II may be used.

TABLE II

DIMENSIONS OF COUPLINGS FOR SHAFTS OVER FOUR INCHES IN DIAMETER

Diameter of Shaft Inches	l Inches	D_2 Inches	Diameter of Shaft Inches	l Inches	D_2 Inches
$4\frac{7}{16}$	$6\frac{1}{4}$	$14\frac{1}{2}$	7	10	22
5	$7\frac{1}{4}$	$16\frac{1}{2}$	8	11	24
$5\frac{1}{2}$	8	$17\frac{1}{2}$	9	12	26
6	$8\frac{1}{2}$	20	10	13	28
$6\frac{1}{2}$	$9\frac{1}{4}$	21			

25. The Sellers Cone Coupling.—The **Sellers cone coupling**, which is shown in Fig. 12, consists of an outer box, or muff, that is cylindrical externally, but has the form of a double truncated cone on the inside. Within the muff are placed two slotted sleeves, which are turned on the outside to fit the muff and are also bored out to fit the shaft. These sleeves are pulled together by three bolts, and as the former are drawn farther into the muff, they grip it and the shaft firmly.

The bolt holes pass through both the sleeves and the muff, and are square in cross-section. The friction between the

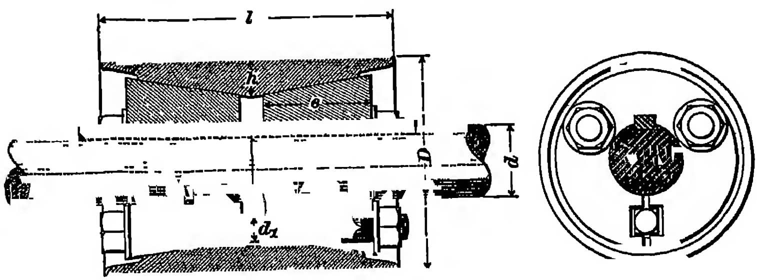


FIG 12

sleeves and the shaft is generally sufficient to prevent slipping, but to be on the safe side, the sleeves are usually keyed to the shaft. The key should have no taper, and should fit at the sides only; its proportions may be obtained by formulas given in *Machine Design*, Part 2. The other proportions may be taken as follows:

d = diameter of shaft;	$d_1 = \frac{1}{3}d$;
$l = 4d$;	$e = 1\frac{1}{2}d$;
$D = 3d$;	$h = \frac{1}{2}d$.

The conical sleeves may be tapered 4 inches per foot of length. In putting up lines of shafting, the couplings, if possible, should be placed near bearings.

LOOSE COUPLINGS

26. Claw Couplings.—**Claw couplings** are used when the shafts are to be alternately connected and disconnected. For large slow-moving shafts, the claw coupling shown in

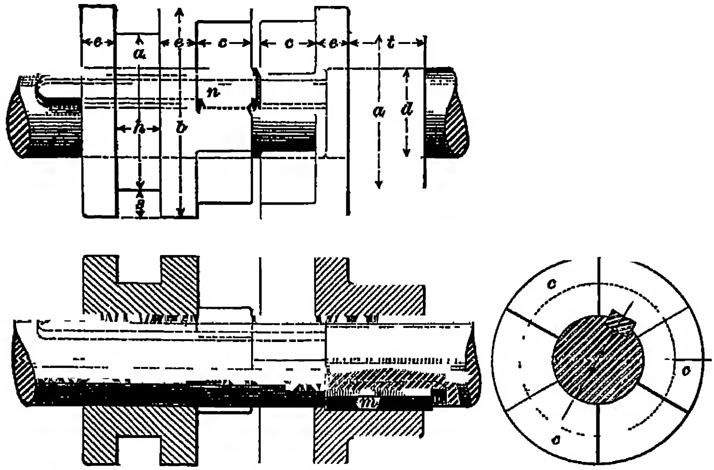


FIG 13

Fig. 13 may be used. This coupling somewhat resembles the flange coupling shown in Fig. 11, except that the flanges, instead of being bolted together, are provided with a set of lugs *c* that interlock. One flange is permanently fastened to the shaft by a sunk key *m*, while the other is fastened to its shaft by a feather key *n*, and may be moved back and forth, thus throwing the coupling, or clutch, in or out of gear. The movement of the clutch is effected by a forked lever fitting into the recess *h*.

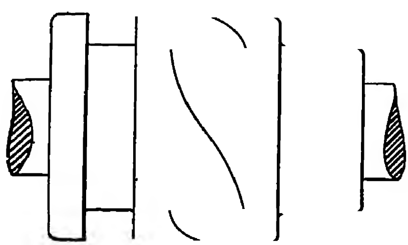


FIG 14

The lugs, or claws, may be given the form shown in Fig. 14, in which case the couplings are more easily put in gear, but will drive in only one direction.

Cast-iron claw couplings may have the following proportions:

d = diameter of shaft;	$e = \frac{3}{8}d$;
$a = 1\frac{3}{4}d$;	$h = \frac{1}{2}d$;
$b = 2\frac{3}{8}d$;	$s = \frac{5}{16}d$;
$c = \frac{5}{8}d$;	$t = \frac{7}{8}d$.

FRICTION CLUTCHES

27. Function of the Friction Clutch.—When it is desired to connect two machine parts—one revolving and the other stationary—without injurious shock to the machinery, the motion must be imparted gradually to the driven part. This gradual starting, however, is impossible without some relative motion between the two parts during the starting period. In a friction clutch, this relative motion can readily be provided, because motion is transmitted from one part to the other solely by means of the friction of the contact surfaces. A gradual increase of this friction is followed by a reduction of the relative motion until both parts revolve at the same speed.

In order to produce relative motion between two bodies in contact, a certain force P is necessary to overcome the friction F between the rubbing surfaces. The amount of this force P depends on the nature of the contact surfaces and on the pressure N normal to them. The ratio between the frictional force F and the normal N, termed *the coefficient of friction*, is generally designated by f; hence,

$$f = \frac{F}{N} \qquad (1)$$

It is also known that if e is the angle of friction, then

$$f = \tan e = \frac{F}{N} \qquad (2)$$

28. For contact between metals, and between metals and wood, f varies between .1 and .5. For cast iron on cast iron it varies from .1 to .15.

From formula **1**, Art. **27**, $F = fN$. As f is always less than 1, it follows that F must always be smaller than N.

For instance, for cast iron on cast iron, $F = fN = .1N$. If the pressure N is 100 pounds, then the resistance F will be $.1 \times 100 = 10$ pounds.

When frictional forces are used to transmit large powers, it is evident that the normal pressure N must be very large. As it is undesirable to produce this directly, a friction clutch must be so designed that only a relatively small pressure will be required to bring the clutch into action so as to produce the necessary friction. This may be attained by different means, one of which is to make the rubbing surfaces parts of cones, depending on their wedge action to produce the pressure required.

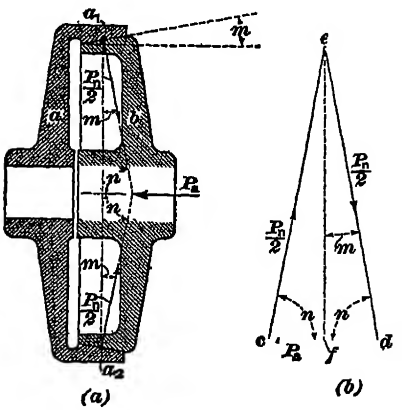


FIG. 15

29. Cone Friction Clutch.—Fig. 15 (*a*) illustrates diagrammatically the two cones of a **cone friction clutch,** *a* being the external and *b* the internal one. The two cones are assumed to be held in contact by means of the axial force P_a, and its components $\frac{P_n}{2}$ exert pressures normal to the contact surfaces of the cones. The angle m indicates the inclination of the cone surface toward its axis, and the angle n is the complement of this angle. A uniform pressure is supposed to be acting normally over the whole circular surface of each cone, but, to facilitate the explanation of its action, this pressure is shown concentrated at the two points a_1 and a_2 located diametrically opposite each other, each component being equal to $\frac{P_n}{2}$.

The magnitude and direction of these components may be found by constructing a triangle as shown in Fig. 15 (*b*), in which the side $c\,d$, drawn to any suitable scale, represents

the direction and value of the force P_a. By drawing the lines $c e$ and $d e$ at angles n to $c d$ and letting them intersect at e, the directions and values of the two forces $\frac{P_n}{2}$ are found. When a perpendicular is drawn from e to $c d$, then $d f = \frac{P_a}{2}$, and $\frac{P_a}{2} = \sin m \frac{P_n}{2}$. Hence, $P_a = \sin m \, P_n$, and

$$P_n = \frac{P_a}{\sin m} \qquad (1)$$

When the cone a revolves, the friction produced at the contact surfaces of cones a and b acts as a tangential force P_t, not shown in the illustration, which tends to rotate the cone b. One-half of this force, that is, $\frac{P_t}{2}$, is supposed to be concentrated at each of the two points a_1 and a_2. Inserting the letters P_t and P_n for F and N, respectively, in formula **1**, Art. **27**, then $f = \frac{P_t}{P_n}$, and

$$P_t = f P_n \qquad (2)$$

The twisting moment T tending to turn the cone b is $R \times 2\frac{P_t}{2} = R P_t$, in which R = distance of points a_1 or a_2 from the axis of the cone b. From formula **2** the tangential force $P_t = f P_n$, and from formula **1**, $P_n = \frac{P_a}{\sin m}$. Hence,

$$P_t = f P_n = f \frac{P_a}{\sin m}$$

and

$$T = R P_t = \frac{f P_a R}{\sin m} \qquad (3)$$

If in place of the moment T it is desired to express the action of the clutch in horsepower, then from the formula $T = PR = 63{,}025 \frac{H}{N}$,

$$T = R P_t = \frac{f P_a R}{\sin m} = \frac{63{,}025 \, H}{N}$$

or,

$$H = \frac{f P_a R N}{63{,}025 \sin m} \qquad (4)$$

Let A = frictional area of a cone or cylinder, in square inches;

p_s = safe pressure per square inch between the surfaces;

w = width of conical or cylindrical surface;

R = radius of a cylindrical surface, and average radius of a conical surface.

Then,
$$A = \frac{P_n}{p_s} \qquad (5)$$

As $A = 2\pi R w$,
$$w = \frac{A}{2\pi R} \qquad (6)$$

An average value for p_s is 50 pounds per square inch. Referring to Fig. 16, the average diameter of the conical

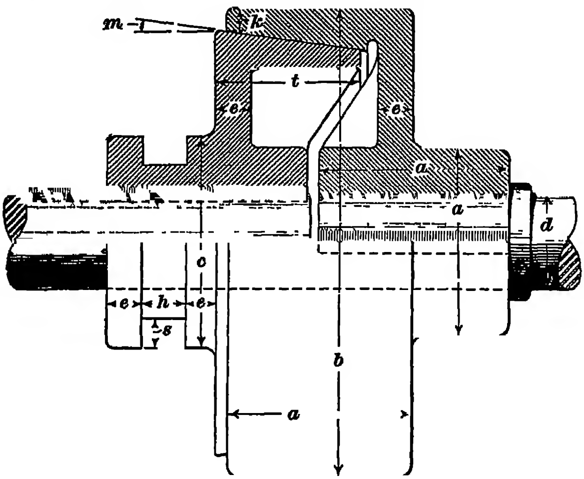


FIG 16

part may be from four to eight times the shaft diameter, according to the amount of power to be transmitted. The angle m of the cone may be from 4° to 10°. The other proportions are as follows:

d = diameter of shaft,

$a = 2\,d$;	$e = \frac{3}{8}\,d$;
$b = 4$ to $8\,d$;	$h = \frac{1}{2}\,d$;
$c = 2\frac{1}{4}\,d$;	$s = \frac{5}{16}\,d$;
$t = 1\frac{1}{2}\,d$;	$k = \frac{1}{4}\,d$.

30. The following example will illustrate the application of the formulas for the cone friction clutch.

EXAMPLE.—Ascertain the width of the cone surfaces in a cast-iron friction clutch intended to transmit 20 horsepower at 200 revolutions per minute, when the mean radius of the cone surface is 5 inches and the pressure per square inch of same is not to exceed 55 pounds.

SOLUTION —Substituting P_t for P in the formula $PR = 63{,}025\frac{H}{N}$,

$$P_t = 63{,}025\frac{H}{RN} = 63{,}025 \times \frac{20}{5 \times 200} = 1{,}260 \text{ lb., nearly}$$

As both cones are made of cast iron, the value of f may be taken as 15. According to the laws of friction, if the angle m is made equal to or a little greater than the friction angle e for cast iron, the two friction surfaces will separate voluntarily when released. As $\tan e = f = .15$, $e = 8$ 5°, nearly. Transposing formula **2**, Art **29**, and solving for P_n, then,

$$P_n = \frac{P_t}{f} = \frac{1{,}260}{.15} = 8{,}400 \text{ lb.}$$

In case f should occasionally assume a greater value than .15, it is well to increase the angle m to prevent the cones from sticking together when released. Let the value m be increased to 11°, corresponding to a value $f = .194$ Then, the axial pressure P_a necessary to produce the pressure P_n is found from formula **1**, Art **29**, from which

$$P_a = P_n \sin m = 8{,}400 \times .191 = 1{,}604 \text{ lb., nearly}$$

From formula **5**, Art. **29**,

$$A = \frac{8{,}400}{55} = 153 \text{ sq. in , nearly}$$

From formula **6**, Art. **29**,

$$w = \frac{153}{2 \times 3.1416 \times 5} = 4\ 87, \text{ or } 4\tfrac{7}{8}, \text{ in., nearly. Ans.}$$

31. Cylinder Friction Clutch.—When the rubbing surfaces of a clutch are cylindrical, formulas **2**, **5**, and **6**, Art. **29**, may be applied, but not formulas **3** and **4**. Eliminating the value sin m, the ratio of P_a to P_n will depend on the mechanism employed for transforming the axial force P_a into the radial forces $\frac{P_n}{2}$.

An example of a **cylinder friction clutch** is shown in Fig. 17. The shaft n carries a flange, or cylinder, A, and the shaft m has keyed to it a ring B. The ring is split and fits inside the flange, or cylinder, A, the split ends being

connected by a screw having a right- and a left-hand thread. The screw is turned by the lever C, which is connected by a link D to the sleeve E. When the sleeve is pushed toward the clutch, the rotation of the screw throws the ends F, F of the ring B apart and thereby causes the ring B to grip the

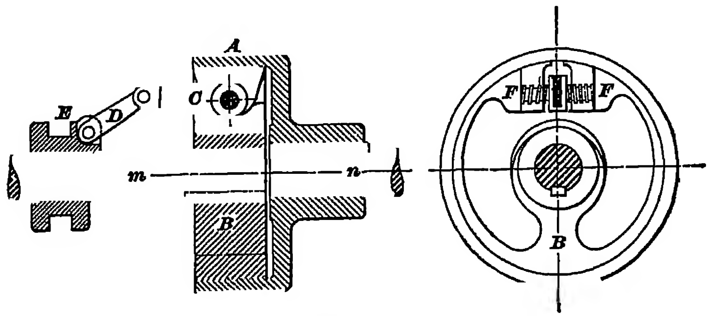


FIG 17

flange A tightly. A clutch of this form is easy to operate and produces no end thrust on the shaft.

Views showing details of the ends of the ring B, Fig. 17, are given in Fig. 18, (a) being an elevation and (b) a section taken on the line $x\,x$. To facilitate the insertion of the

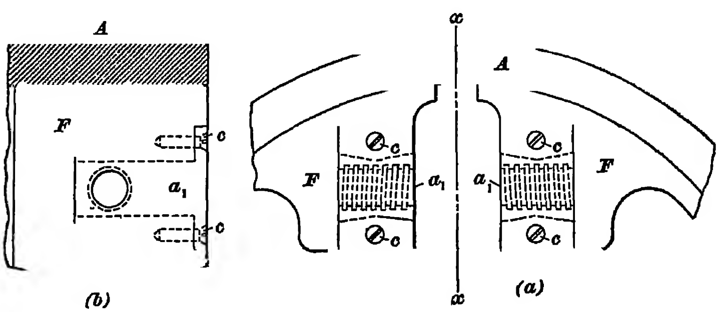


FIG 18

screw shown in Fig. 17, it is necessary to make the nuts a_1, a_1 separate from the ring F. The nuts are dovetailed into recesses in the ring, as indicated After the nuts have been screwed into position on the screw, they are inserted in the recesses and held there by the screws c, c.

Another form of clutch differs from the preceding in having the friction band B on the outside of the flange. The principal proportions of these clutches are about the same as those given for Fig. 16.

32. Disk Friction Clutches.—In order to secure the required frictional surface when conditions limit the diameter of the clutch, it is necessary to extend the dimensions of the clutch lengthwise and to have the friction surfaces at right angles to the shaft. An example of this class is the **disk friction coupling.**

The construction of this clutch when used for coupling a spur gear to a shaft is shown in Fig. 19. The spur gear g,

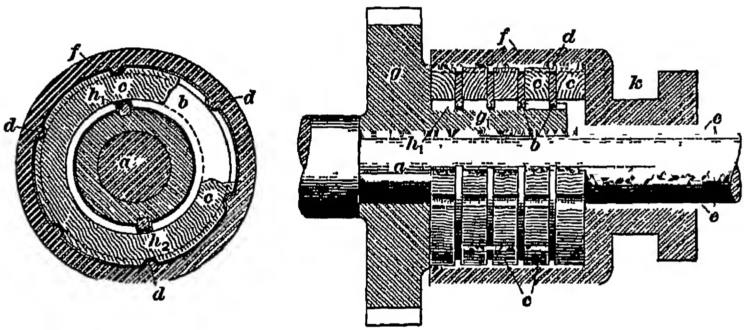


FIG 19

which is provided with a hub g_1, revolves loosely on the shaft a, and on hub g_1 are feathers h_1 and h_2 that engage with wrought-iron rings b, b. The coupling box f is connected to the shaft by means of feathers e, e, which permit the box to be moved lengthwise. The box is provided with six longitudinal ribs d, d that engage with grooves in the wooden rings c, c, and consequently these rings will revolve with the box and the shaft, while the gear g and the rings b, b remain stationary. By means of a forked lever resting in the groove k, the box may be forced to the left, bringing all the rings in contact and compelling the rings b and the gear g to revolve with the shaft.

Let R = mean radius of the rubbing surfaces;

n = number of pairs of rubbing surfaces.

Then, formulas **3** and **4**, Art. **29**, will assume the following forms:

$$T = R P_t = n f P_a R \qquad (1)$$

and

$$H = \frac{n f P_a R N}{63{,}025} \qquad (2)$$

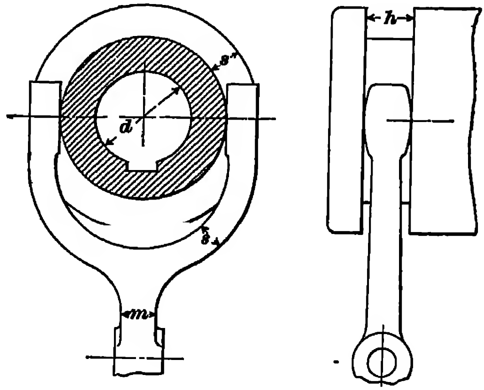


FIG. 20

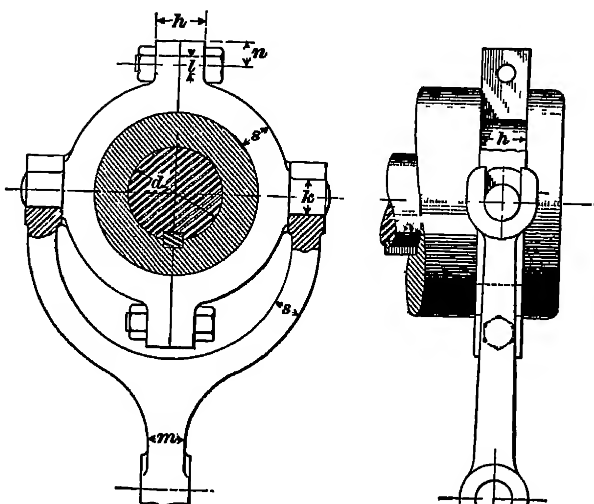


FIG. 21

33. Shifting Gear for Clutches.—A clutch is usually put in or out of gear by means of a **forked lever,** the

prongs of which fit into the groove cut in the sliding part of the clutch. The lever is usually operated by hand, though for large clutches the end of the lever may be moved by a screw and hand wheel. The ordinary design of the forked end of the lever is shown in Fig. 20. To increase the wearing surface, however, a strap as shown in Fig. 21 may be used. The strap completely fills the groove, and is often made of brass. The dimensions h and s are the width and the depth of the groove, respectively, which are to be determined by the rules already given for the proportions of the clutch.

The proportions for the clutch lever are:

$d =$ diameter of shaft;	$s = \frac{5}{16} d$;
$h = \frac{1}{3} d$;	$m = \frac{3}{8} d$;
$k = \frac{3}{8} d$;	$n = \frac{1}{4} d$.
$l = \frac{3}{16} d$;	

SPRINGS

GENERAL FUNCTION OF SPRINGS

34. Most machine parts are constructed so as to show a minimum amount of deflection, but some are designed to give a maximum amount of deflection for a given stress. Such parts are termed **springs**, and their function is either that of providing a yielding connection between two other machine parts, or of providing means for graduating the pressure or pull on a certain machine part in accordance with a certain fixed value. An example of the first kind are the springs under a railroad car, while the springs in safety valves, governors, etc. are examples of the second kind.

Springs are divided into three general classes: *elliptical*, *helical*, and *spiral*.

ELLIPTICAL SPRINGS

35. Plate, or Cantilever, Spring.—The plate spring, whether supported at one end or at the center, is considered as a cantilever, and the formulas for such springs are based on the formulas for cantilevers. In the case of a plate spring supported at the center, one-half of the load is

supposed to be carried at each end of the spring; each half of the spring is thus treated as a cantilever supporting one-half the load.

It was shown in *Strength of Materials*, Part 2, that the bending moment of a beam

$$M = S_4 \frac{I}{c} \qquad (1)$$

if S_4 = safe working stress, in pounds per square inch;
I = moment of inertia of its cross-section;
c = distance of outer fiber from neutral axis, in inches.

It was also shown that the deflection of a beam

$$s = \frac{a W l^3}{E I} \qquad (2)$$

when a = a constant depending on the mode of loading;
W = concentrated load, in pounds;
d = depth of beam, in inches;
l = length of beam, in inches;
E = coefficient of elasticity = $E_t = E_c$.

For a rectangular cross-section $I = \frac{b d^3}{12}$ and $c = \frac{d}{2}$. By inserting these values in formula 1,

$$M = \frac{2 S_4 b d^3}{12 d} = \frac{S_4 b d^2}{6}$$

For a cantilever with a concentrated load at one end, $M = Wl$; hence, $Wl = \frac{S_4 b d^2}{6}$, and

$$W = \frac{S_4 b d^2}{6 l} \qquad (3)$$

If the same values of I and c are inserted in formula 2, it becomes

$$s = \tfrac{1}{3} \frac{12 W l^3}{E b d^3}$$

or,

$$s = \frac{4 W l^3}{E b d^3} \qquad (4)$$

36. Variations of Deflection.—If the dimensions of a spring intended to support a given load W are calculated by means of formula 3, Art. 35, it may be found on applying formula 4, Art. 35, for the purpose of ascertaining the

deflection s resulting from the load W, that this deflection is either too large or too small. As a guide in making the necessary changes in the dimensions of the spring, so as to produce the required deflection, it should be noted that if the length l is changed, it is obvious from formulas **3** and **4**, Art. **35**, that for a given increase in length l the gain in deflection is much greater than the loss in strength. A decrease in the width b for the purpose of increasing the deflection is followed by a corresponding loss in strength. Increasing the depth d will decrease the deflection, and increase the strength of the spring, though not in the same ratio.

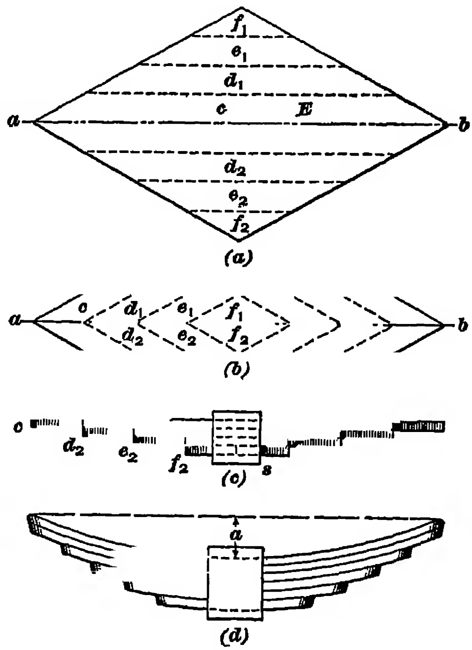


FIG 22

37. Design of Elliptical Springs. The dimensions of a spring need not be uniform throughout its length; it may vary in thickness or in width. If the rate of increase in width is directly proportional to the bending moment, then the stress S_4 will be uniform from end to end. Such a spring is shown in Fig 22 (a), which is a plan view of a flat spring E that is supported along the middle line $f_1 f_2$ and consists of two triangular cantilevers joined end to end. For the purpose of economizing space, it is preferable to subdivide the spring into narrow strips, or *leaves*, as shown by the dotted lines, and to lay these leaves on top of one another

In Fig. 22 (*b*), the central strip *c* is on top, and below it are the narrower strips d_1, d_2, e_1, e_2, etc., which are placed side by side and supposedly united along the center line *a b*.

Fig. 22 (*c*) shows a side view of the spring built up from the strips, which are held together by the strap *s* and form what is termed a *laminated spring*. In practice, the separate leaves are given a curved form before being inserted in the strap, and the spring then assumes the form shown in Fig. 22 (*d*). In this form, it is called a *semielliptic spring*, and if two such springs are joined back to back, it is a *full-elliptic spring*. The distance *a* is called the **set**, or **arch**, of the spring. In general, the upper leaf, whether single or double, is left with square ends, while the others have their ends rounded off.

The deflection calculated for the undivided spring *E*, Fig. 22 (*a*), will not correspond to the actual deflection of the laminated spring, Fig. 22 (*d*). In the latter form, the friction between adjoining plates prevents the deflection from reaching its full value when the spring is straightening out under an increasing load. When the load is decreasing, the friction will also prevent the spring from returning to the position it should occupy in accordance with the load, and will therefore show a deflection that is too large.

The following formulas for elliptical springs, deduced by Reuleaux, have, in general, given satisfactory results:

Let s = deflection at end of spring, in inches;
W = load at one end of spring, in pounds;
E = modulus of elasticity for bending $= E_t = E_c$;
l = one-half length of spring, in inches, when loaded;
b = width of plates, in inches;
t = thickness of one plate, in inches;
n = number of plates in spring;
S_4 = safe maximum fiber stress in plate.

Then,
$$W = \frac{S_4 n b t^2}{6 l} \qquad (1)$$
and
$$s = \frac{6 W l^3}{E n b t^3} \qquad (2)$$

The latter formula gives correct values for s if the plates gradually decrease in length, as shown in Fig. 22 (*c*); but

frequently, as shown in Fig. 22 (*d*), an extra plate of the same length as plate *c* is added. In such cases, the factor 6 of formula **2** is reduced to 5.5 and

$$s = \frac{5.5\ W l^3}{E n b t^3} \qquad (3)$$

In general, for steel, S_4 is taken as 80,000 pounds and E, as 30,000,000 pounds, in which case formulas **1** and **2** assume the forms

$$W = \frac{13{,}333\ n b t^2}{l} \qquad (4)$$

and

$$s = \frac{W l^3}{5{,}000{,}000\ n b t^3} \qquad (5)$$

EXAMPLE.—An elliptical spring, similar to Fig. 22 (*d*), consists of 5 plates 3 inches wide and $\frac{3}{8}$ inch thick. The two upper plates are of equal length. The length of the spring is 40 inches and it supports a load of 1,200 pounds. Find the deflection and the safe load.

SOLUTION.—From formula **5**,

$$s = \frac{600 \times 20^3}{5{,}000{,}000 \times 5 \times 3 \times .375^3} = 1.2 \text{ in. Ans.}$$

From formula **4**,

$$W = \frac{13{,}333 \times 5 \times 3 \times .375^2}{20} = 1{,}406 \text{ lb. for each end of spring. Ans.}$$

HELICAL SPRINGS

38. Springs of Round Wire.—A **helical spring** consists of a wire or rod wound into helical coils, as shown

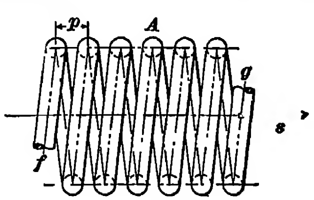


FIG. 23

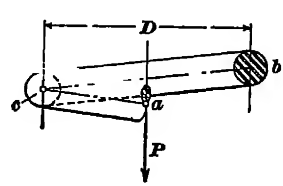


FIG. 24

in Fig. 23, in which *A* is the coiled wire, *fg* its center line, and *p* the pitch, or amount of advance for each turn. The coils may touch one another, or there may be a space between the successive turns, as shown.

In Fig. 24, abc represents part of one coil. If a load P is acting at the point a and the cross-sectional area at c is considered with reference to this load, the wire is subjected to a twisting action, the twisting moment T being $\frac{PD}{2}$, nearly. It is evident that the action of this twisting moment is not limited to the point c, but is also transmitted along the wire, and the effect is the same as if the coil were a straight wire or shaft subjected to simple torsion.

An end view of a spring supposed to be uncoiled into a straight piece of wire is shown at u, Fig. 25. The wire is provided with a pulley B having a diameter D equal to that of the coil in Fig. 24. The action of the load P, Fig. 25, will therefore have the same effect on the wire as the load P, Fig. 24, considered with reference to the point c of the wire. If a load P suspended from the pulley B turns the latter so that the point a_1 moves into the position a_2, then a point b_1 on the wire, while moving into the position b_2, will move through an angle e. The angular deflection of the wire is therefore e°. If s is the length of path through which point a_1 has moved, then

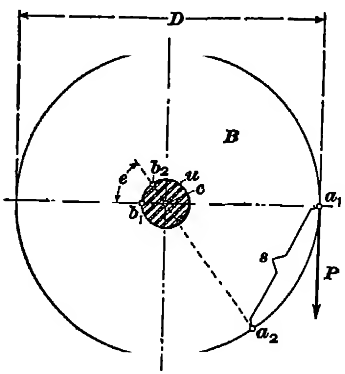


FIG 25

$$s : \pi D = e^\circ : 360^\circ$$

and
$$e = \frac{360\, s}{\pi D} \qquad (1)$$

In Art. 4, it was shown that the twisting moment

$$T = \frac{S_s J}{c} \qquad (2)$$

when S_s = safe fiber stress for shear;

J = polar moment of inertia;

d = diameter of shaft;

c = distance from outer fiber to neutral axis.

For a circular section, $J = \frac{\pi d^4}{32}$ and $c = \frac{d}{2}$. Inserting these values in formula **2**,

$$T = \frac{S_s \pi d^3}{16}$$

If P is the safe load in pounds, the safe twisting moment is $\frac{PD}{2} = \frac{S_s \pi d^3}{16}$; hence,

$$P = \frac{2 S_s \pi d^3}{16 D} = \frac{S_s d^3}{2.55 D}$$

or,

$$P = .3927 \frac{S_s d^3}{D} \qquad (3)$$

39. It has been shown in Art. **5** that the angular deflection of a shaft

$$e = \frac{584 T l}{E_s d^4} \qquad (1)$$

If n is the number of coils in the spring, then the length of the uncoiled spring is $l = n \pi D$, nearly. This value is near enough for most practical purposes.

NOTE —If the correct length is required, the pitch of the coils should be considered, as indicated by p, Fig. 23. In this case $l = n\sqrt{p^2 + (\pi D)^2}$.

Inserting the value $l = n \pi D$ in formula **1**,

$$e = \frac{584 T n \pi D}{E_s d^4}$$

Substituting for e the value given in formula **1**, Art. **38**, and for T the value $\frac{PD}{2}$,

$$\frac{360 s}{\pi D} = \frac{584 P D^2 \pi n}{2 E_s d^4}$$

and

$$s = \frac{8 P D^3 n}{E_s d^4} \qquad (2)$$

The total twist s of the wire u, as represented by the length of the curved path traced by the point a_1 on the pulley B, Fig. 25, will show itself as an axial motion in the coiled spring. This is because each coil is subjected to a twisting action in a direction nearly at right angles to the axis of the

wire. Therefore, the sum of all the resulting twisting motions will also be in a direction at right angles to all the coils; that is, in the direction of the axis of the spring. The direction of this motion for a spring in tension is shown by the arrow s, Fig. 23.

Distinction should be made between the value of P in formula 2 of this article and the value of P in formula 3, Art. 38. In the latter formula, P represents the safe load that the spring may support; in the former, P is the load for which the deflection is to be ascertained. The value used for P in formula 2 must never be greater than the safe load P found by formula 3, Art. 38.

The factor n is not contained in formula 3, Art. 38; consequently, the number of coils n may be varied to give a desired deflection without affecting P.

40. Compression of Springs.—The formulas given in Arts. 38 and 39 apply also to helical springs subjected to compression, since, from Fig. 25, it is immaterial whether the rod u is twisted to the left or to the right. However, the following difference between springs under tension and those under compression should be noted. In a spring under tension, the coils may be in contact while the spring is in its initial position; in a spring under compression, the pitch of the coils, while the spring is in its initial position, must be such that sufficient space is left for the coils to approach the required distance; that is, the pitch p, Fig. 23, must be larger than $\frac{s}{n}$.

EXAMPLE 1.—A helical spring made of $\frac{1}{2}$-inch round steel has a diameter D of 3 inches. (*a*) Calculate the load P that it will carry safely if $S_s = 60{,}000$ pounds. (*b*) Calculate the extension, if the spring has 15 coils and supports a load P of 700 pounds.

SOLUTION.—(*a*) Substituting these values in formula 3, Art. 38,

$$P = \frac{.3927 \times 60{,}000 \times .5^3}{3} = 982 \text{ lb.} \quad \text{Ans.}$$

(*b*) Use formula 2, Art. 39. Thus, if $E_s = 12{,}000{,}000$, then

$$s = \frac{8 \times 700 \times 3^3 \times 15}{12{,}000{,}000 \times .5^4} = 3.024, \text{ or } 3 \text{ in., nearly.} \quad \text{Ans.}$$

EXAMPLE 2.—A helical spring is to be made of $\frac{7}{16}$-inch round steel wire and must show an extension of 5 inches for a load of 500 pounds. Find the diameter of the coils and the required number.

SOLUTION —Applying formula **2**, Art **39**, and solving for the product $D^3 n$,

$$D^3 n = \frac{s E_s d^4}{8 P} = \frac{5 \times 12{,}000{,}000 \times .4375^4}{8 \times 500} = 550$$

Hence, $n = \frac{550}{D^3}$. Inserting a tentative value of $D = 3$,

$$n = \frac{550}{27} = 20 \text{ coils, nearly. Ans.}$$

It is necessary to ascertain whether this value of D will give the spring sufficient strength to support a load of 500 lb By formula **3**, Art. **38**,

$$P = \frac{.3927 \times 60{,}000 \times .4375^3}{3} = 657.7 \text{ lb.}$$

showing that the spring is of ample strength.

41. Working Stress.—Since, owing to its elasticity, a spring is not subjected to sudden shocks, as would be the case with a rigid body under similar conditions, a greater safe working stress may be used. Hence, it is customary to take the value of S_s as one-half the ultimate shearing strength of the material. In preceding examples, the value of S_s for tempered cast steel has been taken as 60,000 pounds. This value is sufficiently safe for conditions in which the spring is required to act only intermittently, as when used in a safety valve or an engine governor, but if the spring is to be in constant action, a lower value of S_s should be used.

42. Table of Strength and Deflection.—In order that the calculations of the strength and deflection of helical springs may be simplified, Table III, computed by J. Begtrup is here inserted. This table is based on the formulas for round steel wire, the derivations of which were given in Arts. **38** and **39.** The symbols used in the table differ in some respects from those employed in the preceding formulas, as will be seen from the list that follows. It should be particularly noted that S_1 represents the deflection of one coil at a load of 100 pounds Thus, in the table,

TABLE III

STRENGTH AND DEFLECTION OF HELICAL SPRINGS OF ROUND STEEL

d = .065 in. No. 16	D_1	.25	.50	.75	1 00	1.25	1 50	1 75	2 00		
	P	35	15	9	7	5	4 5	3 8	3 3		
	s_1	.0276	.3588	1 433	3 562	7.250	12.88	20 85	31 57		
		.0236	.3075	1.288	3 053	6 214	11 04	17 87	27 06		
		.0197	.2562	1.023	2.544	5 178	9 200	14 89	22 55		
d = .120 in. No. 11	D_1	.50	.75	1 00	1 25	1 50	1 75	2 00	2 25	2 50	
	P	107	65	46	36	29	25	22	19	17	
	s_1	.0206	.0937	.2556	.5412	.9856	1 624	2 492	3 625	5 056	
		.0176	.0804	.2191	.4639	.8448	1 392	2.136	3 107	4.334	
		.0147	.0670	.182	.3866	.7040	1 160	1 780	2 589	3 612	
d = .180 in. No. 7	D_1	.75	1.00	1.25	1.50	1 75	2 00	2 25	2 50	2 75	3 00
	P	241	167	128	104	88	75	66	59	53	49
	s_1	.0137	.0408	.0907	.1703	.2866	.4466	.6571	9249	1 256	1 660
		.0118	.0350	.0778	.1460	.2457	3828	.5632	.7928	1 077	1.423
		.0098	.0292	.0648	.1217	.2048	.3190	.4693	.6607	8975	1 186

$d = \frac{1}{4}$ in.	D_1	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	
	P	368	294	245	210	184	164	147	134	123	113	
	s_1	.0199	.0389	.0672	.1067	.1593	.2270	.3109	.4139	.5375	.6835	
		.0171	.0333	.0576	.0914	.1365	.1944	.2665	.3548	.4607	.5859	
		.0142	.0278	.0480	.0762	.1137	.1610	.2221	.2957	.3839	.4883	
$d = \frac{5}{16}$ in.	D_1	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00
	P	605	500	426	371	329	295	267	245	226	209	195
	s_1	.0136	.0242	.0392	.0593	.0854	.1187	.1583	.2066	.2640	.3312	.4089
		.0117	.0207	.0336	.0508	.0732	.1012	.1357	.1771	.2263	.2839	.3505
		.0097	.0173	.0280	.0424	.0610	.0853	.1131	.1476	.1886	.2366	.2921
$d = \frac{3}{8}$ in.,	D_1	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.25
	P	765	663	589	523	473	433	398	368	343	321	301
	s_1	.0169	.0259	.0377	.0528	.0711	.0935	.1200	.1513	.1874	.2290	.2761
		.0145	.0222	.0323	0452	.0610	.0801	.1029	.1297	.1606	.1963	.2367
		.0120	.0185	.0269	.0376	.0508	.0668	.0858	.1081	.1338	.1635	.1972
$d = \frac{7}{16}$ in.	D_1	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.50	5.00
	P	1,263	1,089	957	853	770	702	644	596	544	486	432
	s_1	0081	.0126	0186	.0262	.0357	.0472	.0617	.0772	.0960	.1423	.2016
		.0069	.0108	.0160	.0225	.0306	.0405	.0529	.0661	.0823	.1220	.1728
		.0058	.0090	.0133	.0187	0255	.0337	.0441	.0551	.0686	.1017	.1440

TABLE III—(*Continued*)

$d=\frac{1}{2}$ in.	D_1	2 00	2.25	2.50	2 75	3 00	3.25	3.50	3.75	4 00	4.50	5 00
	P	1,963	1,683	1,472	1,309	1,178	1,071	982	906	841	736	654
	s_1	.0042	.0067	.0099	.0141	.0194	.0259	.0336	.0427	0534	.0796	.1134
		0036	0057	.0085	.0121	.0167	.0222	.0288	0366	.0457	.0683	.0972
		0030	0048	0071	.0101	0139	.0185	0240	0305	.0381	.0569	.0810
$d=\frac{9}{16}$ in.	D_1	2 50	2 75	3 00	3 25	3 50	3 75	4.00	4.25	4.50	5 00	5.50
	P	2,163	1,916	1,720	1,560	1,427	1,315	1,220	1,137	1,065	945	849
	s_1	0056	0081	0112	.0151	0197	.0252	.0316	.0390	0474	.0679	.0935
		0048	0070	0096	0129	0169	.0216	.0271	.0334	0406	.0582	.0801
		0040	0058	0080	0108	0141	.0180	.0225	.0278	0339	0485	0668
$d=\frac{5}{8}$ in.	D_1	2 50	2 75	3 00	3 25	3 50	3 75	4.00	4.25	4.50	5 00	5 50
	P	3,068	2,707	2,422	2,191	2,001	1,841	1,704	1,587	1,484	1,315	1,180
	s_1	0034	0049	0068	.0092	0121	.0155	0196	0243	0297	0427	.0591
		0029	0042	0058	.0079	0104	0133	0168	0208	0254	.0366	.0506
		.0024	0035	.0049	0066	0086	0111	.0140	.0173	.0212	.0305	.0422

$d = 1\frac{1}{16}$ in.												
	D_1	3.00	3 25	3 50	3 75	4.00	4 25	4.50	4.75	5 00	5 50	6 00
	P	3,311	2,988	2,723	2,500	2,311	2,151	2,009	1,885	1,776	1,591	1,441
	s_1	.0043	.0058	.0077	.0100	.0127	.0157	0193	,0233	0279	0388	.0522
		.0037	.0050	.0066	0086	.0108	.0135	0165	.0200	.0239	0333	.0447
		.0030	.0042	0055	.0071	.0090	.0112	.0138	.0167	.0199	0277	0373

$d = \frac{3}{4}$ in.												
	D_1	3 00	3 25	3 50	3 75	4.00	4.25	4 50	4.75	5 00	5 50	6 00
	P	4,418	3,976	3,615	3,313	3,058	2,840	2,651	2,485	2,339	2,093	1,893
	s_1	.0028	.0038	.0051	.0066	.0084	.0105	.0129	.0157	0189	.0264	.0356
		.0024	0033	0044	0057	0072	.0090	.0111	.0135	.0162	.0226	0305
		.0020	.0027	0036	.0047	.0060	.0075	.0093	.0113	0135	.0188	.0254

$d = \frac{7}{8}$ in.												
	D_1	3 50	3 75	4.00	4 25	4.50	4.75	5 00	5 25	5 50	6 00	6.50
	P	6,013	5,490	5,051	4,676	4,354	4,073	3,826	3,607	3,413	3,080	2,806
	s_1	.0021	0027	0035	0045	.0055	.0067	.0081	.0097	.0115	0156	0207
		0018	0024	0030	.0038	.0047	0058	0070	.0083	0098	.0134	.0177
		0015	0020	.0025	.0032	.0039	0048	.0058	.0069	.0082	.0112	.0148

$d = 1$ in.												
	D_1	3 50	3 75	4.00	4.25	4.50	4.75	5 00	5.25	5 50	6 00	6 50
	P	9,425	8,568	7,854	7,250	6,732	6,283	5,890	5,544	5,236	4,712	4,284
	s_1	0012	0016	0021	0026	.0033	0041	0049	0059	0071	.0097	.0129
		0010	.0014	.0018	.0023	0028	.0035	0043	0051	.0061	.0083	0111
		0008	.0011	0015	0019	0023	0029	.0035	.0043	.0051	.0069	0092

D_1 = *outside* diameter of coil;
d = diameter of steel wire or rod;
s_1 = deflection of *one* coil at a load of 100 pounds;
P = safe load in pounds; also the load by which a specified deflection is to be effected;
S = fiber stress in pounds per square inch = 60,000;
E_s = modulus of torsional elasticity.

Comparing the values of D_1 and s_1 with those of D and s used previously, it is seen that

$$D_1 = D + d,$$

also that the *total* deflection s for a given load is

$$s = \frac{n\,s_1\,P}{100} \qquad (1)$$

and

$$n = \frac{100\,s}{s_1\,P} \qquad (2)$$

Table III may also be used in calculations for helical springs made of *square* steel. For square steel, the values given in the table for P must be multiplied by 1.2, and the values for s_1 by .59. Three values of s_1 are given for each separate value of d corresponding to the following values of E_s:

$$E_s = 10{,}000{,}000$$
$$E_s = 12{,}000{,}000$$
$$E_s = 14{,}000{,}000$$

43. Application of Table III.—The use of Table III may be illustrated by the following examples:

EXAMPLE 1.—What is the safe load for a spring made of $\frac{5}{8}$-inch round steel, if the outside diameter D_1 is 3 inches?

SOLUTION.—Descend along the first column of Table III until the value $d = \frac{5}{8}$ in is found. In the second column, an adjacent letter D_1 is found. Proceed to the right along the line headed by D_1 until the value 3 in. is found, and below this, in the line P, is the number 2,422, which is the safe load for the spring.

If this spring is to show a deflection of 2 in for a load of 2,000 lb., ascertain the number of coils required, taking the average value for E_s as 12,000,000 Below 2,422 in the table are three other numbers headed by the letter s_1, the second of these is s_1 = .0058, which corresponds to E_s = 12,000,000. Applying formula **2**, Art. **42**,

$$n = \frac{100 \times 2}{.0058 \times 2{,}000} = 17.24, \text{ or } 17, \text{ coils, nearly. Ans.}$$

EXAMPLE 2.—A spring having an outside diameter of 4 inches is made of 10 coils of $\frac{1}{2}$-inch round steel. Will an extension of 4 inches strain the spring?

SOLUTION.—From Table III, the greatest permissible load P is 841 lb. If $E_s = 12{,}000{,}000$, then $s_1 = .0457$. Inserting these and the other known data in formula **1**, Art. **42**,

$$s = \frac{10 \times .0457 \times 841}{100} = 3.84 \text{ in.}$$

As the spring is extended 4 in., it is supporting an excessive load and will therefore be strained. Ans.

EXAMPLE 3.—Find the safe load for a spring made of $\frac{1}{2}$-inch square steel and having an outside diameter of 3 inches.

SOLUTION.—From the table, $P = 1{,}178$ lb. for round steel, but, as stated in Art. **42**, for square steel, this value should be multiplied by 1.2; hence,

$$P = 1{,}178 \times 1.2 = 1{,}414 \text{ lb. Ans.}$$

EXAMPLE 4.—Find the deflection of the spring in the last example at a load of 1,000 pounds, if it has 15 coils.

SOLUTION.—For square steel, $s_1 = .59\, s_1$, when the latter value of s_1 is that found in Table III for round steel. Applying formula **1**, Art. **42**,

$$s = \frac{15 \times .59 \times .0167 \times 1{,}000}{100} = 1.48 \text{ in. Ans.}$$

SPIRAL SPRINGS

44. **A spiral spring** consists of coils wound spirally on one another in a plane at right angles to the axis of the spring, as shown in Fig. 26. Springs of this class are properly termed *flat spiral springs* to distinguish them from *conical spiral springs*, in which the coils advance longitudinally as well as radially. A railroad buffer spring is an example of the latter class.

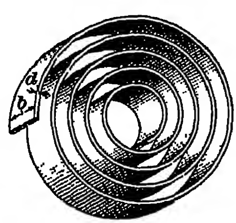


FIG. 26

45. Flat Spiral Springs.—In **a flat spiral spring**, the bending moment is equal to

$$M = Wr = \frac{S_s I}{c}, \qquad (1)$$

in which W = load, in pounds;

r = lever arm of load, in inches, considering center of spring as one end of arm (see Fig. 27);

S_4 = safe outer fiber stress, in pounds per square inch;

d = thickness of bar, in inches;

b = width of bar, in inches.

For a bar or wire of rectangular cross-section, $I = \frac{b\,d^3}{12}$ and $c = \frac{d}{2}$. Inserting these values in formula **1**,

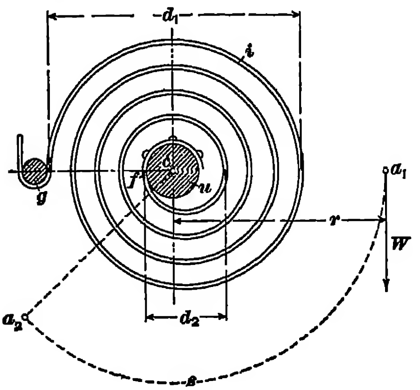


FIG 27

$$W\,r = \frac{S_4\,I}{c} = \frac{S_4\,b\,d^2}{6}$$

and

$$W = \frac{S_4\,b\,d^2}{6\,r} \qquad (2)$$

The parts of the spring that are fastened to the shaft, or that are wound around it so as to be in contact with it, will be unable to bend and are therefore inactive. In Figs. 27 and 28, the active part of the spring is supposed to begin at point f, and the length of the spring or the number of active coils are to be counted from this point.

Let l = developed length, in inches, of all the active coils;

s = distance, in inches, moved through by the point a_1, the point at which the load W is applied (Fig. 27);

$E_t = E_c$ = modulus of elasticity = 30,000,000 for steel;

e = angular deflection in degrees of point a_1.

Then, the following formulas may be evolved:

$$s = \frac{12\,W\,l\,r^2}{E_t\,b\,d^3} \qquad (3)$$

and the angular deflection

$$e = \frac{57.3\, s}{r} \qquad (4)$$

In explanation of Fig. 27, it should be noted that when considering the deflection of the spring i, reference is had to the relative motion between the ends of the spring. In this instance, one end is attached to the stationary stud g, and the other end to the shaft u, which may be turned by the force W acting on the lever arm $r = c\,a_1$. The point a_1 is supposed to move into the position a_2, while the force W acts at right angles to the lever arm $c\,a_1 = c\,a_2$. The linear deflection s is equal to the length of the arc $a_1\,a_2$, while the angular deflection e corresponds to the angle $a_1 c\, a_2$.

46. The length l in formula **3**, Art. **45**, is found by means of the following formula:

$$l = \pi n \frac{d_1 + d_2}{2} \qquad (1)$$

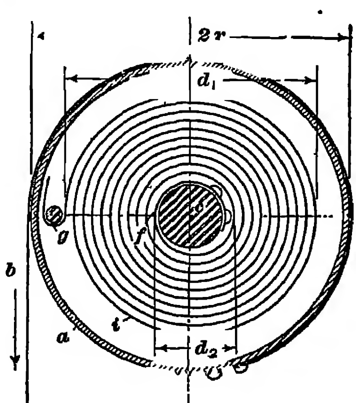


FIG. 28

in which d_1 = diameter of outside coil (Fig. 27), in inches;
d_2 = diameter of inside coil, in inches;
n = number of active coils.

In Fig. 27, for instance, there are four active coils.

If the value of l in formula **1** is inserted in formula **3**, Art. **45**, then

$$s = \frac{18.85\, W r^2 n}{E_t\, b\, d^3}(d_1 + d_2) \qquad (2)$$

EXAMPLE.—A pulley a 18 inches in diameter revolves loosely on the stud c, Fig 28, and supports at intervals a load of 400 pounds by means of the strap b The rotation of the pulley is opposed by a spiral spring i made of a steel band 3 inches wide and $\frac{3}{8}$ inch thick. One end of the spring is attached to the stud g on the pulley, and the other end to the stud c. If the diameters of the outer and inner coils of the

spring are 16 and 4 inches, respectively, and there are 10 active coils, what will be the linear and angular motion of a point on the pulley rim when the load is acting?

SOLUTION.—It is first necessary to ascertain whether the spring will safely support the load of 400 lb. From formula **2**, Art. **45**, $W = \frac{S_4 b d^2}{6r}$. The diameter of the pulley is $2r = 18$ in., hence, $r = 9$ in. Inserting this value of r in the formula and using a value for S_4 of 60,000,

$$W = \frac{60{,}000 \times 3 \times .375^2}{6 \times 9} = 469 \text{ lb.}$$

The spring therefore has sufficient strength to support the load of 400 lb.

From formula **2** of this article,

$$s = \frac{18.85 \times 400 \times 9^2 \times 10}{30{,}000{,}000 \times 3 \times .375^3} \times (16 + 4) = 25.7 \text{ in. Ans.}$$

From formula **4**, Art. **45**,

$$e = \frac{57.3 \times 25.7}{9} = 163.6°. \text{ Ans.}$$

MACHINE DESIGN

Serial 997E (PART 5) Edition 1

DESIGN OF FLEXIBLE GEARING

BELT GEARING

STRESSES IN BELTS

1. Belt Materials.—The material mostly used for belts is *leather* tanned from ox hides. The leather is about $\frac{5}{16}$ inch thick, and is obtained in strips up to 5 feet in length. Belts are made of any required length by joining these strips together. *Single belts* are made of one thickness of leather: *double belts* of two thicknesses of leather.

Cotton may be used for belts that are exposed to dampness. Cotton belts can be made very wide, and without the many joints necessary in leather belts. The required thickness is obtained by sewing together from four to ten plies of cotton duck. Cotton belting is cheaper and stronger than leather belting, but probably less durable.

Rubber belts are made by cementing together plies of cotton duck with india rubber. Rubber belts are more adhesive than leather belts, and hence have greater driving capacity. Rubber belts are considered to be the best to use in damp locations.

2. Belt Friction.—In Fig. 1, the pulleys *a* and *b* are con nected by a belt *c d* subjected to an initial tension T_1, which produces a pressure normal to the contact surfaces at the

arcs of contact of the belt and pulleys. If *a* is the driving pulley revolving in the direction of the arrow, and the lower end of the belt is fastened to the stationary pulley *b*, it is evident that as soon as *a* begins to revolve a change takes place in the tensions of the parts *c* and *d* of the belt. The tension in *c* will increase and that in *d* decrease in a corresponding degree, until, finally, the belt will slip on the pulley *a*. If the normal pressure exerted by the belt against the face of the pulley *a* is P_n, and the force tending to pull the belt along the pulley face is P, then, to cause a slippage of the belt, the value of P must be increased until $P = f P_n$, in which f is the coefficient of friction.

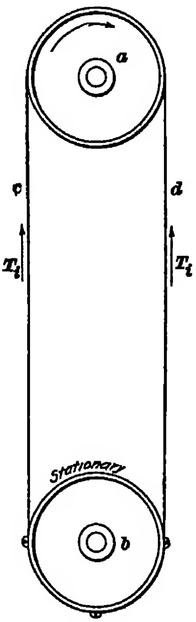


FIG 1

Designating the tensions in the parts *c* and *d* by T_1 and T_2, respectively, it is evident that at any moment the pull P must be equal to $T_1 - T_2$, or

$$T_1 - T_2 = P \qquad (1)$$

At any time, the sum of the tensions T_1 and T_2 is equal to 2 T_i, or $T_1 + T_2 = 2\,T_i$, and

$$\frac{T_1 + T_2}{2} = T_i \qquad (2)$$

It is evident from formula **1**, that for a given value of T_2 the pull P will increase when T_1 is increased. From formula **2**, it is also seen that an increase of T_1 may be secured by an increase of T_i. This would seem to indicate an advantage in increasing the initial tension T_i to a maximum in order to obtain the maximum pull of the belt. However, several factors limit this increase in tension. A belt run under too high tension will deteriorate very quickly and will cause undue wear on the shaft bearings. The proper methods by which to increase the driving power of a belt are to increase the contact area of the belt, or to increase the coefficient of friction f between belt and pulley by applying one of the preparations made for this purpose.

3. In Fig. 2, the pulley a is driving the pulley b. On the shaft of the latter is also a pulley c, from which a weight W is suspended. If P_n is the belt pressure normal to the surface of the pulley b, and P the frictional force developed at the contact surface of the belt, then the driving moment of pulley b is $P \times r$, and the resisting moment of pulley c is $W \times r_1$. As $P = T_1 - T_2$,

$$Pr = Wr_1 = (T_1 - T_2)r$$

4. Effect of Centrifugal Tension. The pressure between a belt and a pulley is not the same at high and at low speeds. As each particle of the belt moves in the curved path around the pulley, centrifugal force tends to move it away from the pulley. The result is that the surface pressure of the belt is *decreased* and that the belt tension is *increased*. Up to a belt speed of 2,000 feet per minute, this centrifugal force may be left out of consideration, but beyond this speed its effects increase very rapidly. These effects are further increased by the stiffness of the belt, which resists the bending action constantly taking place as the belt makes contact with or leaves the pulley. It follows, therefore, that formula **2**, Art. **2**, according to which the sum of T_1 and T_2 is equal to 2 T_i, does not always hold true.

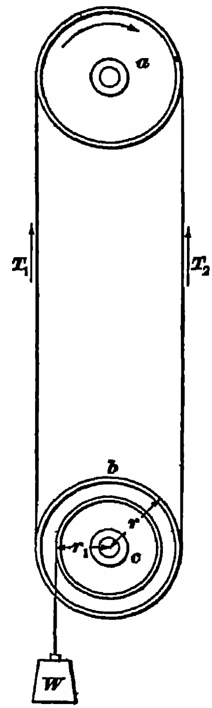


FIG 2

5. Belt Calculations.—To determine the cross-sectional area of a belt required to transmit a given horsepower, it is necessary to ascertain the maximum tension T_1 to which the driving side of the belt is exposed. Formula **1**, Art. **2**, does not give this value, but simply the difference between T_1 and T_2. The value of T_1 is generally ascertained by the following formula, the derivation of which must be omitted, as it involves the use of higher mathematics.

$$\log \frac{T_1}{T_2} = 2.729 f (1 - z) n \qquad (1)$$

In this formula, f is the coefficient of friction between leather and the material of the pulley, and n the fractional part of the circumference of the pulley embraced by the belt. If the angle subtended by the arc of contact is a, then $n = \frac{a}{360}$.

Let w = weight of belt per cubic inch;
V = velocity of belt, in feet per minute;
t = working tension of the belt, in pounds per square inch;
z = a factor depending on the centrifugal force of the belt.

Then, from mechanics, it can be shown that

$$z = \frac{w V^2}{9{,}660 t} \qquad (2)$$

For values of V below 2,000, the factor $1 - z$ may be omitted from formula **1**, when it becomes

$$\log \frac{T_1}{T_2} = 2.729 f n \qquad (3)$$

Formula **1**, Art. **2**, used in combination with formulas **1** or **3**, Art. **5**, will give the value of T_1. The tensile stress T_1 is resisted by the working strength of the belt, which is area $\times t = b \times h \times t$; that is,

$$T_1 = b h t \qquad (4)$$

in which b = width of belt, in inches;
h = thickness of belt, in inches.

When the thickness of the belt is given or assumed, then

$$b = \frac{T_1}{h t} \qquad (5)$$

6. General Remarks on Belting.—The coefficient of friction f for leather belts on cast-iron pulleys varies between .2 and .5, a safe average value being .3.

The working tension t depends on the strength of the joints in the belt. In general, t has the following limiting values in pounds per square inch of area of cross-section: For metal joints, 250; for laced joints, 300; and for cemented joints, 400.

The average weight of good leather per cubic inch is $w =$ 035 pound.

The ultimate tensile strength of leather varies from 3,000 to 5,000 pounds per square inch, but the average working strength for a belt is about 300 pounds per square inch.

The speed of a belt V is preferably taken at from 4,500 to 6,000 feet per minute. There is less stress on a belt and bearings when a belt is running at a high speed and under low tension.

Table I gives the values of z, as found by means of formula **2**, Art. **5**, for various belt speeds, w being taken as .035 and t as 300.

TABLE I

VALUES OF THE FACTOR z FOR VARIOUS SPEEDS

V	2,000	2,500	3,000	3,500	4,000	4,500	5,000	5,500	6,000	6,500	7,000	7,500
z	048	075	.109	148	193	245	.302	.365	434	510	.592	.679

In general, the thickness of belts varies between $\frac{5}{32}$ and $\frac{5}{16}$ inch for single belts, and between $\frac{5}{16}$ and $\frac{7}{16}$ inch for double belts.

7. The following examples illustrate the application of the preceding formulas to belt speeds below and above 2,000 feet per minute:

EXAMPLE 1.—A single belt, 22 inch thick, connects two pulleys, each 24 inches in diameter, making 300 revolutions per minute The arc of contact is 180°, and the working tension is to be 300 pounds per square inch. If the belt is to transmit 30 horsepower, what should be its width?

SOLUTION —From the formula $PR = 63{,}025\,\frac{H}{N}$, given in *Strength of Materials*,

$$P = 63{,}025\,\frac{H}{RN} = \frac{63{,}025 \times 30}{12 \times 300} = 525\text{ lb , nearly}$$

From formula **1**, Art **2**, $T_1 - T_2 = 525$

If the belt speed is greater than 2,000 ft per min , formula **1**, Art. **5**, is used to ascertain the values of T_1 and T_2; if less than 2,000 ft , formula **3**, Art **5**, is used The belt speed is

$$V = \frac{2\,\pi\,RN}{12} = \frac{2 \times 3.1416 \times 12 \times 300}{12} = 1{,}885\text{ ft. per min.}$$

Hence, using a value for $f =$ 3, and applying formula **3**, Art **5**,

$$\log \frac{T_1}{T_2} = 2\ 729 \times .3 \times \tfrac{180}{360} = .40935$$

and $\frac{T_1}{T_2} = 2.567$ lb., nearly, from which $T_2 = \frac{T_1}{2\ 567}$.

However, $$T_1 - T_2 = 525;$$

hence, $$T_1 - \frac{T_1}{2.567} = 525,$$

and $$T_1 = \frac{525 \times 2\ 567}{1\ 567} = 860 \text{ lb.}$$

From formula **5**, Art. **5**,

$$b = \frac{860}{.22 \times 300} = 13 \text{ in., nearly. Ans.}$$

EXAMPLE 2.—A pulley 24 inches in diameter transmits 50 horsepower when making 560 revolutions per minute The belt has an arc of contact of 160° and is $\frac{3}{8}$ inch thick. Assuming that f is equal to .3, the weight per cubic inch of belt is 035 pound, and the working tension t is 300 pounds, what is the required width of the belt?

SOLUTION.—From the formula $PR = 63{,}025 \frac{H}{N}$,

$$P = 63{,}025 \frac{H}{RN} = 63{,}025 \times \frac{50}{12 \times 560} = 469 \text{ lb., nearly}$$

From formula **1**, Art. **2**, $T_1 - T_2 = 469$.

By formula,

$$V = \frac{2 \pi R N}{12} = \frac{2 \times 3.1416 \times 12 \times 560}{12} = 3{,}519 \text{ ft., nearly}$$

Hence, the values of T_1 and T_2 are found by formula **1**, Art. **5**. The value of z is ascertained from Table I, or from formula **2**, Art. **5**; thus,

$$z = \frac{035 \times 3{,}519^2}{9{,}660 \times 300} = .1495$$

Applying formula **1**, Art. **5**,

$$\log \frac{T_1}{T_2} = 2.729 \times .3 \times (1 - .149) \times \tfrac{160}{360}$$
$$= 2.729 \times .3 \times .8505 \times \tfrac{160}{360} = .3094$$

and $$\frac{T_1}{T_2} = 2\ 04.$$

As $T_1 - T_2 = 469$ and $T_2 = \frac{T_1}{2\ 04}$, $T_1 - \frac{T_1}{2\ 04} = 469$; therefore,

$$T_1 = \frac{469 \times 2\ 04}{1.04} = 920 \text{ lb., nearly}$$

From formula **5**, Art. **5**,

$$b = \frac{920}{.375 \times 300} = 8\ 18, \text{ or } 8\tfrac{1}{4}, \text{ in., nearly. Ans.}$$

PULLEYS

8. Stresses in Pulley Rims.—In a revolving pulley there is a centrifugal force acting on each minute part of the pulley rim, tending to force it outwards in a radial direction. The action on the rim as a whole is as if it were part of a cylindrical shell subjected to an internal pressure. This pressure tending to stretch the rim is resisted by the tensile resistance of the rim. The pulley arms are also subjected to centrifugal forces, but the radial stress produced in each is smaller, because the parts *c*, Fig. 3, near the rim revolving at maximum velocity are, relatively, small in mass and therefore exert but little radial pull. Tensile stresses will consequently arise at the places *a*, which stresses will be further increased by the initial stresses existing in the arms, as a result of their contraction in cooling after being cast.

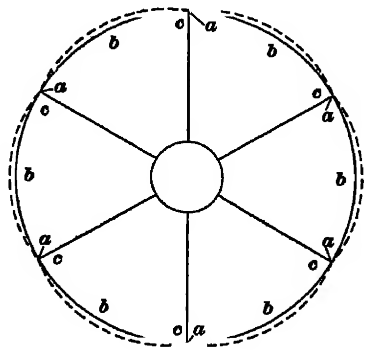


FIG 3

As the parts *a* of the rim are restrained from responding to the centrifugal force, and the parts *b* are free to do so, the rim will have a tendency to assume the form shown by the dotted lines. The parts *a b a* are under stresses similar to those of a beam uniformly loaded and fastened at both ends.

9. Additional stresses are introduced when the pulley is driven by a belt as shown in Fig. 4. In this case, the rim is pulled in the direction of the arrow and tends to bend the arms into the positions indicated by the dotted lines. The arms in bending affect the adjoining parts of the rim, the tendency being to break the latter into separate segments at the points *b*. The positions that these segments would seek to occupy are indicated by the dotted lines c, c_1. As the rim

resists any separation at the points *b*, bending stresses are produced at the points *a*, *b*, and *d*.

Other stresses, such as the compressive stresses, are produced by the belt tension, but for practical designing it is sufficient to consider the predominating stresses and to allow for the others by giving an extra margin of safety. The stresses generally considered are those resulting from the action of the centrifugal force on the rim and the bending forces on the arms.

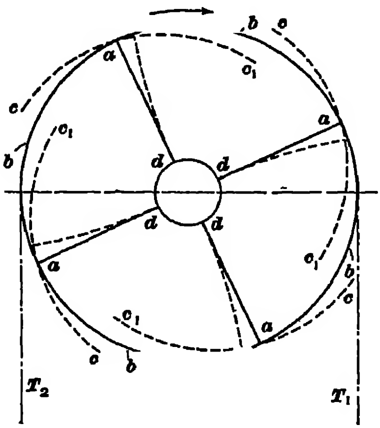


FIG 4

10. Safe Speeds.—Considering the action of the centrifugal force, it is desirable to ascertain that velocity of the rim which will be within safe limits. This velocity may be calculated by a formula from higher mathematics, the results found being given in Table II, in which

V = velocity of rim, in feet per minute;
v = velocity of rim, in feet per second;
s_c = tensile stress per square inch of cross-sectional area of rim, due to centrifugal force.

TABLE II

TENSILE STRESS PRODUCED BY CENTRIFUGAL FORCE

V	4,200	4,800	5,400	6,000	9,000	12,000
v	70	80	90	100	150	200
s_c for cast iron . .	475	620	785	969	2,170	3,876
s_c for wrought iron	511	668	845	1,044	2,348	4,174

If the ultimate strength of cast iron is taken as 20,000 pounds, it may be shown that a speed v of 446 feet per second will cause fracture. In view of the uncertain qualities

of cast iron and the unknown stresses existing in cast-iron pulleys, it is not considered safe to make s_c larger than 1,000 pounds, which corresponds to a velocity of about 100 feet per second. Generally, however, in practice, a rim velocity of 1 mile a minute, or 88 feet per second, is regarded as a safer limit. It can be shown that for a pulley of cast iron $s_c = .09717\ v^2$. For convenience in making calculations, this expression may be shortened to $s_c = .1\ v^2$.

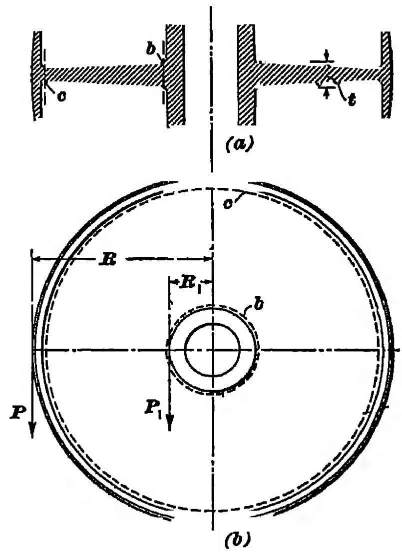


FIG 5

11. Stresses in a Pulley Web.—A pulley with a continuous web is shown in Fig. 5, (*a*) being a transverse section and (*b*) an elevation. A pulley of this form is supposed to be subjected simply to shearing stresses in the web. The resisting area is annular and may be supposed to be located along the dotted circle *b*, as the cross-sectional area represented by the circle *c* is so much in excess that it need not be considered.

Let P = pull at circumference of pulley;
R = radius of same;
R_1 = radius of hub;
P_1 = pull at the hub.

Then, $$PR = P_1 R_1$$

and $$P_1 = \frac{PR}{R_1}$$

If t is the thickness of web at the hub, the cylindrical area of shear is $2\pi R_1 t$. If S_s is the safe shearing stress, then the resistance to shear of the section is $2\pi R_1 t S_s$. Equating the value of the shearing force P_1 and the shearing resistance,

$$\frac{PR}{R_1} = 2\pi R_1 t S_s$$

and
$$t = \frac{PR}{2\pi R_1^2 S_s}$$

S_s may be taken as 1,600 pounds.

12. Stresses in Pulley Arms.—A pulley with arms, as shown in Fig. 6, may be considered as one with the web partly removed. Designating the cross-sectional area of each arm by A and their number by n, the total shearing area is $n \times A$, provided all the arms take equal shares of the turning force exerted by the belt. But, as the rim is very thin, it is probable that some arms work under greater stress than others. If the angle of belt contact is 180°, it is safe to suppose that only $\frac{n}{2}$ arms transmit the turning moment to the hub. The force acting at the outer end of each arm is then

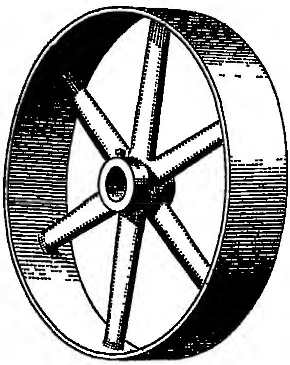


FIG. 6

$$\frac{P}{\frac{n}{2}} = \frac{2P}{n}$$

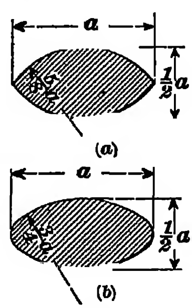


FIG. 7

The arms are usually of elliptical section, as shown in Fig. 7 (a) and (b). It is customary to make the thickness one-half the width. The sectional area of each arm is approximately equal to π times the product of the half diameters, or

$$\pi \times \frac{a}{2} \times \frac{a}{4} = \frac{\pi a^2}{8}$$

To ascertain the shearing resistance of one arm at a section near the hub, as at e, Fig. 8, it is necessary to determine the turning moment at this point.

Let R_1 = distance of section from hub center;
P_1 = turning force on one arm at this point.

Then, $$\frac{2PR}{n} = P_1 R_1$$

and $$P_1 = \frac{2PR}{nR_1}$$

Equating this force and the shearing resistance of the section,

$$\frac{2PR}{nR_1} = \frac{\pi a^2}{8} S_s$$

and

$$a = 2.26\sqrt{\frac{PR}{nR_1 S_s}}$$

13. The shearing resistance of the arms is of less importance than the bending moment at *e*, Fig. 8. A pulley arm may be considered as fixed at the hub and free at the end, at which point it is supposed to support a load of $\frac{2P}{n}$. To be on the safe side, the length of each arm may be considered as being equal to the radius R of the pulley. The bending moment is then $\frac{2PR}{n}$. The moment of resistance is $\frac{I}{c}S_t = \frac{\pi a^3}{64}S_t = .049\,a^3 S_t$.

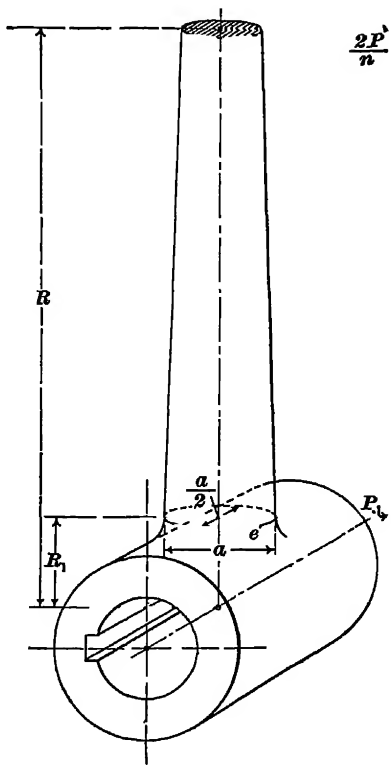


FIG 8

Equating the values of the bending and resisting moments,

$$\frac{2PR}{n} = .049\,a^3 S_t$$

and $$a = 3.44\sqrt[3]{\frac{PR}{nS_t}}$$

S_4, which is the safe unit stress in flexure, may here be taken as 2,500 pounds per square inch for cast iron.

By comparing the formulas for the dimension a in Arts. **12** and **13**, it will be seen that the value of a required to resist bending is the larger. Hence, if the arms are made large enough to sustain the bending stresses, the shearing stresses may be neglected.

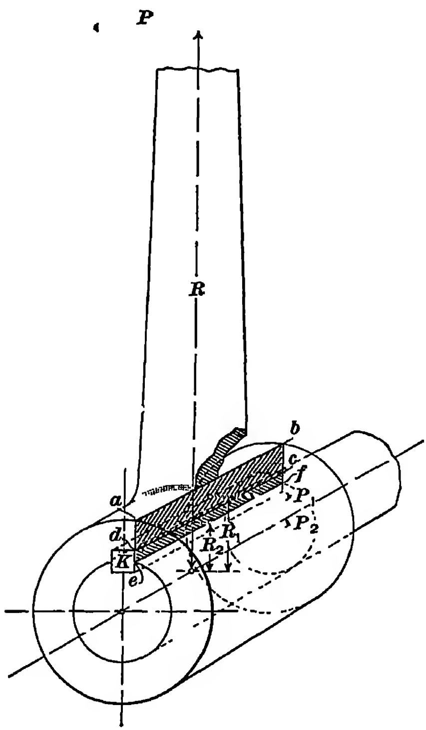


FIG 9

14. Stresses in the Hub.—Fig. 9 shows a diagrammatic view of a pulley hub (supposed to be transparent) with part of one arm, the other parts being broken away. It is assumed that the pulley rim has a radius R and that the force P is acting at the rim in the direction indicated. As the shaft and hub react against the turning force P, it is necessary to find the magnitude of the reactions in various parts of the hub in order to insure against failure of the latter. The key K transmits the turning force of the pulley to the shaft. If P_2 indicates the reaction at the surface of the shaft and R_2 is the radius of the latter, then $PR = P_2 R_2$ and $P_2 = \frac{PR}{R_2}$.

The reaction of the key is exerted against the shaded surface $cdef$ in the hub, subjecting it to a crushing

stress. Formulas for finding the dimensions of a key large enough to prevent crushing are given in *Machine Design*, Part 2.

When the pulley arm is moving in the direction indicated, the key K tends to restrain the part of the hub located to the right of the shaded rectangle $a\,b\,c\,d$ from revolving with the part located to the left of this rectangle. Tensile stresses will therefore arise at this place, and if it is assumed that P_1 represents the location of the resultant of these stresses and R_1 its distance from the center of the shaft, then $P_1 R_1$ represents its moment. Equating the turning moment with the latter moment, then $PR = P_1 R_1$ and $P_1 = \frac{PR}{R_1}$.

The force P_1 is equivalent to the tensile resistance offered by the hub at $a\,b\,c\,d$; the area involved being $a\,b \times b\,c$. If S_1 is the safe fiber stress in tension, and $a\,b = l_1$ and $b\,c = t_1$, then $P_1 = l_1 \times t_1 \times S_1$. Equating the two values of P_1,

$$\frac{PR}{R_1} = l_1\,t_1\,S_1$$

and

$$l_1\,t_1 = \frac{PR}{R_1\,S_1}$$

15. The tensile stresses that the area $a\,b\,c\,d$, Fig. 9, is supposed to withstand in this case are excessive, and would correspond to a case in which a lever keyed to a shaft transmits a given load to the latter. In the present instance, the conditions are more favorable, because the total load is divided among several arms. Likewise, consideration should be given to the connections between the arms and the rim, since they act as braces and materially diminish the tensile stresses in the hub. A material increase of the area $l_1\,t_1$ will be obtained by locating the key under one of the arms, as in the present case, instead of between two arms. By locating the key under one arm, the lower part of the arm itself would constitute part of the area $a\,b\,c\,d$.

16. Stresses in Split Pulleys.—The stresses in the rim of a **split pulley,** shown in Fig. 10, may be considered in the same manner as was done with the pulley

having a solid rim; that is, by leaving the arms out of consideration and assuming that the centrifugal force acts directly on the two rim halves, tending to separate them at the joints. Considered in this manner, the whole centrifugal pull exerted by the rim is taken up by the bolts at the joints, subjecting them to tensile stresses.

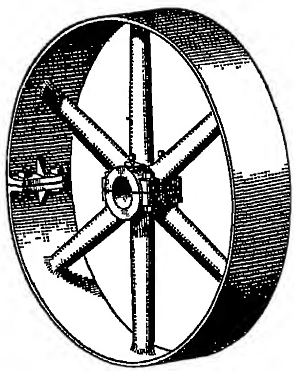


FIG 10

The centrifugal force acting on other parts of the pulley, as the arms and the hub, may be considered as finally acting on the bolts in the hub joint, which joint must also resist the radial pressure of the key or the setscrews, if any are used.

According to the formula of Art. **10**, $s_c = .1\,v^2$. If A is the cross-sectional area of the rim in square inches, the tensile strength of the rim is $A\,s_c = .1\,A\,v^2$ At the rim joint, this stress must be resisted by the bolts. If n is the number of bolts, and d the diameter of a bolt at the root of the thread, the combined area of the bolts is $\frac{n\pi d^2}{4}$, and their safe tensile strength is $\frac{n\pi d^2 S_1}{4}$. In this formula, S_1 is the safe tensile stress. Equating the centrifugal tension and the resistance of the bolts, and neglecting the moment of the lugs,

$$.1\,A\,v^2 = \frac{n\pi d^2 S_1}{4}$$

and

$$d = .357\,v\sqrt{\frac{A}{n\,S_1}}$$

17. By reason of the position of the flange bolts relative to the rim, the flanges tend to tip around the point c. Fig. 11 (a), when the rim is subjected to centrifugal force. Considering each flange as a beam supported at its inner end c, and lifted in a tangential direction at its outer end e,

the pull of the bolts, situated at a distance $\frac{b}{2}$ from either end would constitute the resisting load P. The bolts are located at the distance $\frac{b}{2}$ from the end of the flange, as this is the most unfavorable position in regard to the bending moment. At p_1 and p_2 are two reactions, each equal to $A s_c$, and the load P is equal to the tensile strength of the rim area, or $\frac{n \pi d^2 S_1}{4}$, as already explained.

It is required to ascertain the dimensions of a flange that will support this load. Fig. 11 (b) shows a sectional elevation of a flange taken through the center of the bolt holes d_1. Let w_1 be the width, t_1 the thickness, and b the length of the

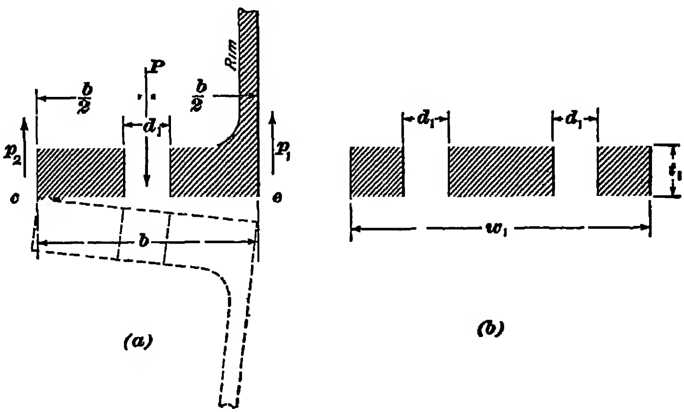


FIG. 11

flange. According to *Strength of Materials*, Part 2, the resisting moment of this rectangular section is $S_4 \frac{I}{c} = S_4 \frac{w_1 t_1^2}{6}$. In this instance, the area under maximum bending stress is not equal to the full width of the flange, it being reduced by the bolt holes. The actual width is therefore $w_1 - n d_1$. The resisting moment is then $S_4 \frac{(w_1 - n d_1) t_1^2}{6}$, if S_4 is the safe stress of the flange material in flexure.

According to *Strength of Materials*, Part 2, the maximum bending moment in a beam loaded in this manner is $\frac{W l}{4}$. The load W in this case is identical with the load P supported by the bolts, and the combined sectional area of the bolts is $\frac{n \pi d^2}{4}$, according to the formula of Art. **16.** Evidently, the area intended to resist the bending moment must be equal in strength to that of the bolts resisting tension. Hence, if the force resisting P is $W = \frac{n \pi d^2 S_1}{4}$ and $l = b$,

then, $$\frac{W l}{4} = \frac{n \pi d^2 S_1 b}{4 \times 4}$$

Equating the values of the two moments,

$$\frac{n \pi d^2 S_1 b}{4 \times 4} = \frac{(w_1 - n d_1) t_1^2 S_4}{6}$$

and $$t_1 = 1.09\, d \sqrt{\frac{n S_1 b}{S_4 (w_1 - n d_1)}}$$

It is evident that the tensile stresses produced in the bolts will be less if the bolts are nearer the rim.

The dimensions found for the rim flanges and the bolts are on the safe side, because the supporting action of the arms and hub has been left out of consideration.

The stresses in the hub bolts are not so easy to determine, but, relatively, they are of less importance, because the greater stress comes on the rim bolts. After suitable dimensions are found for the hub, the hub bolts are so proportioned that they will appear in harmony with the hub.

18. Approximate Formulas for Pulleys.—For ordinary cases, the dimensions of a pulley may be found by empirical formulas that do not give consideration to the stresses involved. In some cases, these formulas give an excess of material; in others, the resulting pulley shows a deficiency in strength at important points.

In the rules and formulas for dimensions of pulleys, the following symbols will be used to represent the various dimensions:

b = width of belt running on pulley;
B = width of pulley rim;
d = diameter of shaft on which pulley is keyed;
t = thickness of pulley rim at edge;
a = width of arm at center of pulley;
w = thickness of hub of pulley;
l = length of hub of pulley;
n = number of arms;
D = diameter of pulley;
R = radius of pulley;
s = swell at center of pulley rim.

All dimensions to be in inches.

NOTE.—To obtain a, the arms are supposed to extend through the hub to the center of the pulley. (See Fig. 17.)

19. The general form of the pulleys shown in Figs. 6 and 10 corresponds very closely to the best modern American practice. The split pulley has the advantage of being more easily put on the shaft, especially when the shaft is in position or has other pulleys already on it.

When the amount of power to be transmitted by a pulley is small, it may be fastened to the shaft by setscrews. Split pulleys are also made so that the bolts through the hub will serve as a clamp to draw the hub tight enough on the shaft to prevent slipping with small loads. When the amount of power to be transmitted is considerable, pulleys should be fastened with keys, and in some cases both keys and setscrews are provided.

20. Rim of Pulley.—The rim of a pulley is usually of the form shown in Fig. 12. If the rim is crowned, the curve may be drawn with a radius of from $3B$ to $5B$, in which case s is about $\frac{B}{20}$ to $\frac{B}{40}$. The width B is made from $\frac{9}{8}b$ to $\frac{5}{4}b$.

For the thickness t, the following formula gives results that agree well with the practice of good shops:

$$t = \frac{B+D}{200} + \tfrac{1}{16} \text{ inch}$$

For double belts, $\frac{1}{8}$ inch may be added to the thickness obtained by the formula just given.

There should be a taper on the inside of the rim amounting to about $\frac{1}{2}$ inch per foot, as indicated by t_1, Fig. 12. This is to facilitate the removal of the pulley from the mold.

21. Flange Pulleys.—When there is a liability of the belt frequently slipping, caused by fluctuations in the power transmitted, the pulley rim may be cast with flanges, as shown in Fig. 13.

22. Arms of Pulleys.—Pulley arms are generally straight, as in Figs. 6 and 10, though curved arms are occasionally used.

The number of arms to be used is largely a matter of

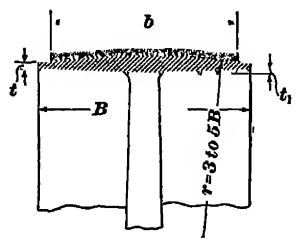


FIG 12

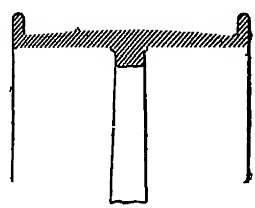


FIG 13

judgment, but in practice, for all sizes of pulleys under 10 feet in diameter, by far the greater number is made with six arms. Eight or ten arms are sometimes used for pulleys above 6 feet in diameter, and for very small sizes, four arms are sufficient.

According to the formula of Art. **13**, the width of the pulley arm at the center of the pulley is $a = 3.44\sqrt[3]{\frac{PR}{nS_4}}$. Inserting a value for $S_4 = 2{,}500$,

$$a = .25\sqrt[3]{\frac{PR}{n}} \qquad (1)$$

$P = T_1 - T_2$, the difference of the belt tension, is usually not known exactly. Its maximum value, however, can easily be found, as follows:

For single belts $\frac{7}{32}$ inch thick, an average tension of 320 pounds per square inch of section may be allowed. Then, $T_1 = \frac{7}{32} \times 320 = 70$ pounds per square inch of width of belt.

Usually, P is not more than one-half of T_1, and its maximum value may be taken at say 50 pounds per inch of width. For double belts, take $P = 100$ pounds per inch of width. Then, $P = 50\,B$ for single belts, and $P = 100\,B$ for double belts.

Substituting these values of P in formula **1**,

$$\left.\begin{aligned} a &= .92\sqrt[3]{\frac{B\,R}{n}}, \text{ for single belts} \\ a &= 1.16\sqrt[3]{\frac{B\,R}{n}}, \text{ for double belts} \end{aligned}\right\} \qquad \textbf{(2)}$$

The section of the arms was shown in Fig. 7. The taper in the width and thickness of the arms toward the rim may be made $\frac{1}{2}$ inch per foot.

For very wide pulleys it is sometimes desirable to use two sets of arms, as shown in Fig. 14. To calculate the size of the arms in this case, the pulley may be considered as being made of two pulleys each having a width of $\frac{1}{2}\,B$. Then, find the dimensions of the arms, as before, and multiply these dimensions by $\sqrt[3]{\frac{1}{2}} = .8$, nearly.

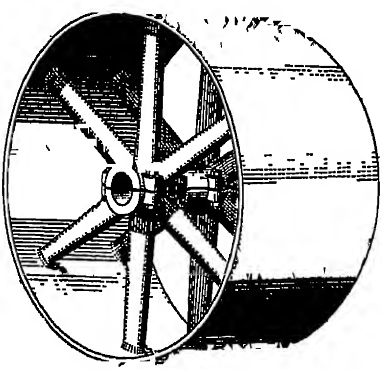


FIG 14

EXAMPLE —Calculate the size of the arms of a six-arm pulley having a diameter of 30 inches and a 6-inch face.

SOLUTION —For a single belt the width at the center of pulley is found by applying formula **2**. Thus,

$$a = 92\sqrt[3]{\frac{6 \times 15}{6}} = 2.27 = 2\tfrac{1}{4} \text{ in}, \text{ nearly. Ans.}$$

The taper of the arms being $\frac{1}{2}$ in. per foot, it follows that at the rim the width is

$$2.25 - (\tfrac{1}{2} \times 1\tfrac{1}{4}) = 2.25 - \tfrac{5}{8} = 1\tfrac{5}{8} \text{ in. Ans.}$$

The thickness of the arms at the center is $2.25 \times \frac{1}{2} = 1\frac{1}{8}$ in. Ans.
The thickness at the rim is $1\frac{5}{8} \times \frac{1}{2} = \frac{13}{16}$ in. Ans

23. Hub of Pulley.—The thickness of the hub may be obtained from the following formula:

$$w = \frac{B + R}{32} + \frac{1}{8} \text{ inch}$$

The length of the hub may be $l = \frac{2}{3} B$ to B.

The hub should taper toward each end at the rate of about $\frac{1}{2}$ inch per foot.

The keyway may be calculated by the rules given in *Machine Design*, Part 2.

24. Loose Pulleys.—Pulleys that run loose on a shaft should have longer hubs than those fastened with keys. The hubs may also be lined with brass bushings if desired.

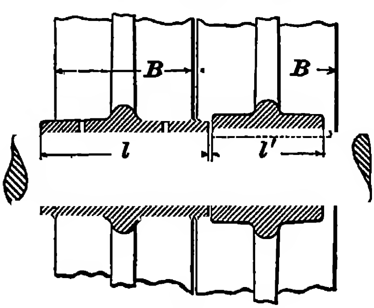


FIG 15

Where a fast and a loose pulley are placed together on a shaft, as shown in Fig. 15, the length l of the hub of the loose pulley may be $l = 1.2 B$, and that of the fixed pulley $l' = .8 B$. The thickness of the hub of the loose pulley may be less than that of the fast pulley on account of its increased length.

Some arrangement must be provided for oiling the loose pulley; generally, the hub has one or two oil holes drilled through it.

25. Split Pulleys.—For split pulleys the size of the bolts at rim and hub may be determined in the following manner:

Let A_1 = area of cross-section of rim;
A_2 = area of cross-section of hub along the line of division,
a_1 = section at root of thread of bolt or bolts at rim;
a_2 = section at root of thread of bolt or bolts at hub.

Then, $$a_1 = \frac{A_1}{4} + \tfrac{1}{4} \text{ square inch}$$

$$a_2 = \frac{A_2}{4} \text{ square inch}$$

EXAMPLE —The hub of a split pulley is 4 inches long and $1\frac{1}{4}$ inches thick. If the hub is held by four bolts, what should be the diameter of the bolts?

SOLUTION.— $A_2 = 4 \times 1\frac{1}{4} = 5$ sq. in. By the formula,

$$a_2 = \frac{A_2}{4} = \tfrac{5}{4} \text{ sq. in} = \text{net section of four bolts}$$

$$\tfrac{5}{4} \div 4 = \tfrac{5}{16} \text{ sq in} = \text{net section of one bolt}$$

Hence, the diameter of bolt is $\frac{3}{4}$ in. Ans.

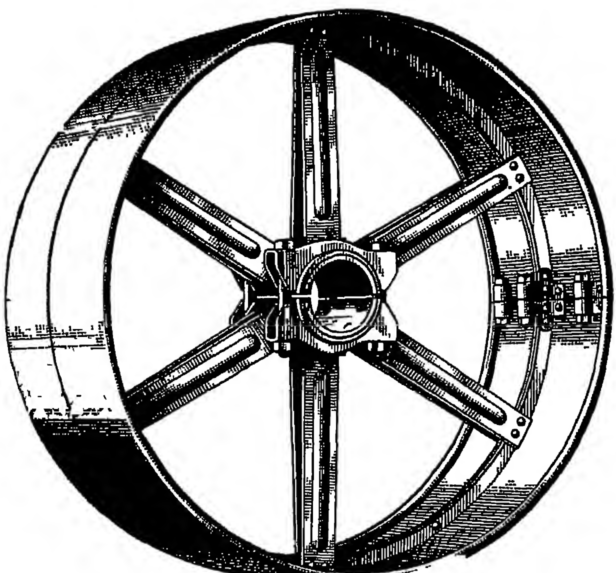


FIG. 16

26. Steel and Wrought-Iron Pulleys.—Pulleys made of **wrought iron** are coming into extensive use, and they possess important advantages over those made of cast iron. They are lighter and stronger, and are free from the initial stresses to which cast-iron pulleys are liable. Owing to the stresses due to centrifugal force, cast-iron pulleys cannot be

safely run at very high speeds; wrought-iron pulleys, however, may be used at almost any reasonable speed, because of the greater tenacity of the material of which they are made. These pulleys are usually split.

A further advance along these lines is the combination of **steel** and cast iron, the latter metal being used for the hub and the arms, and steel for the rim. Pulleys are also now made entirely of steel stampings, an example of which is shown in Fig. 16.

27. Counterbalance.—Pulleys that run at high speeds must be carefully balanced; that is, the center of gravity of the

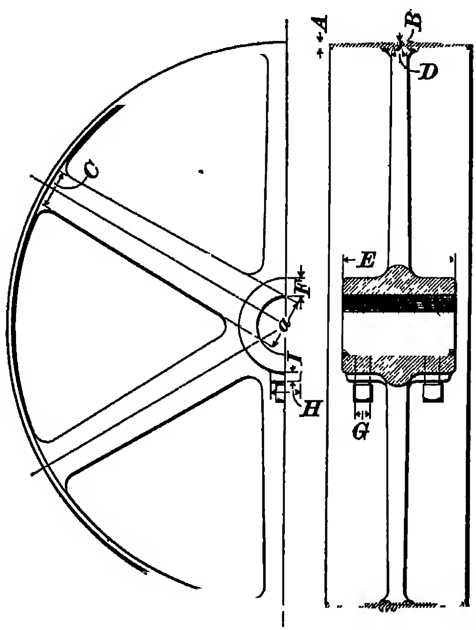


FIG. 17

pulley must correspond with the center of the shaft, otherwise there will be a heavy stress on the shaft and bearings. Since it is seldom possible to make the pulley exactly symmetrical, the difference in weight of the heavy side is compensated for by riveting weights to the inside of the rim on the light side.

28. Examples of Belt Pulleys.—Table III gives the dimensions of a set of cast-iron belt pulleys ranging from 6 to 72 inches in diameter, as made by a well-known manufacturing company. These pulleys are so designed that the number of patterns may be kept within reasonable limits, and at the same time have the dimensions correspond as nearly as possible with well-established rules.

The letters over the columns of dimensions given in the table correspond to the letters in Fig. 17.

In all cases the number of arms is six, and the arms increase in size toward the hub with a taper of $\frac{1}{2}$ inch per foot. All dimensions in the table are in inches.

29. Application of Formulas for Pulleys.—The following examples will show how the formulas referring to pulleys are used and also how some of the required dimensions may be ascertained from Table III.

EXAMPLE 1 —A belt 13 inches wide exerts a pull of 860 pounds on a 24-inch pulley The pulley has six arms and revolves at a speed of 300 revolutions per minute If the pulley is made of cast iron and the safe fiber stress is taken at 2,500 pounds, find dimensions of the pulley.

SOLUTION —The rim speed in this instance is 1,885 ft. per min., which, according to Table II, is a safe value

According to the formula of Art. **20**, the width of the rim is $B = \frac{9}{8}$ to $\frac{5}{4}\,b$. Selecting the fraction $\frac{9}{8}$, then

$$B = \frac{9}{8} \times 13 = 14.75 \text{ in}, \text{ nearly}$$

A pulley rim cannot ordinarily be cast so thin that there is any danger of its breaking, except under the tension due to centrifugal force. The rim is therefore generally made as thin as it can be cast successfully. As a guide, Table III may be consulted, here, for a rim 24 in. in diameter and 16 in wide, the dimensions at the center and edge of rim are $\frac{3}{8}$ and $\frac{1}{4}$ in., respectively.

The important dimension to consider is the width of the arms at the hub. This is found by the formula of Art. **13**, in which

$$a = 3.44\sqrt[3]{\frac{PR}{nS_4}} = 3.44\sqrt[3]{\frac{860 \times 12}{6 \times 2{,}500}} = 3 \text{ in., nearly}$$

The total taper of the arm being $\frac{1}{2}$ in. per ft., as stated in Art **22**, the width of the arm near the rim is $3 - \frac{1}{2} = 2\frac{1}{2}$ in It is understood that the corners at both hub and rim are to be well filleted.

The arms are supposed to be elliptical in section, as shown in Fig 7 (*b*), and also to taper in thickness at the rate of $\frac{1}{2}$ in. per ft.

TABLE III
DIMENSIONS OF PULLEYS

Diameter	Face	Rim		Arm		Hub		Boss		
		A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
6	4	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{7}{16}$	3	$\frac{3}{8}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{7}{16}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	8	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{7}{16}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	10	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{7}{16}$	4	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	12	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{3}{4}$	$\frac{7}{16}$	4	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
8	4	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{13}{16}$	$\frac{7}{16}$	3	$\frac{3}{8}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{13}{16}$	$\frac{7}{16}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	8	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{1}{16}$	$\frac{9}{16}$	$4\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	10	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{1}{16}$	$\frac{9}{16}$	$5\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	12	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{1}{16}$	$\frac{9}{16}$	$5\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
10	4	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{15}{16}$	$\frac{9}{16}$	3	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{1}{16}$	$\frac{9}{16}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	8	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{1}{16}$	$\frac{9}{16}$	$4\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	10	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{5}{16}$	$\frac{5}{8}$	$5\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	12	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{5}{16}$	$\frac{5}{8}$	$5\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
12	4	$\frac{5}{32}$	$\frac{1}{4}$	1	$\frac{7}{16}$	$3\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{3}{4}$	$\frac{1}{2}$	4	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	8	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{3}{4}$	$\frac{1}{2}$	5	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	10	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{1}{2}$	$\frac{3}{4}$	$5\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	12	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{1}{2}$	$\frac{3}{4}$	$6\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
14	4	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{1}{8}$	$\frac{1}{2}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{1}{8}$	$\frac{1}{2}$	$4\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	8	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{5}{16}$	$\frac{9}{16}$	5	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	10	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{5}{16}$	$\frac{9}{16}$	6	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	12	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{11}{16}$	$\frac{13}{16}$	$6\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
16	4	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{3}{8}$	$\frac{9}{16}$	$3\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{4}$
	6	$\frac{5}{32}$	$\frac{1}{4}$	$1\frac{3}{8}$	$\frac{9}{16}$	$4\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{3}{8}$
	8	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{7}{16}$	$\frac{5}{8}$	5	$\frac{5}{8}$	$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{3}{8}$
	10	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{7}{16}$	$\frac{5}{8}$	6	$\frac{5}{8}$	$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{3}{8}$
	12	$\frac{7}{32}$	$\frac{11}{32}$	$1\frac{7}{16}$	$\frac{5}{8}$	$6\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{3}{8}$
	16	$\frac{7}{32}$	$\frac{11}{32}$	$1\frac{7}{8}$	$\frac{15}{16}$	$8\frac{1}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{4}$	$\frac{3}{8}$
18	4	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{5}{16}$	$\frac{9}{16}$	4	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	6	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{5}{16}$	$\frac{9}{16}$	$4\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	8	$\frac{7}{32}$	$\frac{11}{32}$	$1\frac{1}{2}$	$\frac{11}{16}$	$5\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	0	$\frac{7}{32}$	$\frac{11}{32}$	$1\frac{1}{2}$	$\frac{11}{16}$	6	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	12	$\frac{7}{32}$	$\frac{11}{32}$	$1\frac{1}{2}$	$\frac{11}{16}$	$7\frac{1}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{8}$
	16	$\frac{1}{4}$	$\frac{3}{8}$	$2\frac{1}{4}$	$1\frac{1}{4}$	8	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{8}$
	20	$\frac{1}{4}$	$\frac{3}{8}$	$2\frac{1}{4}$	$1\frac{1}{4}$	9	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{8}$	$\frac{3}{8}$
20	4	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{3}{8}$	$\frac{5}{8}$	4	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	6	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{3}{8}$	$\frac{5}{8}$	$4\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	8	$\frac{3}{16}$	$\frac{5}{16}$	$1\frac{3}{8}$	$\frac{5}{8}$	5	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	10	$\frac{7}{32}$	$\frac{11}{32}$	$1\frac{5}{8}$	$\frac{3}{4}$	6	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	12	$\frac{7}{32}$	$\frac{11}{32}$	$1\frac{5}{8}$	$\frac{3}{4}$	7	$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$\frac{3}{8}$
	16	$\frac{9}{32}$	$\frac{7}{16}$	$2\frac{1}{4}$	$1\frac{1}{8}$	8	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{3}{4}$	$\frac{3}{8}$
	20	$\frac{9}{32}$	$\frac{7}{16}$	$2\frac{1}{4}$	$1\frac{1}{8}$	10	1	$\frac{3}{4}$	$1\frac{3}{4}$	$\frac{3}{8}$

TABLE III—(*Continued*)

Diameter	Face	Rim A	Rim B	Arm C	Arm D	Hub E	Hub F	Boss G	Boss H	Boss I
22	4	3/16	5/16	1 1/2	5/8	4	5/8	5/8	1 1/4	3/8
	6	3/16	5/16	1 1/2	5/8	4 1/2	3/4	5/8	1 1/4	3/8
	8	3/16	5/16	1 1/2	5/8	5	3/4	5/8	1 1/4	3/8
	10	7/32	11/32	1 3/4	13/16	6 1/2	3/4	5/8	1 1/4	3/8
	12	7/32	11/32	1 3/4	13/16	6 1/2	7/8	3/4	1 3/8	3/8
	16	7/32	11/32	1 3/4	13/16	8 3/4	1	3/4	1 3/8	3/8
	20	9/32	7/16	2 1/2	1 1/4	11	1 1/8	7/8	1 1/2	3/8
24	4	7/32	11/32	1 9/16	11/16	4	5/8	5/8	1 1/4	3/8
	6	7/32	11/32	1 9/16	11/16	4 3/4	3/4	5/8	1 1/4	3/8
	8	7/32	11/32	1 9/16	11/16	5 1/2	3/4	5/8	1 1/4	3/8
	10	1/4	3/8	1 7/8	11/16	7	7/8	3/4	1 3/8	3/8
	12	1/4	3/8	1 7/8	11/16	7	7/8	3/4	1 3/8	3/8
	16	1/4	3/8	1 7/8	11/16	9 1/2	1	3/4	1 3/8	3/8
	20	5/16	15/32	2 3/4	1 3/8	9 1/2	1 1/8	7/8	1 1/2	3/8
	24	5/16	15/32	2 3/4	1 3/8	11	1 1/8	7/8	1 1/2	3/8
26	4	7/32	11/32	1 11/32	3/4	4 1/4	3/4	5/8	1 1/4	3/8
	6	7/32	11/32	1 11/32	3/4	5	3/4	5/8	1 1/4	3/8
	8	7/32	11/32	1 11/32	3/4	6	7/8	3/4	1 3/8	3/8
	10	1/4	3/8	2	7/8	7	7/8	3/4	1 3/8	3/8
	12	1/4	3/8	2	7/8	7 1/2	7/8	3/4	1 3/8	3/8
	16	1/4	3/8	2	7/8	10	1 1/8	7/8	1 1/2	3/8
	20	5/16	15/32	2 15/16	1 7/16	10 1/2	1 1/8	7/8	1 1/2	3/8
	24	5/16	15/32	2 15/16	1 7/16	11	1 1/8	7/8	1 1/2	3/8
28	4	7/32	15/32	1 3/4	3/4	4 1/2	3/4	5/8	1 1/4	3/8
	6	7/32	15/32	1 3/4	3/4	5 1/2	7/8	3/4	1 3/8	3/8
	8	7/32	15/32	1 3/4	3/4	7	7/8	3/4	1 3/8	3/8
	10	7/32	15/32	1 3/4	3/4	7 1/2	7/8	3/4	1 3/8	3/8
	12	1/4	3/8	2 1/8	13/16	8	1	3/4	1 3/8	3/8
	16	1/4	3/8	2 1/8	13/16	10	1	3/4	1 3/8	3/8
	20	5/16	15/32	2 1/8	13/16	11	1 1/8	7/8	1 1/2	3/8
	24	11/32	1/2	3 1/2	1 1/2	11	1 1/8	7/8	1 1/2	3/8
30	4	7/32	11/32	1 7/8	13/16	4 1/2	3/4	5/8	1 1/4	3/8
	6	7/32	11/32	1 7/8	13/16	5 1/2	7/8	5/8	1 1/4	3/8
	8	7/32	11/32	1 7/8	13/16	6 1/4	7/8	3/4	1 3/8	3/8
	10	7/32	11/32	1 7/8	13/16	6 1/2	7/8	3/4	1 3/8	3/8
	12	9/32	7/16	2 1/4	1	8	1	3/4	1 3/8	3/8
	16	9/32	7/16	2 1/4	1	8 1/2	1	3/4	1 3/8	3/8
	20	9/32	7/16	2 1/4	1	11 1/2	1 1/4	7/8	1 1/2	3/8
	24	3/8	9/16	3 5/16	1 5/8	13	1 1/4	7/8	1 1/2	3/8
32	4	1/4	3/8	2 1/8	15/16	4 1/2	7/8	3/4	1 3/8	3/8
	6	1/4	3/8	2 1/8	15/16	5 1/2	7/8	3/4	1 3/8	3/8
	8	1/4	3/8	2 1/8	15/16	6 1/2	1	3/4	1 3/8	3/8
	10	1/4	3/8	2 1/8	15/16	7 1/2	1	3/4	1 3/8	3/8
	12	5/16	15/32	2 7/16	1 1/16	8	1 1/8	7/8	1 1/2	3/8
	16	5/16	15/32	2 7/16	1 1/16	9 1/2	1 1/8	7/8	1 1/2	3/8
	20	5/16	15/32	2 7/16	1 1/16	11	1 1/4	7/8	1 1/2	3/8
	24	5/16	15/32	2 7/16	1 1/16	13	1 1/4	7/8	1 1/2	3/8

TABLE III—(*Continued*)

Diameter	Face	Rim		Arm		Hub		Boss		
		A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
34	4	1/4	3/8	2 1/8	15/16	4 1/2	7/8	3/4	1 3/8	3/8
	6	1/4	3/8	2 1/8	15/16	5 1/2	7/8	3/4	1 3/8	3/8
	8	1/4	3/8	2 1/8	15/16	6 1/2	1	3/4	1 3/8	3/8
	10	1/4	3/8	2 1/8	15/16	7 1/2	1	3/4	1 3/8	3/8
	12	5/16	15/32	2 7/16	1 1/16	7 3/4	1 1/8	7/8	1 1/2	3/8
	16	5/16	15/32	2 7/16	1 1/16	9 1/2	1 1/8	7/8	1 1/2	3/8
	20	5/16	15/33	2 7/16	1 1/16	12	1 1/4	7/8	1 1/2	3/8
	24	5/16	15/32	2 7/16	1 1/16	13	1 1/4	7/8	1 1/2	3/8
36	4	1/4	3/8	2 3/16	15/16	4 1/2	7/8	3/4	1 3/8	3/8
	6	1/4	3/8	2 3/16	15/16	5 1/2	7/8	3/4	1 3/8	3/8
	8	1/4	3/8	2 3/16	15/16	6 3/4	7/8	3/4	1 3/8	3/8
	10	1/4	3/8	2 3/16	15/16	7 1/4	7/8	3/4	1 3/8	3/8
	12	5/16	15/32	2 3/16	15/16	7 3/4	1 1/8	7/8	1 1/2	3/8
	16	5/16	15/32	2 9/16	1 1/8	10 1/4	1 1/4	7/8	1 1/2	3/8
	20	5/16	15/32	2 9/16	1 1/8	12	1 1/4	7/8	1 1/2	3/8
	24	5/16	15/32	2 9/16	1 1/8	13 1/2	1 3/8	1	1 3/4	1/2
40	8	5/16	15/32	2 5/16	1	6 3/4	1	3/4	1 3/8	3/8
	12	5/16	15/32	2 5/16	1	7 3/4	1 1/8	7/8	1 1/2	3/8
	16	11/32	1/2	2 3/4	1 1/4	10	1 1/4	7/8	1 1/2	3/8
	20	11/32	1/2	2 3/4	1 1/4	11 1/2	1 3/8	1	1 3/4	1/2
	24	11/32	1/2	2 3/4	1 1/4	15 1/4	1 1/2	1	1 3/4	1/2
44	8	9/32	7/16	2 1/2	1 1/4	6 3/4	1 1/8	7/8	1 1/2	3/8
	12	9/32	7/16	2 1/2	1 1/4	8	1 1/8	7/8	1 1/2	3/8
	16	11/32	1/2	3	1 5/16	10	1 1/4	7/8	1 1/2	3/8
	20	11/32	1/2	3	1 5/16	12	1 3/8	1	1 3/4	1/2
	24	11/32	1/2	3 1/2	1 5/16	15	1 1/2	1	1 3/4	1/2
48	8	9/32	7/16	2 3/4	1 3/4	7 1/2	1 1/8	7/8	1 1/2	3/8
	12	9/32	7/16	2 3/4	1 3/4	8 3/4	1 1/4	7/8	1 1/2	3/8
	16	3/8	9/16	3 1/4	1 7/16	10	1 3/8	1	1 3/4	1/2
	20	3/8	9/16	3 1/4	1 7/16	12	1 3/8	1	1 3/4	1/2
	24	3/8	9/16	3 1/4	1 7/16	15	1 1/2	1	1 3/4	1/2
54	12	5/16	15/32	3	1 5/16	9 3/4	1 3/8	1	1 3/4	1/2
	16	5/16	15/32	3	1 5/16	11 1/4	1 1/2	1	1 3/4	1/2
	20	13/32	19/32	3 5/8	1 5/8	11 1/4	1 1/2	1	1 3/4	1/2
	24	13/32	19/32	3 5/8	1 5/8	15	1 3/4	1 1/4	2	1/2
60	12	11/32	1/2	3 5/16	1 7/16	10	1 3/8	1	1 3/4	1/2
	16	11/32	1/2	3 5/16	1 7/16	11 1/4	1 1/2	1	1 3/4	1/2
	20	7/16	5/8	3 13/16	1 3/4	12 1/2	1 3/4	1 1/4	2	1/2
	24	7/16	5/8	3 13/16	1 3/4	15	1 3/4	1 1/4	2	1/2
66	12	11/32	1/2	3 9/16	1 9/16	10	1 1/2	1	1 3/4	1/2
	16	11/32	1/2	3 9/16	1 9/16	11 1/2	1 5/8	1 1/4	2	1/2
	20	1/2	3/4	4 1/4	1 15/16	13 1/2	1 7/8	1 1/4	2	1/2
	24	1/2	3/4	4 1/4	1 15/16	15	1 7/8	1 1/4	2	1/2
72	12	3/8	9/16	3 7/8	1 11/16	10 1/2	1 5/8	1 1/4	2	1/2
	16	3/8	9/16	3 7/8	1 11/16	12 1/2	1 3/4	1 1/4	2	1/2
	20	9/16	13/16	4 5/8	2 1/16	13 1/2	1 7/8	1 1/4	2	1/2
	24	9/16	13/16	4 5/8	2 1/16	15	2	1 1/4	2	1/2

The length of the hub may be made according to the formula $l = \frac{2}{3} B$ to B, as stated in Art. **23.** Selecting the ratio of $\frac{3}{4} B$,

$$l = \frac{3}{4} \times 14\frac{3}{4} = 11 \text{ in., nearly}$$

The thickness of the hub may be made according to the formula of Art. **23**; thus,

$$w = \frac{B + R}{32} + \frac{1}{8} \text{ in.} = \frac{14.75 + 12}{32} + \frac{1}{8} \text{ in} = .96, \text{ or } 1, \text{ in , nearly}$$

The dimensions of the key, if one is used, are found from the formulas given in *Machine Design*, Part 2 In important or doubtful cases, it may be advisable to ascertain whether the area $a\,b\,c\,d$, Fig. 9, is sufficiently large, this is determined by means of the formula of Art. **14**, that is, $l_1\, t_1 = \frac{P R}{R_1 S_1}$.

Example 2.—A split cast-iron pulley 24 inches in diameter and 9 inches wide revolves at a circumferential speed of 3,517 feet per minute. The sectional area of the rim is 2 5 square inches. Assuming that there are two wrought-iron bolts in each rim joint and two bolts on each side of the hub, find their diameters Also find the thickness of the rim-joint flange

Solution.—The velocity of the rim in ft. per sec. is $v = \frac{3{,}517}{60}$ = 58 6 ft. Assuming the bolts to have a safe tensile stress $S_1 = 4{,}000$ lb., and inserting these values in the formula of Art. **16**, that is, $d = .357\, v\sqrt{\frac{A}{n\, S_1}}$, then, the diameter of the rim bolts at the root of the thread is

$$d = .357 \times 58\ 6\sqrt{\frac{2\ 5}{2 \times 4{,}000}} = .37 \text{ in.}$$

Consulting the table of bolt dimensions, the nearest outside bolt diameter is $\frac{1}{2}$ in.

Assume that the flange width w_1, Fig. 11, is equal to that of the rim, or 9 in , that b is 3 in., and that the hole diameters $d_1 = d + \frac{1}{16}$ in. $= \frac{9}{16}$ in. From the formula of Art. **17**, $t_1 = 1.09\, d\sqrt{\frac{n\, S_1\, b}{S_4\,(w - n\, d_1)}}$. Selecting a value for $S_4 = 2{,}500$, then

$$t_1 = 1\ 09 \times .5\sqrt{\frac{2 \times 4{,}000 \times 3}{2{,}500\,(9 - 2 \times .5625)}} = .602, \text{ or } \tfrac{5}{8}, \text{ in., nearly}$$

The diameter of the hub bolts may be found from the formula of Art **25**, that is, $a_2 = \frac{A_2}{4}$ sq. in. In order to find the value A_2, it is necessary to find the thickness of the hub w from the formula of Art. **23**; thus,

$$w = \frac{B + R}{32} + \frac{1}{8} \text{ in.} = \frac{25}{32} \text{ in.}$$

If the length of the hub is made $\frac{3}{4}B = \frac{3}{4}\times 9 = 7$ in., nearly, then $A_s = 7\times\frac{25}{32}$ and $a_s = \frac{7\times\frac{25}{32}}{4} = 1.37$ sq. in for four bolts, and $1.37 \div 4 = .34$ sq. in. for one bolt. Therefore, the diameter of the bolts is $\frac{7}{8}$ in., nearly. Ans.

BAND BRAKE

30. A **strap, or band, brake** may be considered as a variation of the belt-and-pulley combination in which the friction between the belt and the pulley may be great enough to prevent rotation.

The coefficient of friction f between wood and cast iron, which are the materials mostly used, may vary, being $f = .1$ when the surfaces are oiled, from .15 to .2 for basswood when dry, and greater when oak is used. Basswood is generally preferred, because it will stand hard service better than most other woods. This wood is mounted with the grain in the direction in which the pulley revolves.

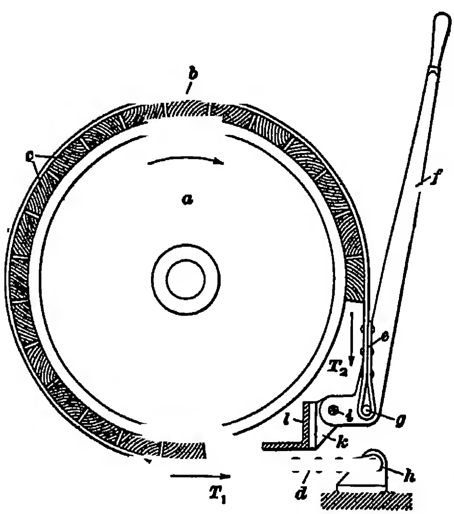


FIG 18

Fig. 18 shows a general arrangement of a band brake. A pulley, or drum, a revolves in the direction indicated by the arrow. Encircling the drum is a metal band b lined with wooden blocks c. The end d of the band is held stationary by means of the bolt h, while the end e is attached to the pin g on the lever f, which is pivoted at i; by means of this lever the band may be tightened. The bracket k is fastened to the cross-beam l. It should be noted that the pulley

revolves toward the loose end of the band, thus aiding in tightening the band.

Considering the relative motion of the band and pulley, the end d may be assumed to be the pulling end with a tension T_1, as indicated, and the end e as the loose end with the tension T_2.

Formula **1**, Art. **2**, $T_1 - T_2 = P$, also applies here; likewise formula **3**, Art. **5**, that is, $\log \frac{T_1}{T_2} = 2.729\ fn$. In regard to pulley calculations, the formula, as given for belt pulleys, will apply.

EXAMPLE.—A pulley 25 inches in diameter is subjected to a tangential pull of 6,500 pounds. Keyed to the pulley shaft is a brake drum 30 inches in diameter and 5 inches wide. A wrought-iron strap 5 inches wide and lined with basswood blocks is to serve as a brake; it has an angle of contact of 270°. Ascertain the tensions at both ends of the band and determine its thickness.

SOLUTION —The turning moments of the pulley and brake must be equal. Hence, if P is the pull of the brake band, $P \times 30 = 6{,}500 \times 25$, and $P = \frac{6{,}500 \times 25}{30} = 5{,}417$ lb. Hence, $T_1 - T_2 = P = 5{,}417$ lb. If $f = .2$ and $n = \frac{270°}{360°} = .75$, then, applying formula **3**, Art. **5**, $\log \frac{T_1}{T_2} = 2.729\ fn = 2.729 \times .2 \times .75 = .4094$, and $\frac{T_1}{T_2} = 2.567$, from which, $T_2 = \frac{T_1}{2.567}$. Inserting this value in the formula $T_1 - T_2 = 5{,}417$, $T_1 - \frac{T_1}{2.567} = 5{,}417$. Hence, $T_1 = \frac{5{,}417 \times 2.567}{1\ 567} = 8{,}874$ lb., and $T_2 = 8{,}874 - 5{,}417 = 3{,}457$ lb.

The strap, therefore, has to sustain a maximum stress of 8,874 lb. The width w of the strap being 5 in., it is required to find the thickness of the strap. Let the safe fiber stress S_1 be taken as 10,000 lb. The product of the sectional area and S_1 must equal T_1 Therefore,

$$w \times t \times S_1 = 8{,}874$$

and $$t = \frac{8{,}874}{w S_1} = \frac{8{,}874}{5 \times 10{,}000} = 177, \text{ or } \tfrac{3}{16}, \text{ in., nearly}$$

The end e sustaining the tension T_2 could be reduced in area, but this would be impractical and the strap is therefore made the same size throughout.

This thickness would be sufficient if the strap throughout its length retained its full sectional area, but if the ends are overlapped and

riveted, as indicated in Fig. 18, a certain amount of area is lost, and the thickness must be increased and one or more trial calculations made to ascertain whether the increase is sufficient. Let the thickness be increased to $\frac{1}{4}$ in. Assuming that there are six rivets at each end arranged in two longitudinal rows, there will then be two rivets in each transverse row. Each of the six rivets will sustain a load of $8,874 \div 6 = 1,480$ lb., nearly. Taking a safe fiber stress against a shear of 7,000 lb., the area of one rivet is $1,480 \div 7,000 = .21$ sq. in., which corresponds to a diameter of about $\frac{5}{8}$ in. With two $\frac{5}{8}$-in. rivets, the width of the strap would be reduced to $5 - 1\frac{1}{4} = 3\frac{3}{4}$ in., and the area is now $3\frac{3}{4} \times \frac{1}{4} = \frac{15}{16}$ sq. in. The original area was $\frac{3}{16} \times 5 = \frac{15}{16}$ sq. in., a $\frac{1}{4}$-in. strap will therefore be sufficient.

It may also be well to ascertain the crushing strength of the rivets. With a $\frac{5}{8}$-in. rivet in a $\frac{1}{4}$-in. strap, the crushing area is $\frac{5}{8} \times \frac{1}{4} = .156$ sq. in. As each rivet sustains a load of 1,480 lb., its fiber stress is $1,480 \div .156 = 9,487$ lb. per sq. in. A safe crushing stress would be 10,000 lb., hence, the diameter of the rivets is sufficiently large.

Knowing the tension T_2 at e, the proportions of the lever for a moderate pull at the handle may be calculated by means of the formula for beams. Formulas for the distances between the rivets in a row and between a rivet and the adjoining edge of the strap have already been given in *Machine Design*, Part 2.

ROPE BELTING

MULTIPLE AND CONTINUOUS SYSTEMS

31. Advantages of Rope Transmission.—In large factories and shops, there is a growing tendency toward the substitution of hemp and cotton ropes for belting and line shafting as a means of transmitting power. The advantages claimed for the rope-driving system are:

1. Economy; for a rope system is cheaper to install than either leather belting or shafting.

2. In the rope system, there is less loss of power by slipping.

3. Flexibility; that is, the ease with which the power is transmitted to any distance, and in any direction.

32. The English System.—There are two systems of rope transmission in common use. In the first, known as the

multiple, or **English, system,** the transmission is effected by several parallel independent ropes, which pass around the flywheel of the engine and the pulley or pulleys to be driven. Each rope is made quite taut at first, but stretches until it slips, after which it is respliced.

A good example of a rope transmission of this character is shown in outline in Fig. 19. The flywheel *d* carries thirty-five parallel ropes, which distribute power to the pulleys *a*, *b*, *c*, *d*, *e*, and *f* located on the five floors of the mill. The ropes are distributed as follows: *a*, four ropes; *b* and *c*, five

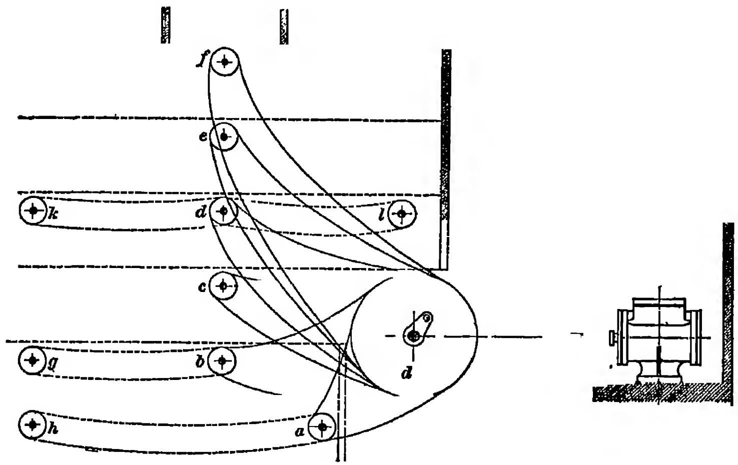


FIG. 19

ropes each; *d*, *e*, and *f*, seven ropes each. A secondary system of ropes drives the pulleys *g*, *h*, *k*, and *l*.

33. The American System.—In the second system of rope transmission, known as the **continuous,** or **American, system,** a single rope is carried around the pulley as many times as is necessary to produce the required power, and the necessary tension is obtained by passing a loop of the rope around a weighted pulley.

An example of the American system is shown in Fig 20. The rope is wrapped continuously around the flywheel *d* and the driven pulley *e*. From the last groove of *e* the rope is

led over idlers f and g, which are set at such an angle as to lead it back to the first groove in d. The weight w is attached to the pulley f, which is movable along the rod h. The movement of the pulley f therefore takes up the stretch of the rope and keeps it always at the same tension. Rope pulleys may be attached to the shaft of the pulley e, and the power received by e may thus be transmitted to any desired points.

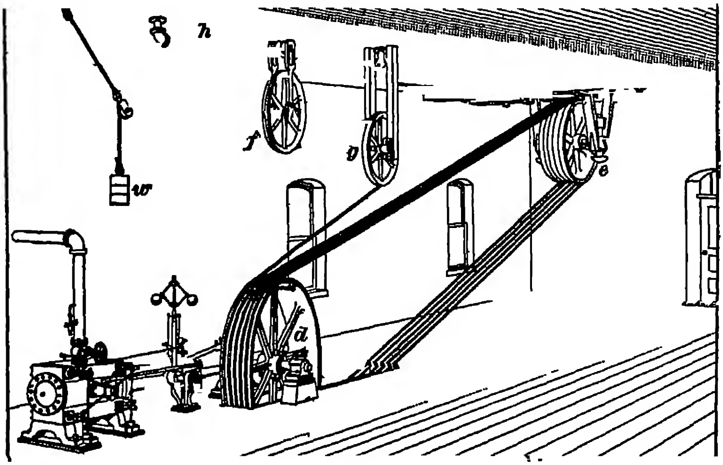


FIG 20

The first of the systems of transmission just described is used chiefly in Europe; the second, in the United States.

34. Ropes.—The **ropes** used in rope transmission are either of hemp, manila, or cotton. Manila ropes are mostly used in the United States. They consist of three strands, and may be from $\frac{1}{2}$ inch to 2 inches in diameter.

The weight of ordinary manila or cotton rope is about $.3\,D^2$ pounds per foot of length, where D represents the diameter of the rope in inches. Letting w_1 equal the weight per foot of length,

$$w_1 = .3\,D^2$$

The breaking strength of the rope varies from 7,000 to 12,000 pounds per square inch of cross-section. The average

value may be taken as 7,000 D^2, when D is the diameter of rope in inches.

For a continuous transmission, it has been determined by experiment that the best results are obtained when the tension in the driving side of the rope is about $\frac{1}{35}$ the breaking strength; that is,

$$T_1 = \text{tension in tight side} = \frac{7{,}000\, D^2}{35} = 200\, D^2$$

35. Friction of the Pulley Groove.—Ropes running on groove pulleys have an advantage over belts in regard to the pressure required between rope and pulley to produce a given pull. A belt is pressed against the flat surface of a pulley solely by the radial force resulting from the tension in the belt. With a rope supported by the walls of a groove, as in rope pulleys, the wedging action of the rope comes into play, greatly increasing the pressure between the rope and the pulley. This will be more fully explained in Fig. 21 (*a*),

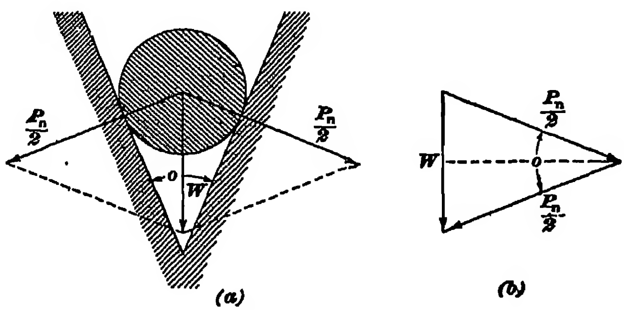


FIG. 21

which represents a rope resting in a groove. If the pressure by which the rope is pulled downwards is W, and the resulting pressure acting normally against each side of the groove is $\frac{P_n}{2}$, then the magnitude of these forces may be determined by the force diagram, Fig. 21 (*b*). In both illustrations, the angle o is that between the walls of the groove. Dividing the triangle, Fig. 21 (*b*), by the dotted line normal to W

gives two triangles, in each of which $\frac{P_n}{2} = \frac{\frac{W}{2}}{\sin \frac{o}{2}}$. The friction of the rope is produced by both forces $\frac{P_n}{2}$; hence,

$$P_n = \frac{W}{\sin \frac{o}{2}}$$

If f is the coefficient of friction, and P the frictional pull made possible by the pressure P_n, then, at the moment when slipping begins,

$$P = f P_n = \frac{f W}{\sin \frac{o}{2}}$$

If the value $\frac{f}{\sin \frac{o}{2}}$ is designated by f_1, then,

$$P = f_1 W$$

Table IV gives various values of f with the corresponding values of f_1 for the angles indicated.

TABLE IV

COEFFICIENT OF FRICTION f_1 FOR ROPE PULLEYS

Values of f	Angle of Groove Degrees = o			
	30°	35°	40°	45°
.12	.46	.40	.35	.31
.15	.58	.50	.44	.39
.20	.77	.66	.58	.52
.25	.97	.83	.73	.65
.30	1.16	1.00	.88	.78
.35	1.35	1.16	1.02	.91

Table IV is used in the following manner: Suppose that the value of f for the materials of the rope and pulley is .2, and that it is desired to find the corresponding value of f_1 for a rope running in a groove of 40°. Descend along the first

column of the table until the value .20 is found; then proceed to the right of this value to the column headed 40°, where the number .58 is found. This is the value required.

36. Power Transmitted by Ropes.—The method used for calculating the power transmitted by ropes is identically the same as that for belts. As the centrifugal force here also affects the tension in the rope, formula **1**, Art. **5**, that is, $\log \frac{T_1}{T_2} = 2.729 f (1 - z) n$, is used by inserting the value f_1 in place of f, or

$$\log \frac{T_1}{T_2} = 2.729 f_1 (1 - z) n \qquad (1)$$

The value of z is found by formula **2**, Art. **5**, or

$$z = \frac{w V^2}{9{,}660 t}$$

The value of w is about $.0268 D^2$ pounds per linear inch. In determining the value n in formula **1**, the angle of contact of the rope with the smaller pulley should be chosen in case the pulleys are of different diameters.

The safer fiber stress t of the rope is found by means of the formula of Art. **34**, or $T_1 = 200 D^2$. Inserting the values of w and $t = T_1$ in the formula $z = \frac{w V^2}{9{,}660 t}$,

$$z = \frac{.0268 D^2 V^2}{9{,}660 \times 200 D^2}$$

or

$$z = .0000000139 V^2 \qquad (2)$$

TABLE V

VALUES OF THE FACTOR $1 - z$ FOR ROPES UNDER A FIBER STRESS OF $200 D^2$ POUNDS

V . . .	1,000	1,500	2,000	2,500	3,000	3,500	4,000
$1 - z$ . . .	.99	.97	.94	.91	.87	.83	.78
V	4,500	5,000	5,500	6,000	6,500	7,000	7,500
$1 - z$ . . .	.72	.65	.58	.50	.41	.32	.22

The values of z for various velocities were calculated from formula **2**, and the values of $1 - z$ were then determined. The values of $1 - z$ are given in Table V.

EXAMPLE.—A series of manila ropes $1\frac{1}{4}$ inches in diameter transmits 250 horsepower between two pulleys. The angle o of the grooves is 45°, and $f = 12$. If the velocity of the rope is 4,500 feet per minute and the angle of contact is 165°, find the number of ropes required.

SOLUTION.—From the formula $H = \frac{PV}{33{,}000}$,

$$P = \frac{33{,}000\,H}{V} = \frac{33{,}000 \times 250}{4{,}500} = 1{,}833 \text{ lb.}$$

Therefore, $T_1 - T_2 = 1{,}833$

From Table IV it is found that for values $f = .12$ and $o = 45°$, the value f_1 is .31; from Table V, $1 - z = .72$. From formula **1**,

$$\log\frac{T_1}{T_2} = 2.729 \times 31 \times .72 \times \tfrac{165}{360} = .2792$$

Hence,
$$\frac{T_1}{T_2} = 1.902$$

As $T_2 = \frac{T_1}{1.902}$ and $T_1 - T_2 = 1{,}833$, $T_1 - \frac{T_1}{1.902} = 1{,}833$, and $T_1 = 3{,}865$ lb. $T_2 = T_1 - 1{,}833 = 2{,}032$ lb.

From Art. **34**, the allowable tension T_1 in each rope is $200\,D^2 = 200 \times (1\frac{1}{4})^2$. Hence, the number of ropes required is

$$\frac{3{,}863}{200 \times (1\frac{1}{4})^2} = 12.37, \text{ or } 13. \quad \text{Ans}$$

The horsepower that each rope transmits is found by dividing the total horsepower by the number of ropes. Thus, $250 - 13 = 19\ 23$, or $19\frac{1}{4}$, horsepower, nearly. The horsepower that each rope could actually transmit is $250 - 12.37 = 20.2$.

37. Horsepower Diagram.—The horsepower transmitted by ropes of different diameters running at different velocities may be calculated in the manner shown by the preceding example and plotted on cross-section paper. The accompanying diagram, Fig. 22, shows the horsepower transmitted at various velocities by $\frac{3}{4}$-, 1-, $1\frac{1}{4}$-, $1\frac{1}{2}$-, $1\frac{3}{4}$-, and 2-inch ropes running in grooves of 45° with an angle of contact equal to 165°. The vertical distances represent the horsepower transmitted by a single rope, and the horizontal distances the velocities in feet per minute.

The diagram shows that the maximum power is obtained at a speed of about 5,500 feet per minute. For higher velocities, the centrifugal force becomes so great that the power is decreased. When the speed reaches about 8,500 feet per minute, the centrifugal force just balances the

tension, so that no frictional contact can be established and therefore no power transmitted.

In practice, the velocity is generally from 3,000 to 5,000 feet per minute. Some investigators claim that, if first cost and relative wear are taken into consideration, the most economical speed is about 4,500 feet per minute.

Practically the same result as that found by calculation in the preceding example may be obtained directly from the diagram. To ascertain how many 1¼-inch ropes are required to transmit 250 horsepower, follow the curve representing

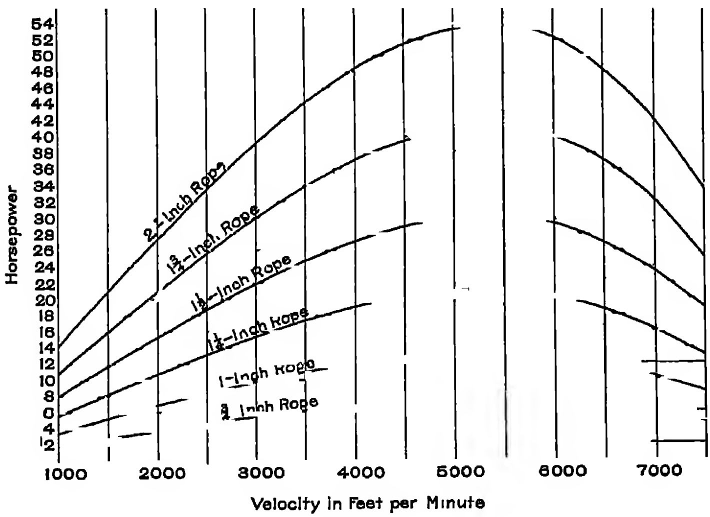


FIG 22

the 1¼-inch rope until it intersects the line representing 4,500 feet velocity. At this point, the curve indicates a value of about 20.2 horsepower. The number of ropes required is $250 \div 20.2 = 12.37$ or 13 nearly as before.

38. Deflection of the Rope.—Occasionally, it is desired to ascertain the deflection of a hemp rope. In such cases, the following formula may be used, provided both pulleys are on the same level:

$$h = \frac{T_1}{2 w_1} - \sqrt{\frac{T_1^2}{4 w_1^2} - \frac{a^2}{2}},$$

in which h = deflection, or sag, in feet at the lowest point of rope;

a = one-half the distance, in feet, between points of support;

w_1 = weight of rope per foot of length;

T_1 = tension in driving part of rope;

T_2 = tension in driven part of rope.

EXAMPLE.—Using the same dimensions given in the example of Art **36**, determine the sag in the driving and driven parts of the rope when the distance between the points of support is 50 feet.

SOLUTION.—The tension T_1 in one rope is

$$\frac{\text{total tension}}{\text{number of ropes}} = \frac{3{,}865}{13} = 297 \text{ lb.}$$

From Art **34**, $w_1 = .3\,D^2 = .3 \times 1.25^2 = 47$. Inserting these values in the formula just given,

$$h = \frac{297}{2 \times .47} - \sqrt{\frac{297^2}{4 \times 47^2} - \frac{25^2}{2}} = .46 \text{ ft.} = 5\tfrac{1}{2} \text{ in., nearly}$$

For ascertaining the deflection in the driven part of the rope, the same formula is used, except that the value of T_2 is used in place of T_1. The value of T_2 is $2{,}032 \div 13 = 156$ lb. Hence,

$$h = \frac{156}{2 \times .47} - \sqrt{\frac{156^2}{4 \times .47^2} - \frac{25^2}{2}} = .94 \text{ ft.} = 11 \text{ in., nearly}$$

39. Pulleys for Rope Gearing.—A section of what

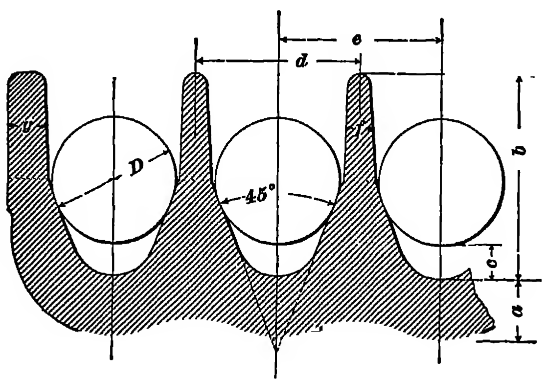


FIG 23

is known as the English form of grooved rim is shown in Fig. 23. The grooves are made circular at the bottom, and

are polished or smoothed to increase the adhesion, and to decrease the wear of the rope. The angle between the sides is generally made 45°. The following proportions, in which D is the diameter of the rope, in inches, may be used:

$a = \frac{1}{2}D$; $e = \frac{4}{3}D$;
$b = 1\frac{5}{8}D$; $f = \frac{1}{4}D$;
$c = \frac{1}{4}D$; $g = f + \frac{1}{4}$ in.
$d = \frac{4}{3}D$;

40. An objection to the groove shown in Fig. 23 is that when the ropes are worn down to a smaller diameter they are subjected to a greater stress than those possessing the

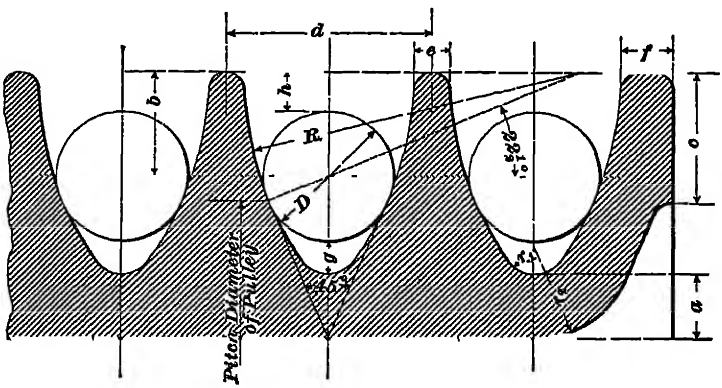


FIG 24

full diameter. The reason for this is that a rope of decreased diameter will occupy a lower position in the groove and thus virtually run on a pulley of a smaller diameter. The rope will then tend to drive the pulley at a greater speed than that produced by the ropes of thicker diameter. The thicker ropes will resist this increase in speed, and the old ropes are therefore subjected to an extra stress, which they are less able to bear Provision must therefore be made to relieve the smaller ropes. This is accomplished by giving the groove a form that will facilitate the sliding of a rope of reduced diameter when the stress is increased beyond the normal.

Fig. 24 shows the section of a rim constructed with this aim in view. The sides are formed by circular arcs so located as to increase the angle of the groove near the bottom. For this reason, a rope that is decreasing in diameter will gradually fall into positions where the coefficient of friction is decreasing and where, therefore, a sliding of the rope is more and more facilitated.

The proportions for this rim are as follows, using the diameter D of the rope as a unit:

$a = \frac{1}{2} D$;	$e = \frac{1}{4} D + \frac{1}{16}$ in.;
$b = \frac{3}{4} D + \frac{1}{16}$ in.;	$f = \frac{1}{4} D + \frac{3}{16}$ in.;
$c = D$;	$g = \frac{1}{8} D$;
$d = 1.6\ D$;	$h = \frac{1}{4} D + \frac{1}{16}$ in.

The radii r_1 and r_2 are found by trial; they should be of such lengths as to make the curves drawn by them tangent to the required lines.

The long radius R is determined by drawing a line through the center of the rope at an angle of $22\frac{1}{2}°$ with the horizontal, and producing this line until it intersects a line drawn through the tops of the dividing ribs; then, with this point of intersection as a center, the curve forming the side of the groove tangent to the circumference of the rope may be drawn.

Another advantage claimed for this groove is that the rope will turn more freely in it, thus presenting new sets of fibers to the sides of the groove, which will increase the life of the rope. In the continuous system, the rope will turn as a result of the pulley arrangement.

The pitch diameter of the pulley is taken at the point of tangency between the rope and the groove, as shown in Fig. 24.

41. Another method of adjusting the difference between the stresses in the ropes is to make the groove angle on the larger pulley smaller; for instance, 30° instead of 45°. This method is also advocated for the continuous system where a difference in tension exists in the different turns of the rope, and where more equality of tension may be established by this means.

42. Guide Pulleys, Idlers, and Tension Pulleys.—In general, guide pulleys, idlers, and tension pulleys do not have **V** grooves, but the rope rests on the bottom of a circular groove. Fig. 25 shows an idle pulley as used for the continuous and multiple systems. The grooves for pulleys in the multiple system are made somewhat deeper, as a single rope may have more sag than a single turn in the continuous system. The groove for a tension pulley is made still deeper. When the rope is outside and exposed to the wind, a deep,

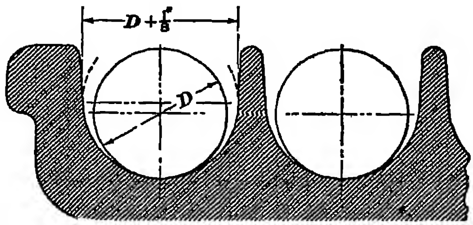


FIG 25

circular groove should be used for the guide pulleys and idlers.

43. Diameters of Pulleys.—The diameter of a rope pulley should be at least thirty times the diameter of the rope. Good results are obtained when the diameters of pulleys and idlers on the driving side are forty times and those on the driven side thirty times the rope diameter. Idlers used simply to support a long span may have diameters as small as eighteen rope diameters, without injuring the rope.

When possible, the lower side of the rope should be the driving side, for in that case the rope embraces a greater portion of the circumference of the pulley and increases the arc of contact.

When the continuous system of rope transmission is used, the tension pulley should not act on too large an amount of rope. The total length of the rope should not exceed 3,000 feet. If the power to be transmitted requires an increase in the number of turns of the rope, and, consequently, in the rope length, then it is advisable to use two or more separate

rope drives, each of which may operate side by side on the same pulleys, but having independent tension carriages. It is good practice to use a tension pulley and carriage for every 1,200 feet of rope. The advantages derived are a more equalized tension, a longer life of the rope, and more independence in case one of the rope drives fails.

Aside from the grooved rim, rope pulleys are constructed the same as other pulleys. They may be cast solid, in halves, or in sections. The pulley grooves must be turned to exactly the same diameter; otherwise, the rope will be severely strained.

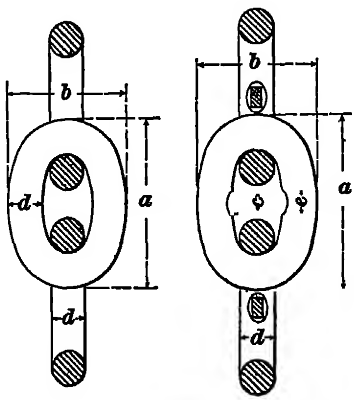


FIG 26

CHAIN GEARING

CHAINS

44. Chains may be used as simple fastenings or as belts for transmitting power. The ordinary, or open-link, and the stud-link round iron chains are shown in Fig. 26. The links are made from round iron bars cut off at the proper length and then bent and welded. The links should be made as small as possible, on account of both strength and flexibility. Ordinary chain proportions are as follows, the dimensions referring to Fig. 26:

$$d = \text{diameter of iron};$$

$$\text{For open link}\begin{cases} a = 4\tfrac{1}{2}\,d \text{ to } 5\,d; \\ b = 3\tfrac{1}{2}\,d; \end{cases}$$

$$\text{For stud link}\begin{cases} a = 5\,d \text{ to } 6\,d; \\ b = 3\tfrac{1}{2}\,d \text{ to } 3\tfrac{3}{4}\,d; \\ c = .6\,d; \\ e = .7\,d. \end{cases}$$

Link chains used merely to support loads, as in suspension bridges, etc., have links from 3 to 9 feet or more in length.

As such chains do not belong properly to the subject of machine design, they will not be considered here.

45. Strength and Weight of Chains.—The **strength of a chain** is less than that of the iron composing it, on account of the weld and also because of the presence of bending action.

Formulas given in *Strength of Materials*, Part 2, may be used to find the safe load in ordinary cases. For crane

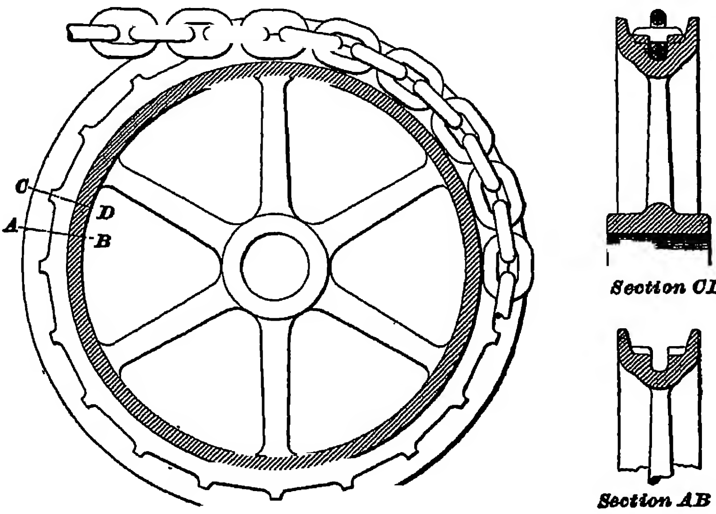


FIG. 27

chains, which require a large factor of safety, Towne gives the following as the safe load:

$$P = 3.3\,d^2 \text{ tons} = 6{,}600\,d^2 \text{ pounds}$$

The **weight of chains** (open and stud link) may vary from $9\,d^2$ to $9\frac{1}{2}\,d^2$ pounds per foot.

46. Chain Drums.—When a chain must be coiled, as in the case of cranes and derricks, a grooved drum may be used. The groove passes spirally around the drum and is just wide enough to receive the edge of a link of the chain. The drum may have a diameter of from $24\,d$ to $30\,d$ or more; the length should be such that the total amount of chain may

be coiled on in one layer; because, if one layer is wound over another, the chain is injured.

Instead of a drum, a wheel or sheave having pockets for receiving the alternate links of the chain may be used. By means of a suitable guide, the chain is held against the sheave for one-half or three-quarters of the circumference. Such a wheel requires less space than the drum, and injures the chain less. A form of chain wheel largely used for transmitting power, especially on cranes, chain blocks, etc., is shown in Fig. 27. The rim of the wheel is grooved for the links, and pockets, into which the links that lie parallel with the axis of the wheel rest, are provided. The pitch of the pockets must, of course, be the same as the pitch of the links.

47. Flat-Link Chains.—For driving machinery where very heavy resistances are to be overcome, as, for example,

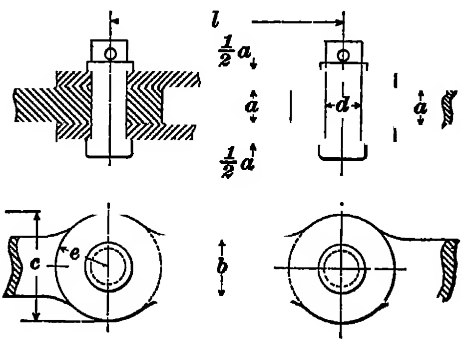


FIG 28

in wire-drawing machines, cranes, and dredging machines, **flat-link chains** are used.

When the chain merely supports a load, it may have the form shown in Fig. 28. This chain consists of flat plates connected by pins. The pins are evidently in shear, and the plates are in direct tension. Since each of the two parallel plates carries one-half the load, it should have one-half the thickness of the single plate to which it is pinned.

The links may be of any length desired, but the shortest convenient length is about $l = 3\,d$. The cross-section is $a\,b$

$= \frac{P}{S_1}$, where P is the load and S_1 the safe stress in tension. Taking both P and S_1 in tons, and assuming $S_1 = 5$ tons, then,

$$a\,b = .2\,P$$

The shearing section of a pin is $2 \times \frac{1}{4}\pi d^2 = \frac{1}{2}\pi d$. Therefore, when S_2 is the safe shearing stress,

$$\frac{1}{2}\pi d^2 = \frac{P}{S_2}$$

Assuming that $S_2 = 4$ tons,

$$d = \sqrt{\frac{P}{2\pi}}$$

The following proportions may be used in ordinary cases:

$$b = \frac{4}{3}d;\ c = \frac{8}{3}d;\ e = \frac{c}{2} = b$$

When the links are short, the width b may be the same

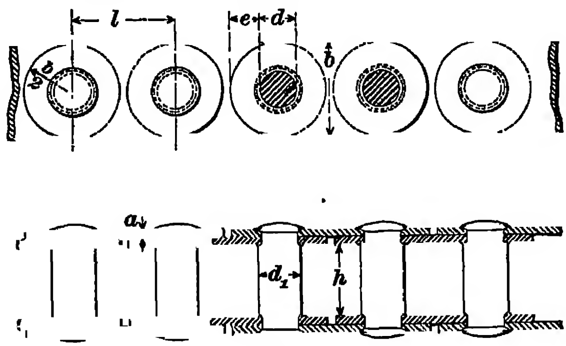


Fig 29

throughout. The pin connecting the plates may be riveted over or secured by a washer and a split pin.

A flat-link chain may be used for transmitting power somewhat after the fashion of a belt. The chain passes over wheels provided with teeth, which engage with the links of the chain. Such a wheel is known as a *sprocket wheel*. Examples of chains used in this way are met with in agricultural machinery, bicycles, automobiles, coal-mining machinery, dredges, etc.

The flat-link gearing chain, Fig. 29, consists of two series of flat links, which are kept some distance apart by the pins

that connect them. These pins engage with the teeth of the sprocket wheel; they are enlarged between the series of plates, so as to form a shoulder to prevent the plates from slipping, and also to give a greater wearing surface.

Let n = number of plates on one side of chain;
a and b = thickness and breadth of plates, respectively;
d = diameter of ends of pin;
d_1 = diameter of center of pin;
h = length of enlarged part of pin;
l = length of link between centers of pins;
P = total load on chain in pounds.

All dimensions are taken in inches.

Then, the following formulas and proportions are generally used:

$$n = .13\sqrt[3]{P}; \qquad h = 1.7\,d + .5;$$
$$d = .0115\sqrt{\frac{P}{\sqrt{n}}}; \qquad l = 2.9\,d;$$
$$b = 2.5\,d; \qquad a = \frac{.85\,d}{n+l};$$
$$d_1 = 1.2\,d; \qquad e = .75\,d.$$

EXAMPLE.—Calculate the dimensions of a flat-link gearing chain for a working load of 8,000 pounds.

SOLUTION.—Applying the formulas just given,

$$n = .13\sqrt[3]{8{,}000} = 2\ 6, \text{ say 3, plates},$$
$$d = .0115\sqrt{\frac{8{,}000}{\sqrt{3}}} = .78, \text{ say } \tfrac{3}{4}, \text{ in.};$$
$$d_1 = 1\ 2 \times \tfrac{3}{4} = \ 9, \text{ say } \tfrac{29}{32}, \text{ in.};$$
$$b = 2\ 5 \times \tfrac{3}{4} = 1\tfrac{7}{8} \text{ in };$$
$$h = 1\ 7 \times \tfrac{3}{4} + .5 = 1.775 \text{ in.};$$
$$l = 2.9 \times \tfrac{3}{4} = 2\ 175 \text{ in.};$$
$$a = \frac{.85 \times \tfrac{3}{4}}{3 + 2.175} = .12, \text{ say } \tfrac{1}{8}, \text{ in.};$$
$$e = \ 75 \times \tfrac{3}{4} = \tfrac{9}{16} \text{ in.}$$

A defect in the chain shown in Fig 29 is the limited bearing surface of the pin, which results in rapid wear In the forms shown in Figs. 30 and 31 this defect has been overcome by extending the rubbing surface of the pin nearly the whole width of the chain

48. Roller and Block Chains.—Other chains are the *roller chain* and the *block chain*, both are extensively used on

automobiles. There are some variations in the details of each type, but these variations have little effect on the main features. One form of **roller chain** is shown in Fig. 30, in which a is the main pin connecting the outer side plates b, b. The pin has two shoulders, as indicated, against which the side plates are riveted by means of the pin ends. The tubular pin f serves as a rivet for the inner side plates c, c; shoulders on this pin also hold these plates a given distance apart. On the pin f, the roller e may revolve. The length l is termed the *pitch* of the chain.

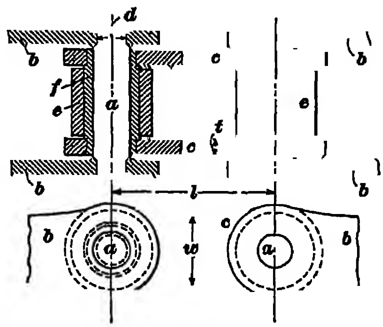


FIG. 30

A **block chain** is shown in Fig. 31. The side links c, c are riveted together by means of the pins a, a, which also pass through the blocks e.

The side plates in a chain should be designed so as not

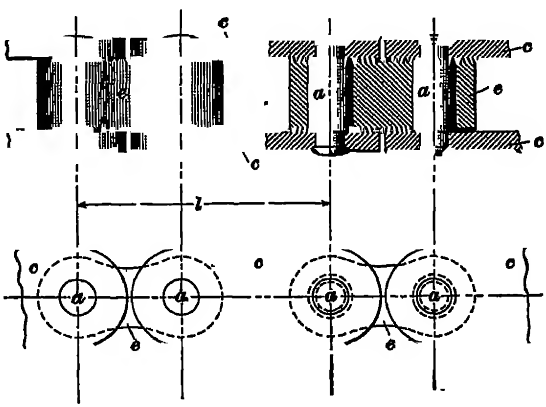


FIG. 31

to be subjected to bending, but to tension only. When the plates or links are curved at the top and bottom, as in Fig. 30, the width w should be considered as the width of the link. If t is the thickness of the link, its area is tw.

If P is the total load on the chain, in pounds, and S_1 the safe tensile stress per square inch, then

$$t\,w = \frac{P}{2\,S_1}$$

When d is the diameter of the end of the pin, its area is $\frac{\pi d^2}{4}$. With a load P and a shearing stress per square inch of S_2, $\frac{\pi d^2}{4} = \frac{P}{S_2}$, and

$$d = 1.128\sqrt{\frac{P}{S_2}}$$

In designing machinery requiring the use of chains, it is customary to use some standard size made by a company engaged in manufacturing chains.

SPROCKET WHEELS

49. Function of Sprocket Wheels.—The purpose of **sprocket wheels** is to transmit power by means of link chains. The sprocket wheel in Fig. 32 has teeth a, similar to those on a spur gear, but they serve only to prevent irregular motion between the wheel and the chain. The wheel would still transmit motion if the extensions a were omitted and the teeth shortened as at d; but as the pitch of a chain will increase by use and as the chain is liable to sway sidewise, while in motion, the rollers would often miss the recesses intended for them, and mount, or ride, the arcs d. This would increase the virtual pitch diameter and cause the chain to break. The extensions a are therefore added to guide the rollers into position.

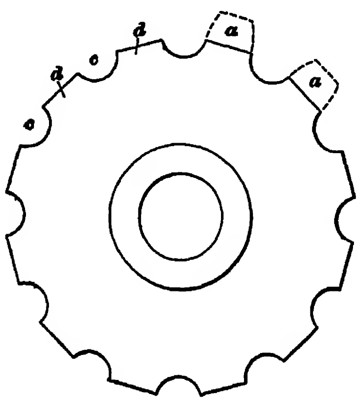


FIG 32

50. Relative Motion Between Chain and Sprocket Wheel.—The motion of the pin center in a chain relative to the sprocket wheel is shown in Fig. 33. If the pitch of the chain were infinitely small, that is, if the chain were perfectly elastic, the pin centers would describe involutes. However, as the chain is made up of links that swing around pins as fulcrums, the relative motion of pin and wheel will be represented by a series of circular arcs. In Fig. 33, $e\,a\,b\,c\,d$ represents a roller chain resting on a sprocket wheel A, the centers of the pins being indicated by the letters a, b, c, d. It will be assumed that the wheel is stationary and that the chain is unwound. In raising the end e of the chain, the

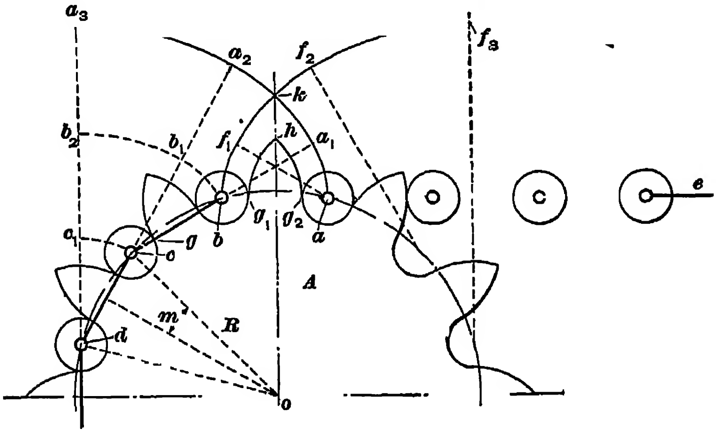


FIG. 33

latter will swing around the pin b until the point a has reached the position a_1 where it is in line with the points b, c. Then c will be the fulcrum until the points a_1 and b occupy the positions a_2 and b_1, respectively. The next center will be d, around which the chain will swing into the position $a_3\,b_2\,c_1\,d$. If the other end d is swung outwards and the chain is unwound in the opposite direction, then the point b will describe a similar curve $b\,f_1\,f_2\,f_3$.

It is evident that if the rollers were only points, the outline of the tooth would have to conform to the curves b, f_1, k, and k, a_1, a. With rollers of the radii indicated, the largest

tooth will be one in which the various curve radii are diminished by a length equal to the radius of the roller. The outline of the largest tooth will therefore be that of the curves g_1, h and h, g_2.

51. Pitch.—The polygon formed by the center lines of the links $a\,b$, $b\,c$, $c\,d$, etc., Fig. 33, is termed the **pitch polygon,** and the circle shown in dotted lines circumscribing the polygon is generally called the **pitch circle.** The distance between similarly located points on adjacent teeth, such as g, g_1, is termed the **pitch.**

Let R = radius of pitch circle;

p = pitch;

n = number of teeth on sprocket.

Then,
$$\text{angle } c\,o\,d = \frac{360}{n} \text{ degrees}$$

and
$$\text{angle } m = \frac{360}{2n} = \frac{180}{n} \text{ degrees}$$

$$R = o\,c = \frac{\frac{p}{2}}{\sin\frac{180}{n}}$$

and
$$R = \frac{p}{2\sin\frac{180}{n}}$$

52. Form of Tooth.—When a tooth is given the form shown in Fig. 33, the roller a or b will make contact with

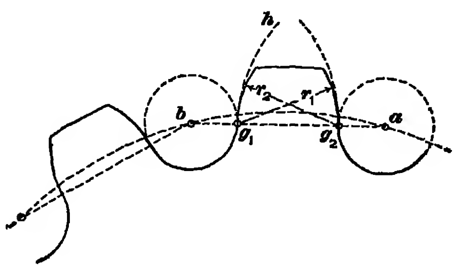


FIG 34

the tooth along the whole length of its side while moving toward or away from it. In order that this length of contact

may be decreased, the tooth curve is generally drawn as shown in Fig. 34. Instead of using a radius ag_1, the tooth is outlined by circular arcs with radii r_1 and r_2, having g_1 and g_2 as centers. The point g_1 on roller b will describe an arc $g_1 h$, having a as a center; contact with the tooth will therefore begin to take place only at the point g_1. This is also the case with the roller a.

53. Stretching of the Chain.—So far, no provision has been made for a possible stretching of the chain. With a sprocket wheel such as shown in Fig. 33, trouble will arise

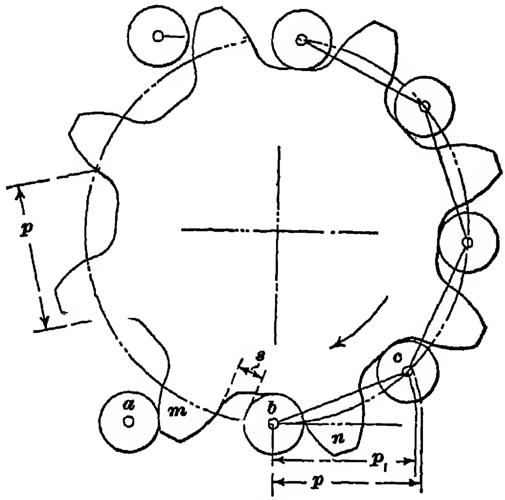


FIG. 35

as soon as the chain pitch is increased. The rollers will no longer be able to occupy their true positions, and rubbing between the rollers and teeth will ensue while they move into or out of contact. To obviate this trouble, the spaces between the teeth may be enlarged by an amount s, as indicated in Fig. 35. By means of this clearance s, it is possible to let the chain reach a certain maximum elongation before it is unfit for service. This result is obtained by letting a new chain have a *smaller* pitch than that of the wheel, and continuing its use until this pitch is *larger* by a similar amount.

If the total number of teeth on the wheel is n, then the number of teeth in action as a rule, is, $\frac{n}{2}+1$. By distributing the clearance s equally over this number, it is possible to decrease the chain pitch by an amount equal to $\frac{s}{\frac{n}{2}+1}$. A diagram of a chain with this decreased pitch is shown in Fig. 35, the wheel being a driver revolving in the direction of the arrow. The length p is the pitch of the wheel, and p_1 that of the chain; therefore,

$$p_1 = p - \frac{s}{\frac{n}{2}+1}$$

In the illustration, roller a has left the pitch line of the wheel, and, while rolling along tooth m, is gradually letting roller b make contact with tooth n. It will be noticed that, by reason of the difference in pitch, roller c and each of the succeeding rollers are unable to make contact with the teeth, and that the tangential pull of the wheel is transmitted to the chain by one tooth only, except for a relatively short period.

54. After the chain has been running for some time, its pitch has increased until $p_1 = p$, when contact is established with all of the teeth included in the number $\frac{n}{2}+1$. As the stretching of the chain continues, the relative position of chain and wheel will eventually be that shown in Fig. 36. Here, the chain pitch has increased from p_1 to p_2, a pitch that is *greater* than p by the amount $\frac{s}{\frac{n}{2}+1}$. Hence,

$$p_2 = p + \frac{s}{\frac{n}{2}+1}$$

At present, roller f is the only one in contact with the wheel, that is, at tooth n. Tooth m is moving toward roller g; when it makes contact with roller g, roller f will be released. It will be noticed that the clearances on the driven sides of the

rollers are gradually increasing from tooth n toward tooth d. A further elongation of the chain will result in roller g making contact with m above the pitch circle. When the pitch is increased to this extent, the rollers will tend to mount the teeth.

If conditions are reversed and the wheel is driven by the chain, the relative position of chain and wheel may be ascertained by simply reversing the arrows in Figs. 35 and 36.

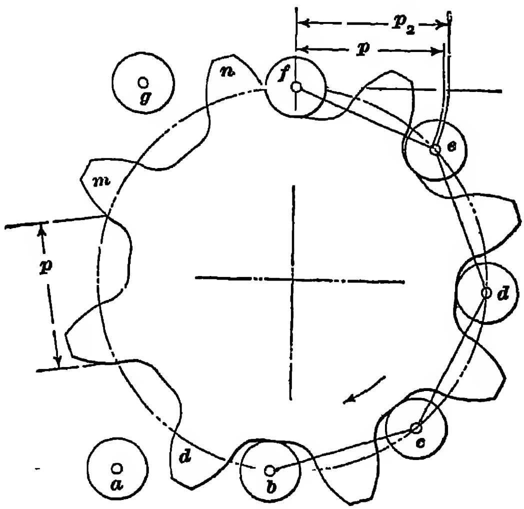


FIG 36

55. Construction of Sprocket Wheels for Roller Chains.—Assume that a roller chain of a certain type and pitch is given, and that its actual pitch is to be ascertained. In order that this may be determined with great accuracy, it is best to suspend the chain, or otherwise stretch it so as to take up all looseness, and then measure the distance between the first and the last roller center. Let this distance be L and the number of links n, then the pitch $p = \frac{L}{n}$. The radius R of the pitch circle may be found by the formula of Art. 51, that is, $R = \dfrac{p}{2 \sin \frac{180°}{n}}$.

With o as a center and R as a radius, draw the pitch circle, part of which is shown at $x x_1$, Fig. 37. On this circle lay off the lengths $a b$, $b c$, etc , each equal to the pitch p. Then, with points a, b, c, etc., as centers, and with a radius r equal to that of the chain rollers, draw the circles shown in full lines, indicating the positions of the rollers. With a radius

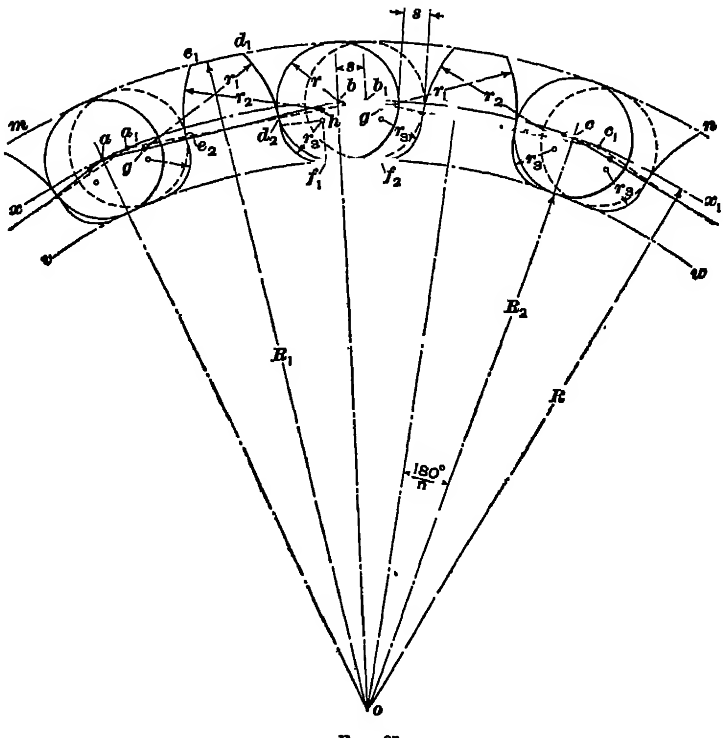


FIG. 37

$R_1 = R + r$, draw the addendum circle $m n$, and with a radius $R_2 = R - r$, the root circle $v w$. The amount of clearance s between the rollers and the teeth, given in Table VI, is now laid off on the pitch circle to the right of the roll centers a, b, c, etc., thus determining the positions of the centers a_1, b_1, c_1, etc., of the rollers when occupying the extreme right-hand positions. Draw the outlines of the

rollers in these positions in dotted lines, and then draw the supplementary pitch polygon by connecting the points a_1 and b_1, b_1 and c_1, etc. with dotted lines.

The profiles of the teeth may now be constructed in the following manner: Draw the arc $d_1 d_2$ with a radius r_1, the maximum length of which is $p - r$; it may be made as short or shorter than $p - 2r$. In the present instance, the arc is somewhat larger, in order to make the drawing clearer. The position of the center g is found by cutting off toward the left, from point d_2 on the chord ab, a length equal to r_1. The arc $e_1 e_2$ on the other side of the tooth is drawn in a similar manner, but here its center is found by laying off the radius r_2 from the point e_2 toward b_1 on the chord $a_1 b_1$. It is preferable to make the radius r_3 of the fillets about two-thirds the roller radius r. The roller will then touch the tooth at only two points and will find its true position more readily if foreign materials are present between the tooth and the roller. The outlines of the teeth are completed by drawing the arcs $e_1 d_1$ and $f_1 f_2$ in full lines.

TABLE VI

VALUES OF CLEARANCE s

Pitch p Inches	Clearance s Inches	Pitch p Inches	Clearance s Inches
$\frac{1}{2}$	$\frac{1}{32}$	$1\frac{1}{2}$	$\frac{5}{32}$
$\frac{3}{4}$	$\frac{1}{16}$	$1\frac{3}{4}$	$\frac{3}{16}$
1	$\frac{3}{32}$	2	$\frac{7}{32}$
$1\frac{1}{4}$	$\frac{1}{8}$		

56. Clearance.—The amount of clearance s varies with the pitch. Some roller chains have a shorter space between the rollers than others, resulting in a narrower sprocket tooth. Such teeth will not allow much reduction of the thickness of the tooth on the pitch line, but standard American and English sprockets may be given the amount of clearance indicated in Table VI.

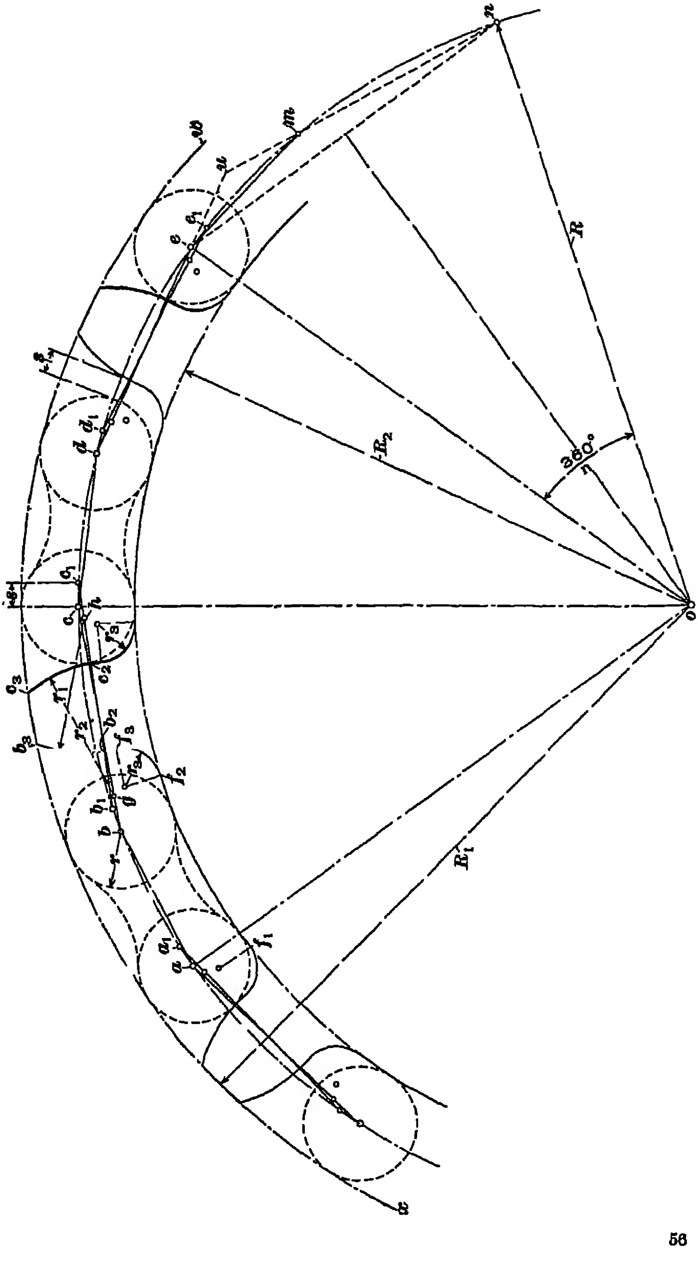

n
m
w
u
e
e1
s
d
d1
R
R2
360°/n
c1
c
h
c2
c3
r1
r2
r3
b3
b2
b1
b
f3
f2
f1
g
r
a1
a
R1
x
o

Fig. 38

An average number of teeth for a sprocket wheel is thirty-six. This number may be increased to fifty or decreased to fourteen, but it is not wise to go below this number. In transmitting a given power, the aim should be to have the chain speed high and the pull low. The speed is often as high as 2,000 feet per minute, but a speed of 1,000 to 1,200 is safer.

For sprocket wheels with more than seventeen teeth, the outside radius R_1 is generally equal to the pitch radius $R + r$. If the number of teeth is less than seventeen, the radius R_1 should be somewhat reduced so as to allow the teeth to clear the rollers at high speed. The maximum reduction is about $\frac{1}{8}$ inch for a sprocket wheel with from 8 to 12 teeth and a pitch of from 1 to 2 inches.

57. Construction of Sprocket Wheels for Block Chains.—The method pursued in constructing sprocket wheels for block chains is similar to that used for a roller chain. The difference between the two methods is due to the fact that the distance between the pin centers in one block is not equal to that between adjoining pins in two adjoining blocks. The pitch polygon will therefore be one in which the sides are alternately short and long.

The total number of sides in the pitch polygon is $2n$, when n is the number of teeth in the wheel. The centers of the curved ends of the blocks are indicated in Fig. 38 by the letters a, b, c, d, e, etc. If ab indicates the distance between the centers of the pins in one block and bc the distance between the centers of adjacent pins in the blocks ab and cd, then, in general, the ratio $\frac{ab}{bc} = \frac{2}{3}$. As the points a, b, c, d, e, m, and n are on the pitch circle, it can be proven by geometry that if two sides, such as de and mn, are produced until they intersect at u, the angles uem and ume are equal. The angle eon is $\frac{360}{n}$ degrees, and the angles uem and ume are each equal to $\frac{360}{2n} = \frac{180}{n}$ degrees. As $en = p$, the radius

of the pitch circle may be readily found by the formula

$$R = \frac{p}{2 \sin \frac{180}{n}},$$

and from the principles of trigonometry,

$$p = \sqrt{\overline{mn}^2 + \overline{em}^2 + 2em \times mn \cos \frac{180}{n}}$$

$$\text{Hence, } R = \frac{\sqrt{\overline{mn}^2 + \overline{em}^2 + 2em \times mn \cos \frac{180}{n}}}{2 \sin \frac{180}{n}}$$

The clearance s is laid off to the right of the points $a, b, c, d,$ etc., giving the points $a_1, b_1, c_1,$ etc. Lines $b_1 c_1$, $d_1 e_1$, etc. are sides in the supplementary pitch polygon, indicating the positions of the blocks in their extreme right-hand positions.

The radii chosen for the faces of the teeth may now be laid off on their respective polygons in the following manner: To draw the arc $c_2 c_3$, lay off the radius r_1 on the side bc from the point c_2 toward b, thereby determining the center g. The arc $b_2 b_3$ is drawn with a radius $r_2 = r_1$, laid off from the point b_2 toward c_1 on the side $b_1 c_1$. Both arcs should be continued below their corresponding pitch lines, so as to connect with the fillets $f_2 f_3$ tangent to the root circle. The radius r_2 has here been made $\frac{2}{3} r$. The addendum circle is drawn with a radius $R_1 = R + r$, and the root circle with a radius $R_2 = R - r$.

EXAMPLE.—In a block chain, the distance em, Fig 38, is 53 inch, and mn is .78 inch The chain is to be used on a sprocket wheel having 30 teeth, find its radius.

SOLUTION —As $\frac{180}{n} = 6$ and $n = 30$, from the formula of this article,

$$R = \frac{\sqrt{.78^2 + .53^2 + 2 \times .53 \times .78 \times \cos 6^\circ}}{2 \times \sin 6^\circ} = 6\ 26 \text{ in. Ans.}$$

58. Sprocket-Teeth Profiles.—Several end views of sprocket teeth are shown in Fig. 39. Practice differs as to the amount of curvature given to the sides of the tooth; in some cases the sides are parallel, the top being semicircular, as in (a), or flat with rounded corners, as in (b). The form

shown in (c) is preferable, because more allowance is made for the possible swaying of the chain, the tooth being so made as to guide the links safely on to the tooth and prevent their mounting the top. The upper half of the tooth is outlined by curves of radius r, the centers of the curves being located on the pitch line. The length of the radius should be such as to make the flat top surface equal to one-half the total width w of the tooth.

Some sprockets have flanges at the sides, as indicated by dotted lines in Fig. 39 (c). In this case, the position of the flange must be such as not to come in contact with the

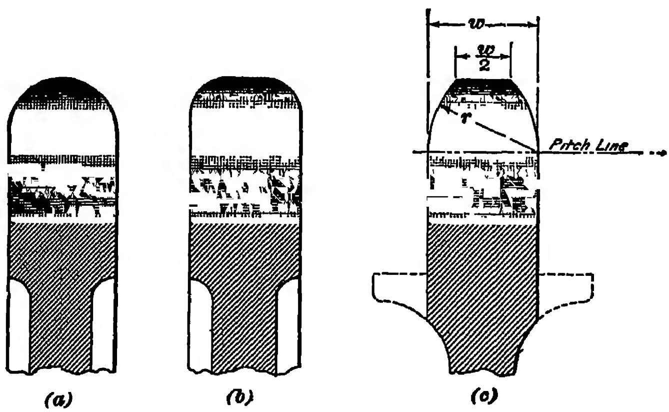


FIG. 39

chain links and thereby prevent the rollers from seating properly.

In most sprocket wheels, the rim is connected to the hub by means of a web provided with variously shaped openings, so as to make it lighter. In some cases, the sprocket wheel consists of nothing but the rim, which is fastened to some revolving part of the mechanism.

59. Speed Ratio of Sprocket Wheels.—The speed ratio of two sprocket wheels in gear is not constant. Fig. 40 shows a roller chain $a\,b$ in such a position as to be tangential to a circle with a radius r_1 inscribed in the pitch polygon. When the roller a moves into the position a_1, the chain is

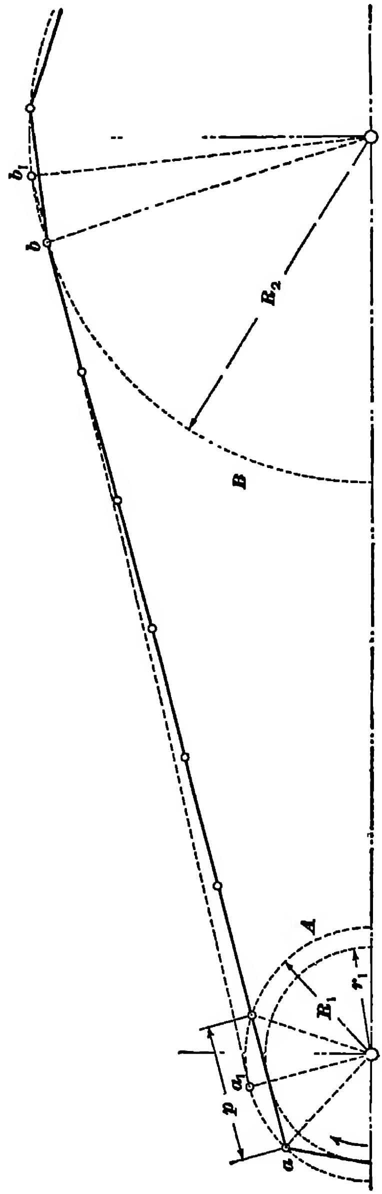


FIG 40

tangential to a pitch circle with a radius R_1. The virtual pitch radius of the sprocket wheel will therefore change twice during an angular motion of $\frac{360}{n}$ degrees

In a block chain, the pitch radius will change four times during a motion of $\frac{360}{n}$ degrees.

In Fig. 40, A is the driver and B the driven wheel. If the number of teeth on the latter is considerably greater than that of A, and the distance between centers is relatively great, then the virtual pitch radius of B remains nearly constant.

In the special case where the wheels A and B are of equal diameter and the distance between wheel centers is an exact multiple m of the chain pitch, then the periodical changes in the virtual pitch diameters will occur simultaneously and the speed ratio will remain constant. On the other hand, if the distance between

centers is increased or decreased by a length equal to $\frac{p}{2}$, the total distance being $(m + \frac{1}{2})p$, then the speed variation will attain its maximum.

The variation in speed ratio for various sizes of sprockets has been calculated and inserted in Table VII. The ratio V_1 is for a driven sprocket wheel having a relatively large number of teeth and located at a considerable distance from the driver. The ratio V_2 refers to a driven sprocket wheel having a number of teeth equal to that of the driver, and where the distance between wheel centers is $(m + \frac{1}{2})p$.

TABLE VII

RATIO OF MAXIMUM TO MINIMUM SPEED OF A DRIVEN SPROCKET WHEEL

No. of Teeth on Driver	V_1	V_2	No. of Teeth on Driver	V_1	V_2
6	1.155	1.333	10	1.051	1.105
7	1.110	1.231	11	1.042	1.086
8	1.082	1.172	12	1.035	1.072
9	1.064	1.133			

The speed variations with a block chain are smaller. For instance, for a driver with twelve teeth, $V_1 = 1.012$ and $V_2 = 1.025$.

MACHINE DESIGN

Serial 997F (PART 6) Edition 1

FRICTION AND TOOTH GEARING

FRICTION GEARING

1. Cylindrical Friction Gears.—In **friction gearing**, the smooth faces of a pair of wheels are pressed together, and one drives the other by means of the friction between the surfaces in contact. Friction gearing may be considered as the connecting link between belt driving and toothed gears. As with belt and pulleys, in friction gears there is a metal surface in contact with one of fibrous material, and a tangential force is transmitted from the driving to the driven gear, depending on the normal pressure at the place of contact and on the coefficient of friction between the surfaces. In common with toothed gears, the velocity ratio of friction gears is determined by the diameters of the pitch surfaces; also, theoretically, there is only line contact between these surfaces. Actually, this line is broadened out to a rectangular surface by reason of the flexibility of the material in the driver and of the relatively high pressure exerted.

Friction gearing should not be used for heavy work, but may be used for the transmission of small powers, where machines are frequently started and stopped, and where the speed of the gears is so high that ordinary gears would be too noisy. If the resistance to turning offered by the driven wheel is less than the frictional resistance between the

surfaces of the two wheels, motion will be transmitted; otherwise, the surfaces will slide over each other, and the gearing will not work.

The usual practice is to face the rim of one of the wheels with leather, wood, or paper. Fig. 1 shows a friction wheel *a* thus faced with wood driving a cast-iron wheel *b*. The driving wheel should be made of the fibrous material, so as to prevent flat places from forming in case of slippage; the driven pulley is usually made of cast iron. As shown, the grain of the wood lies

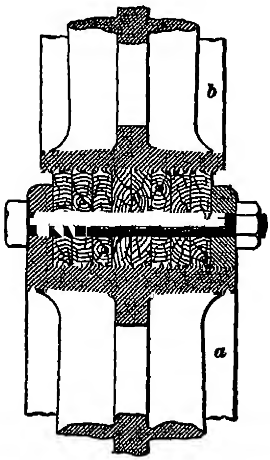


FIG. 1

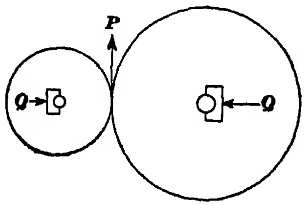


FIG 2

along the working surface. When leather or paper is used, the edges of the layers are used as the driving surface.

Let P = driving force at circumference of friction wheels, Fig. 2;

Q = force acting on the bearings to press the wheels together;

f = coefficient of friction between surfaces.

Then,
$$P = fQ \qquad (1)$$
or,
$$Q = \frac{P}{f} \qquad (2)$$

If the horsepower of a gear is to be calculated by means of the force P, the radius R of the wheel, and the number of revolutions per minute N, then the formula from *Machine Design*, Part 2, is applied; namely,

$$H = \frac{PRN}{63{,}025} \qquad (3)$$

The coefficient of friction f has about the following values:

Metal on metal	$f = .15$ to $.25$
Wood on metal	$f = .25$ to $.4$
Paper on metal	$f = .2$
Leather on metal	$f = .25$ to $.3$

Experiments by Goss show that the value of f for a paper wheel running on a metal wheel is not constant, but varies when there is a certain amount of slip between the two wheels.

At a 1-per-cent. slip	$f = .14$
At a $1\frac{1}{2}$-per-cent. slip	$f = .18$
At a 2-per-cent. slip	$f = .2$

The latter is the maximum value of f. If the slip is increased beyond 3 per cent., motion of the driven wheel ceases. The value of f was found to be constant at pressures up to 200 pounds per inch of width of the face of the wheels and for velocities of the contact surfaces varying from 400 to 2,800 feet per minute.

The power transmitted by a pair of cylindrical friction gears can be easily calculated. The following example shows the method:

EXAMPLE—Suppose that a friction wheel faced with leather drives another wheel $2\frac{1}{2}$ feet in diameter at 300 revolutions per minute. The force pressing the wheels together is 450 pounds. Assuming $f = .3$, calculate the horsepower transmitted.

SOLUTION—By formula **1**,

$$P = .3 \times 450 = 135 \text{ lb.}$$

R, the radius of wheel in inches, is equal to $\frac{2\frac{1}{2} \times 12}{2} = 15$ in.

From formula **3**,

$$H = \frac{135 \times 15 \times 300}{63{,}025} = 9.64 \text{ H. P.} \quad \text{Ans.}$$

2. Width of Face.—Sometimes, it is desirable to ascertain the width of the face of a friction wheel required to transmit a given horsepower. In general, it is assumed that a friction wheel may transmit the same power as a single belt when both are of the same width. In this case, the maximum value of P is taken at about 50 pounds per inch of width.

For a paper pulley running with a metal wheel, f is about .2. If P is to be 50 pounds, then from formula **2**, Art. **1**,

$$Q = \frac{50}{.2} = 250 \text{ pounds}$$

Let q = pressure per inch of width of wheel;
w = width of wheel, in inches.

Then,
$$Q = w\,q = \frac{P}{f}$$

and
$$P = w\,f\,q \qquad (1)$$

Inserting this value of P in formula **3**, Art. **1**,

$$H = \frac{w\,f\,q\,R\,N}{63{,}025},$$

from which
$$w = \frac{63{,}025\,H}{f\,q\,R\,N} \qquad (2)$$

EXAMPLE —A paper pulley is to transmit 12 horsepower to a 24-inch cast-iron pulley making 520 revolutions per minute Find the width of face of the wheels if the pressure is to be 250 pounds per inch of width.

SOLUTION.—Substituting the given values in formula **2**,

$$w = \frac{63{,}025 \times 12}{.2 \times 250 \times 12 \times 520} = 2\ 42, \text{ say } 2\tfrac{1}{2}, \text{ in} \quad \text{Ans.}$$

3. Bevel Friction Gears.—In **bevel friction gearing**, the pressure necessary to bring the friction surfaces in contact acts in a direction *parallel* to the axis of each gear, instead of *normal* to the axis, as in cylindrical gears. In Fig. 3 (a), the heavy lines $a\,b$ and $b\,c$ represent the mean diameters of the two bevel gears shown by dotted lines. The angle between the face of $a\,b$ and its axis is m. This angle is equal to that between the normal force Q and the diameter $a\,b$. If $b\,c$ is the driving gear, it is necessary to push the shaft A in the direction of the arrow L_1 in order to produce a pressure Q sufficient to obtain a tangential driving force P. To ascertain the relation between the forces Q and L_1, the force Q may be resolved into the two components L_1 and N_1, as shown in Fig. 3 (b), in which diagram N_1 represents a force acting in a direction normal to the gear $b\,c$ and, hence, parallel to the shaft B. N_1 will therefore produce the reaction L_2. Then,

$$N_2 = L_1 = Q \sin m \qquad (1)$$

and
$$N_1 = L_2 = Q \cos m \qquad (2)$$

The force N_1 acts along the shaft B, producing an end thrust that is counteracted by the reaction L_2 of its bearings. The force L_1 acts as a side thrust on the shaft B, resulting in the reaction N_2 of its bearings.

4. To transmit a given tangential force P at the line of contact at b from the driving gear bc to the gear ab, there is a certain minimum value that L_1 must equal or exceed if slippage is to be prevented. According to formula **2**, Art. **1**, $Q = \frac{P}{f}$. Inserting this value of Q in formula **1**, Art. **3**, $L_1 = Q \sin m$; then, the minimum value of L_1 is

$$L_1 = \frac{P}{f} \sin m \qquad (1)$$

Similarly, from formula **2**, Art. **3**,

$$L_2 = \frac{P}{f} \cos m \qquad (2)$$

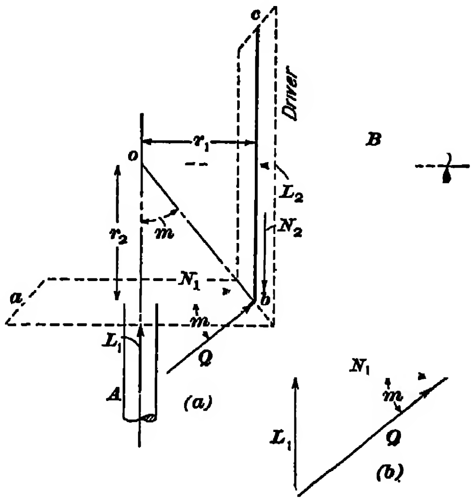


FIG. 3

EXAMPLE.—The driver of a pair of bevel friction gears has a mean diameter of 12 inches and is provided with a wooden rim. The driven gear is of cast iron and has a mean diameter of 10 inches. Assuming that the tangential force P is to be 40 pounds and that the coefficient of friction f is .3, find the amount of end thrust necessary to prevent the driven gear from sliding, also, the end thrust and side thrust existing in the driving shaft. If the pressure per inch of width of face is 200 pounds, find width of face required.

SOLUTION.—Referring to Fig. 3 (*a*), in which r_1 and r_2 represent the mean radii of the two gears, $r_1 = 5$ in. and $r_2 = 6$ in. Then, $\tan m = \frac{r_1}{r_2} = \frac{5}{6}$, corresponding to an angle $m = 39° 48'$, and $\sin m = .6401$. From formula **1**,

$$L_1 = \frac{40}{.3} \times .6401 = 85.35 \text{ lb. Ans.}$$

The side thrust on the shaft is

$$B = N_2 = L_1 = 85.35 \text{ lb} \quad \text{Ans.}$$

The end thrust on the shaft $B = L_2 = N_1$. From formula **2**,

$$L_2 = \frac{40}{3} \cos 39° 48' = \frac{40}{3} \times .7683 = 102.4 \text{ lb} \quad \text{Ans}$$

The required width of face w may be found from formula **1**, Art. **2**. Solving for w,

$$w = \frac{P}{fq} = \frac{40}{.3 \times 200} = \tfrac{2}{3}, \text{ say } \tfrac{11}{16}, \text{ in.} \quad \text{Ans.}$$

SPUR GEARS

NOTE —Before reading this section on spur gears, the section on *Gearing* should be carefully reviewed.

5. Materials of Gearing.—Gearing is ordinarily made of cast iron. If great strength is required, steel may be used. Gears that are called on to resist shocks may be made of gun metal or phosphor-bronze. Fast-running gears are sometimes made with *wooden cogs*, or the gears are made of *rawhide*, or *fiber*, instead of *metal*.

Gear-wheels are either cast with teeth all complete or with a blank rim in which the tooth spaces are afterwards cut with a milling cutter. Gears with cast teeth are less expensive; those with cut teeth are more accurate. Gear-cutting machines that can easily cut teeth on both spur and bevel gears are now made; hence, cut gears are quite generally used, except on rough work.

STRENGTH OF GEAR-TEETH

6. Application of the Load.—In considering the **strength of gear-teeth,** it is customary to assume that the load is applied at the end of the tooth, and that it is evenly distributed along the *whole width* of the face. Objection has been made to this assumption on the ground that the teeth may possibly make contact at one corner only, by reason of irregularities in the teeth or from bending of the shaft, and that, therefore, the whole load should be considered as being applied at *one corner* only. However, considering the present facilities for making gears and presuming that reasonable care has been taken in designing the shafts,

the yielding of the latter should not be so great as to decrease the amount of contact to any serious extent, especially when it is considered that the width of a tooth is rarely more than two or three times the pitch.

The load transmitted from one gear to another is carried by the teeth while passing through the arc of action. The most severe stress on a tooth during this period of contact is while the contact is made near its point. It is supposed that even then one tooth does not carry the load alone, but is assisted by at least one more tooth. While this is true, theoretically, it cannot be depended on in practice. It is therefore safe to assume that the load is supported by one tooth only, and that the point of contact is near its end. Formulas developed by Wilfred Lewis, based on these assumptions, have been extensively used in practice and have given good results.

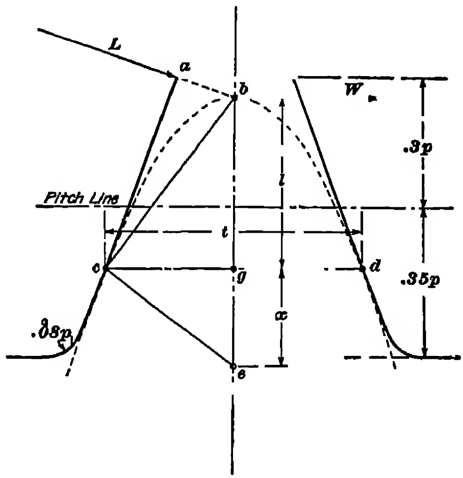


FIG. 4

7. The Lewis Formula.—The principle on which the **Lewis formulas** are based is explained by means of Fig. 4, which represents an involute rack tooth. The load L acts in the direction of the arrow, normal to the face of the tooth, at the point a. If this force is resolved into two components, one acting in the direction of motion, indicated by the arrow W, and the other along the center line $b\,e$ of the tooth, then the effect of the latter will be to put the tooth under compressive stress. Neglecting this force as being of minor importance, there remains the force W acting at the point b; this is the

point of intersection of the force L, produced, and the center line of the tooth.

One of the features of a cantilever of parabolic outline is that it is of uniform strength throughout its length. Such a cantilever may be constructed with one end at b, tangent to W and with its sides tangent to the sides of the tooth. The parabola, shown by dotted lines, is in this instance tangent to the tooth profile at the points c and d. The line cd therefore indicates the place where the tooth has its minimum strength. It is evident that these points of tangency will vary in position with the different tooth forms and that they may serve as a means for indicating their relative strengths. In order that this ratio may be indicated graphically, draw the line bc, and at right angles to this line the line ce, intersecting the center line at e. By reason of the similarity of the triangles bcg and cge, $ge : cg = cg : bg$, or $ge = \frac{cg^2}{bg}$. Designating the thickness of the tooth, $2cg$ by t, the length bg by l and ge by x, then the formula $ge = \frac{cg^2}{bg}$ will assume the following form:

$$x = \frac{\left(\frac{t}{2}\right)^2}{l} = \frac{t^2}{4l}$$

and
$$t^2 = 4lx \qquad (1)$$

It can be shown that the length x is proportional to the strength of the tooth and that it will vary with the curvature of the tooth.

From *Strength of Materials*, Part 2, the bending moment M of a cantilever loaded at the free end is Wl, and the moment of resistance is $S_4 \frac{I}{c}$. For a rectangular section, $I = \frac{1}{12} b d^3$, and $c = \frac{1}{2} d$; hence, the moment of resistance is $S_4 \frac{b d^2}{6}$.

Equating the bending and resisting moments,

$$Wl = S_4 \frac{bd^2}{6}$$

and
$$W = S_4 \frac{b d^2}{6l}$$

In this case, the width of the tooth is $f = b$, and the thickness of the tooth is $t = d$; hence,

$$W = S_4 \frac{f t^2}{6 l}$$

But, from formula **1**, $t^2 = 4 l x$; therefore,

$$W = S_4 \frac{f 4 l x}{6 l} = S_4 \frac{f 2 x}{3},$$

and by multiplying both numerator and denominator by the circular pitch p_1,

$$W = S_4 f p_1 \frac{2 x}{3 p_1}$$

Designating the value $\frac{2 x}{3 p_1}$ by y,

$$W = S_4 f p_1 y \qquad (2)$$

Table I gives the various values that y may assume, depending on the number of teeth in the gear and whether they are involute with an angle of obliquity of 15° or 20°, cycloidal, or rack teeth. Definite data as to safe fiber stress at different speeds are not at hand, but the values given in Table II have been used for a number of years with good results, and if they err, it is on the side of safety.

NOTE —Tables I and II differ somewhat from the original tables of Mr Lewis. On representing the latter tables, graphically, by means of curves, certain irregularities were found and smoothed out. These corrected values are given in the two tables

In the first column of Table III are given common values of the diametral pitch, and in the second column, the corresponding values of the circular pitch. In the third column, common values of the circular pitch are given, and in the fourth column, the corresponding values of the diametral pitch.

To make the calculations of tooth proportions less difficult, Table II has been worked out in the form of a diagram, Fig. 5, in which are given the values of the working stresses for cast iron and steel at various velocities. These values apply to gears of accurate workmanship running without severe shocks. For gears of less accurate workmanship or subjected to shocks, the values given in the diagram should be multiplied by $\frac{3}{4}$.

TABLE I

VALUES OF THE FACTOR y FOR DIFFERENT TOOTH FORMS

Number of Teeth n	Involute 20° Obliquity	Involute 15° Obliquity and Cycloidal	Teeth With Radial Flanks
12	.077	.065	.0520
13	.084	.069	.0533
14	.088	.072	.0545
15	.092	.076	.0555
16	.094	.079	.0565
17	.096	.082	.0575
18	.098	.084	.0585
19	.100	.087	.0593
20	.102	.089	.0600
21	.104	.091	.0603
23	.106	.095	.0620
25	.109	.098	.0633
27	.111	.100	.0640
30	.114	.103	.0650
32	.116	.104	.0655
35	.119	.106	.0665
40	.124	.108	.0675
42	.126	.109	.0680
45	.127	.111	.0685
50	.130	.112	.0690
55	.132	.113	.0695
60	.134	.114	.0700
75	.138	.116	.0710
100	.142	.118	.0720
150	.146	.120	.0730
300	.150	.122	.0740
Rack	.154	.124	.0750

TABLE II

SAFE WORKING STRESS S_4 FOR DIFFERENT SPEEDS AND MATERIALS

Speed of Teeth Feet per Minute V	0	100	200	300	600	900	1,200	1,800	2,400
Cast iron . .	8,000	6,900	6,000	5,300	4,000	3,200	2,700	2,000	1,600
Steel . .	20,000	17,000	15,000	13,000	10,000	8,000	6,700	5,000	4,000
Bronze . . .	15,000	12,800	11,300	9,800	7,500	6,000	5,000	3,800	3,000

TABLE III

CIRCULAR AND DIAMETRAL PITCHES

Diametral Pitch p	Circular Pitch p_1 Inches	Circular Pitch p_1 Inches	Diametral Pitch p
1	3.142	2	1.571
$1\frac{1}{4}$	2.513	$1\frac{7}{8}$	1.676
$1\frac{1}{2}$	2.094	$1\frac{3}{4}$	1.795
$1\frac{3}{4}$	1.795	$1\frac{5}{8}$	1.933
2	1.571	$1\frac{1}{2}$	2.094
$2\frac{1}{4}$	1.396	$1\frac{7}{16}$	2.185
$2\frac{1}{2}$	1.257	$1\frac{3}{8}$	2.285
$2\frac{3}{4}$	1.142	$1\frac{5}{16}$	2.394
3	1.047	$1\frac{1}{4}$	2.513
$3\frac{1}{2}$	.898	$1\frac{3}{16}$	2.646
4	.785	$1\frac{1}{8}$	2.793
5	.628	$1\frac{1}{16}$	2.957
6	.524	1	3.142
7	.449	$\frac{15}{16}$	3.351
8	.393	$\frac{7}{8}$	3.590
9	.349	$\frac{13}{16}$	3.867
10	.314	$\frac{3}{4}$	4.189
11	.286	$\frac{11}{16}$	4.570
12	.262	$\frac{5}{8}$	5.027
14	.224	$\frac{9}{16}$	5.585
16	.196	$\frac{1}{2}$	6.283
18	.175	$\frac{7}{16}$	7.181
20	.157	$\frac{3}{8}$	8.378

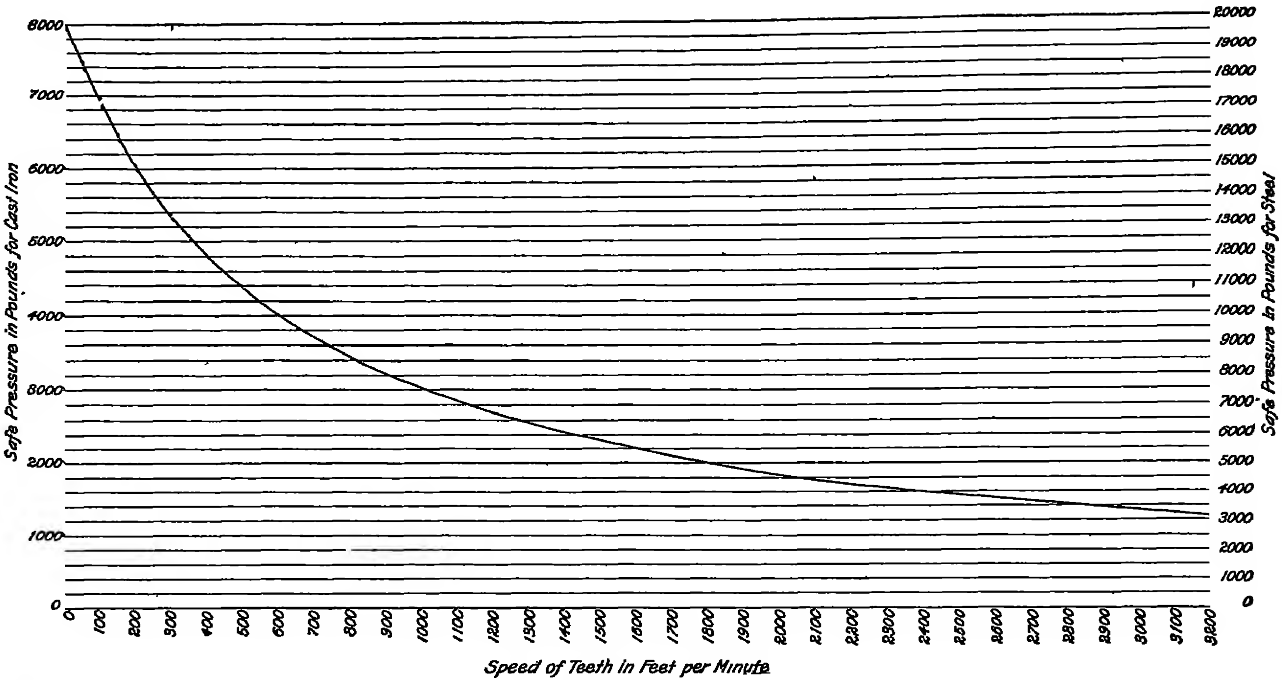

Safe Pressure in Pounds for Cast Iron
8000
7000
6000
5000
4000
3000
2000
1000
0
Safe Pressure in Pounds for Steel
20000
19000
18000
17000
16000
15000
14000
13000
12000
11000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0 100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500 1600 1700 1800 1900 2000 2100 2200 2300 2400 2500 2600 2700 2800 2900 3000 3100 3200
Speed of Teeth in Feet per Minute

FIG. 5

The application of Fig. 5 is explained by means of the following example:

EXAMPLE —(*a*) It is required to find the safe working stress of a cast-iron gear running at a speed of 700 feet per minute. (*b*) Find the working stress if the gear is made of steel.

SOLUTION.—(*a*) Find the value 700 at the foot of Fig. 5 and follow the vertical line at this point until it intersects the curve At the point of intersection, proceed to the left along a horizontal line that may intersect at this point; if none intersects, then parallel with the next horizontal line. In the present instance, the point of intersection is about equidistant from two horizontal lines Proceeding to the left, the working stress S_4 is found to be 3,700 lb. Ans.

(*b*) To find the working stress for steel, proceed to the right instead of to the left, where the value of S_4 is found to be 9,250 lb. Ans.

8. Radius of Fillet.—The values given in Fig. 5 and Table II are based on the assumption that the fillets at the root of the teeth are made as large as possible and yet clear the engaging tooth. For the involute or the cycloidal system, in which there is a root clearance of .05 p_1, it is safe to give the twelve-tooth pinion a fillet with a radius of .15 p_1. This radius will gradually decrease as the number of teeth increases, until, for a twenty-tooth pinion, the radius becomes .08 p_1, which value remains constant for any number of teeth above twenty. For internal gearing, these fillets will have to be smaller.

9. Values of the Factor y.—When the number of teeth n in a gear is unknown, the value of y cannot be found from Table I. In such cases, the following three formulas replace formula **2**, Art. **7**.

For the involute 20° system,

$$W = S_4 p_1 f\left(.154 - \frac{.912}{n}\right) \qquad (1)$$

For the involute 15° and the cycloidal gearing systems,

$$W = S_4 p_1 f\left(.124 - \frac{.684}{n}\right) \qquad (2)$$

For teeth with radial flanks,

$$W = S_4 p_1 f\left(.075 - \frac{.276}{n}\right) \qquad (3)$$

10. Horsepower.—The horsepower that a gear will transmit is found by the following formula, in which V = velocity in feet per minute at the pitch line:

$$H = \frac{W V}{33{,}000} \qquad (1)$$

Then, by inserting the value for W given in formula **2**, Art. **7**, the horsepower is

$$H = \frac{S_4 f p_1 y V}{33{,}000} \qquad (2)$$

The values for W given in formulas **1**, **2**, and **3**, Art. **9**, may also be used.

In general, it is less important to know the horsepower of the gearing than the working strength. This is particularly so when the gearing is part of machines that intermittently consume great power, such as punching and shearing machines, in which case the maximum, and not the average, load should be considered.

11. Width of Face.—The width f of the tooth face is seldom less than 1.5 p_1, or $\frac{4.7}{p}$. The following widths are generally used in practice:

For wheels moving slowly or intermittently, as in hoisting apparatus,

$$f = \begin{Bmatrix} 2 p_1 \text{ to } 2\tfrac{1}{2} p_1 \\ \dfrac{6.3}{p} \text{ to } \dfrac{7.9}{p} \end{Bmatrix} \qquad (1)$$

For more rapidly moving cast gears, for example, transmission gears,

$$f = \begin{Bmatrix} 2\tfrac{1}{2} p_1 \text{ to } 3 p_1 \\ \dfrac{7.9}{p} \text{ to } \dfrac{9.4}{p} \end{Bmatrix} \qquad (2)$$

For gears moving more rapidly and having cut teeth,

$$f = \begin{Bmatrix} 3 p_1 \text{ to } 3\tfrac{1}{2} p_1 \\ \dfrac{9.4}{p} \text{ to } \dfrac{11}{p} \end{Bmatrix} \qquad (3)$$

For very rapidly moving gears, with small pitch and with cut teeth,

$$f = \begin{Bmatrix} 3\frac{1}{2} p_1 \text{ to } 4 p_1 \\ \frac{11}{p} \text{ to } \frac{12.6}{p} \end{Bmatrix} \qquad (4)$$

12. Application of the Formulas.—The following example will illustrate the use of the Lewis formulas:

EXAMPLE.—A 16-tooth pinion of $1\frac{1}{4}$-inch circular pitch and $3\frac{1}{4}$-inch face drives a 75-tooth wheel at a speed of 200 feet per minute The tooth form is involute of 15° obliquity (*a*) If both gears are made of cast iron, find their working strengths. (*b*) Find the strength if the pinion is made of steel. (*c*) Find the horsepower of each combination.

SOLUTION —(*a*) From Table I, the value of y for the pinion is .079, and from Table II or Fig. 5, S_4 = 6,000, hence, from formula **2**, Art **7**,

$W = 6{,}000 \times 3.25 \times 1.25 \times .079 = 1{,}926$ lb. Ans

For the wheel, the value of y from Table I is 116; hence,

$W = 6{,}000 \times 3.25 \times 1.25 \times 116 = 2{,}828$ lb. Ans

(*b*) If the pinion is made of steel, then, from Fig. 5, S_4 = 15,000, and $W = 15{,}000 \times 3\ 25 \times 1\ 25 \times .079 = 4{,}814$ lb. Ans.

By making the pinion of steel, it is changed from the weaker into the stronger one of the pair In any case, the weaker one determines the strength of the combination.

(*c*) By formula **1**, Art. **10**, $H = \frac{W V}{33{,}000}$ Then, for the cast-iron pinion,

$$H = \frac{1{,}926 \times 200}{33{,}000} = 11.67$$

With the steel pinion, the horsepower possible to be transmitted is limited by the strength of the wheel; hence, the horsepower is

$$H = \frac{2{,}828 \times 200}{33{,}000} = 17.14, \text{ or } 17, \text{ nearly. Ans.}$$

13. It is seen from the preceding example that when the gear dimensions are known the application of the formulas is simple. If, however, these dimensions are not given, and, for instance, only the distance between centers of shafts, the velocity ratio, the number of revolutions of one of the gears, and the horsepower are known, then the solution must be made partly by trial, as will be seen from the following example:

EXAMPLE —A pair of 20° involute gears is to transmit 8 horsepower. The distance between centers is 12 inches and the velocity ratio is 3 to 5 If the pinion shaft makes 200 revolutions per minute, find the pitch and the number of teeth on the gears

SOLUTION.—The velocity ratio being 3 to 5 and the distance between centers 12 in., the radii of the pitch circles will be as 5 is to 3 and their sum will be 12 in. If x is the radius of the pinion, $(12 - x) : x = 5 : 3$, and $36 - 3x = 5x$, hence, $x = 4.5$ in., and the radius of the wheel is $12 - 4.5 = 7.5$ in.

In any combination in which both gears are made of the same material, the teeth on the pinion are always the weaker; consequently, the calculation for the working strength of the combination may be limited to the pinion. The pitch diameter of the pinion is 9 in., and therefore its velocity is

$$V = \frac{200 \times \pi \times 9}{12} = 471 \text{ ft. per min.}$$

In formula **1**, Art. **10**, $H = \frac{W V}{33,000}$; hence,

$$W = \frac{H \times 33,000}{V} = \frac{8 \times 33,000}{471} = 561 \text{ lb., nearly}$$

If the pinion is to be made of cast iron, the value for S_4, from Fig. 5, is found to be about 4,500 Making f tentatively equal to 1.75 in. and using formula **1**, Art. **9**, then

$$W = 561 = 4,500 \times p_1 \times 1.75 \left(.154 - \frac{.912}{n}\right)$$

As $n = \frac{\pi d}{p_1} = \frac{3.1416 \times 9}{p_1} = \frac{28.27}{p_1}$, then, by inserting this value in the preceding equation,

$$561 = 4,500 \times p_1 \times 1.75 \left(.154 - \frac{.912 p_1}{28.27}\right)$$

Clearing of fractions and transposing p_1,

$$p_1^2 - 4.77 p_1 + 2.208 = 0$$

and

$$p_1 = 2.385 \pm 1.865$$

If the smaller value is taken, then $p_1 = .520$ in.

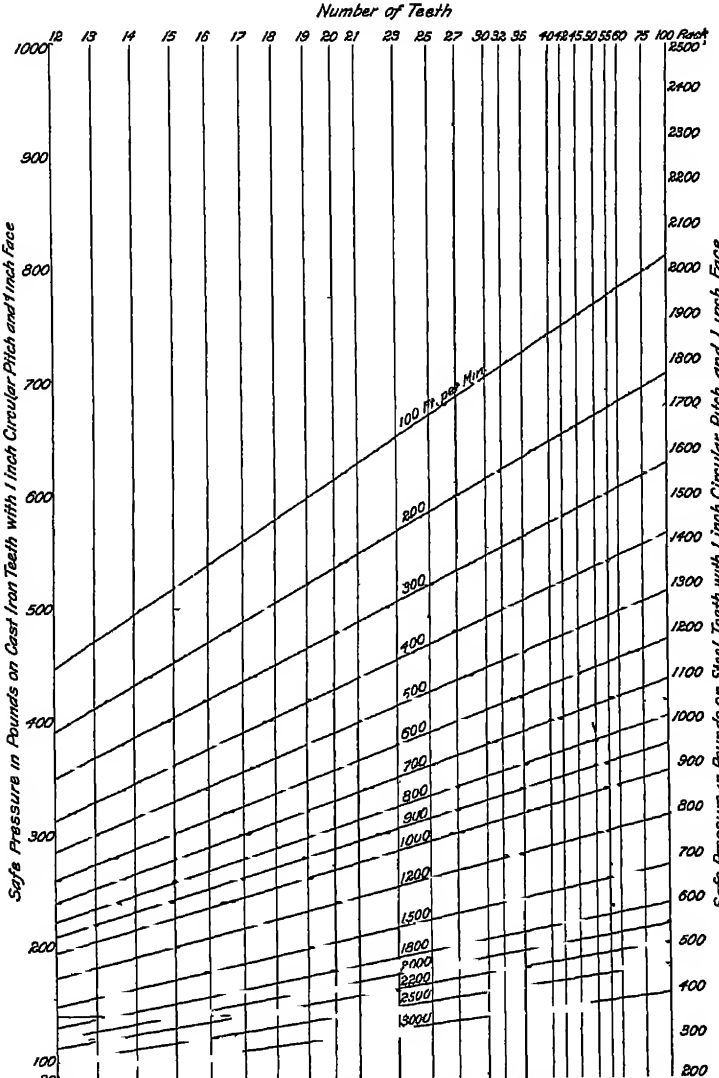
If it is preferable to use diametral pitch, then, from Table III, the nearest equivalent value is 6

To ascertain whether these values of f and p are suitable, they should be tested by means of formula **3**, Art **11**; that is, $f = \frac{9.4}{p}$ to $\frac{11}{p}$. As $f = 1.75$ and $p = 6$, $f p = 1.75 \times 6 = 10.5$. As this value is between 9.4 and 11, it is satisfactory

The value n for the pinion is $p d = 6 \times 9 = 54$, if d is the pitch diameter of the gear. For the wheel, $n = p d = 6 \times 15 = 90$.

In this example, the approximate diametral pitch is so near the circular pitch found by calculation that the variation in the center distance between gears is negligible In some cases, however, this variation may be so great that the values of f and p_1 must be readjusted.

14. The calculation of gear proportions may be greatly simplified by means of the diagram shown in Fig. 6, which



Number of Teeth
12 13 14 15 16 17 18 19 20 21 23 25 27 30 32 35 40 42 45 50 55 60 75 100 Rack
Safe Pressure in Pounds on Cast Iron Teeth with 1 inch Circular Pitch and 1 inch Face
1000
900
800
700
600
500
400
300
200
100
80
Safe Pressure in Pounds on Steel Teeth with 1 inch Circular Pitch and 1 inch Face
2500
2400
2300
2200
2100
2000
1900
1800
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100 Ft. per Min.
200
300
400
500
600
700
800
900
1000
1200
1500
1800
2000
2200
2500
3000
12 13 14 15 16 17 18 19 20 21 23 25 27 30 32 35 40 42 45 50 55 60 75 100 Rack
Number of Teeth

Fig. 6

represents the strength of teeth having 1-inch circular pitch and 1-inch face, the teeth being parts of gears with teeth numbering from 12 to 300, and running at speeds varying from 100 to 3,000 feet per minute.

The diagram is based on the values given in Table I for cycloidal or involute teeth with 15° obliquity, while the working stresses are based on the values given in Fig. 5. For cast-iron teeth, the values at the left side of the diagram are used; for steel, those at the right side.

The diagonals represent the speeds in feet per minute marked near their centers. The point of intersection between a diagonal and a vertical line, representing a given number of teeth, gives what may be termed the *unit strength* of the tooth. The value of this unit strength, in pounds, is indicated by the horizontal line passing through the point of intersection.

For example, to find the unit strength of a tooth in a thirty-tooth gear running at 700 feet per minute, follow the 700-foot diagonal until it intersects the vertical line marked *30*. The horizontal line passing through the point of intersection indicates a strength of 380 pounds for cast iron and 950 pounds for steel.

15. In this special case, where $p_1 = 1$ and $f = 1$, formula **2**, Art. **7**, $W = S_4 f p_1 y$, assumes the form $W = S_4 y$. This value is given in Fig. 6, and the special value termed unit strength may be designated by W_u; hence,

$$W_u = S_4 y \qquad (1)$$

and

$$W = W_u p_1 f \qquad (2)$$

Solving this formula for $p_1 f$,

$$p_1 f = \frac{W}{W_u} \qquad (3)$$

The application of these formulas may be shown by means of the following examples:

EXAMPLE 1 —Find the working strength of a steel gear with 60 teeth of 3 diametral pitch and 3-inch face running at a speed of 1,500 feet per minute.

SOLUTION —From the diagram, Fig 6, the value W_u is about 650 lb. From Table III, the diametral pitch 3 = 1.047 in. circular pitch $= p_1$. From formula **2**,

$$W = 650 \times 1\ 047 \times 3 = 2{,}042 \text{ lb. Ans}$$

EXAMPLE 2 —A cast-iron gear with 20 teeth runs at a speed of 400 feet per minute. Find the pitch and width of the teeth if the load W is 1,200 pounds.

SOLUTION —From Fig. 6, $W_u = 430$ lb. From formula **3**,

$$p_1 f = \frac{1{,}200}{430} = 2\ 79$$

Of the factors p_1 and f, either one may be assumed, in which case the other may be found by one of the following equations.

$$p_1 = \frac{2.79}{f}, \text{ or } f = \frac{2.79}{p_1}$$

Or, the ratio of p_1 to f may be assumed. For instance, if $f = 3\,p_1$, then $2.79 = p_1 f = p_1 \times 3 p_1 = 3 p_1^2$, $p_1 = \sqrt{\frac{2.79}{3}} = 9644$ in., and

$$f = .9644 \times 3 = 2\ 89, \text{ say } 2\tfrac{7}{8}, \text{ in. Ans.}$$

PROPORTIONS OF SPUR GEAR-TEETH

16. The following formulas are commonly used in practice for the proportions of **spur gear-teeth** that are cut with a milling cutter. Referring to Fig. 7, let

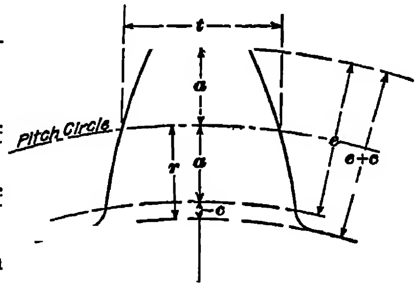


FIG. 7

a = addendum;
c = clearance at bottom of tooth;
$r = a + c$ = root;
e = working depth of tooth;
$a + r$ = whole depth of tooth;
t = thickness of tooth on pitch line;
d = diameter of pitch circle;
D = outside diameter of gear = blank diameter;
p = diametral pitch;
p_1 = circular pitch;
n = number of teeth in gear.

Then, $$p = \frac{\pi}{p_1} = \frac{n}{d} \qquad (1)$$

$$p_1 = \frac{\pi}{p} \qquad (2)$$

$$a = \frac{1}{p} = \frac{p_1}{\pi} \qquad (3)$$

$$a = \frac{d}{n} \qquad (4)$$

$$t = \tfrac{1}{2} p_1 = \frac{1.57}{p} \qquad (5)$$

$$d = \frac{n}{p} = \frac{n p_1}{\pi} \qquad (6)$$

The clearance c varies in practice from $.04\,p_1$ to $.1\,p_1$, the most common practice for cut gearing being from .04 to $.05\,p_1$.

If $c = .04\,p_1 = \text{approximately } \frac{1}{8p} = \frac{.125}{p} = \frac{t}{12}$; (7)

then $$r = \frac{1.125}{p} = .358\,p_1 \qquad (8)$$

and $$a + r = \frac{2.125}{p} = .676\,p \qquad (9)$$

$$\text{If } c = .05\,p_1 = \frac{t}{10} = \frac{.157}{p}; \qquad (10)$$

then $$r = \frac{1.157}{p} = .368\,p_1; \qquad (11)$$

and $$a + r = \frac{2\,157}{p} = .687\,p_1 \qquad (12)$$

In the succeeding pages of this Section a clearance of $.05\,p_1$ will be used.

PROPORTIONS OF SPUR WHEELS

17. Rims.—Detailed calculation of the stresses in a gear-wheel rim are not necessary, as there may be accidental stresses that cannot be estimated. Hence, the rim is made amply strong for all possible contingencies. Various shapes of rim section are shown in Fig. 8, the part e in (a) and (b) being a rib that gives additional strength. It is good practice to make the rim thickness h equal to that of the tooth at the pitch circle if the circular pitch is greater than $1\frac{1}{2}$ inches.

For gears of smaller pitch, the thickness h may be found by the following formulas:

For circular pitch,

$$h = .4p_1 + \tfrac{1}{8} \text{ inch} \qquad (1)$$

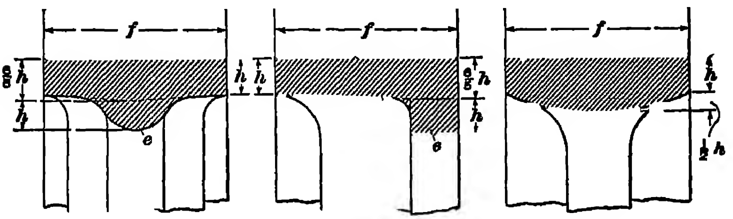
For diametral pitch,

$$h = \frac{5}{4p} + \tfrac{1}{8} \text{ inch} \qquad (2)$$

EXAMPLE.—Calculate the thickness of the rim of a gear-wheel having a diametral pitch of 4.

SOLUTION.—By formula 2,

$$h = \tfrac{5}{16} + \tfrac{1}{8} = \tfrac{7}{16} \text{ in. Ans.}$$



(a) (b) (c)

FIG. 8

18. When wooden cogs are mortised into a gear-wheel, the rim may have the proportions shown in Fig. 9. Usually only one of the wheels in a pair that gear together is furnished with wooden teeth, or cogs, in which case the wooden cogs may be made $1\tfrac{1}{2}$ times as thick as the iron

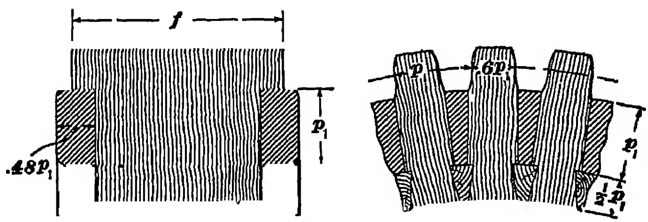


FIG. 9

teeth meshing with them; that is, for the wooden cogs, $t = .6p_1$; for the iron teeth, $t = .4p_1$, or less.

Fig. 9 shows two common methods of fastening the cogs to the rim. The unit of the dimensions is the circular pitch p_1.

19. Shrouded Gear-Teeth.—The teeth of a gear-wheel are said to be **shrouded** when the rim is made wider than the tooth and carried outwards so as to unite the ends of the teeth.

Three methods of shrouding are shown in Fig. 10. In (*a*),

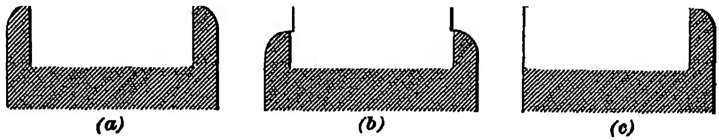


Fig. 10

the shrouding is carried on both sides to the end of the tooth. In this case it is evident that only one of the pair of wheels gearing together can be shrouded. In (*b*), the shrouding is carried to the pitch line, and, consequently, both wheels of the pair may be shrouded. In (*c*), the shrouding is carried up on one side only.

Pinions with few teeth are most benefited by shrouding, since the teeth in that case are weak at the roots, and also because more wear comes on the pinion.

20. Arms of Wheels.—The form of the cross-section of the arms of a gear-wheel depends on the form of rim used.

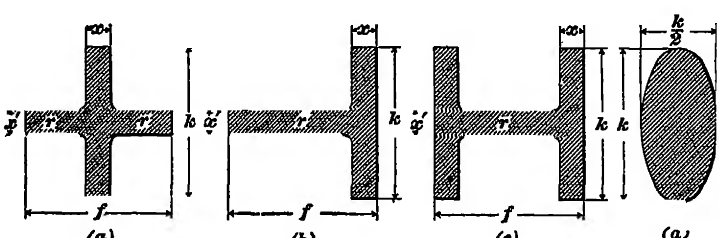


Fig 11

The forms in ordinary use are shown in Fig. 11. The cross-shaped form in (*a*) is the one mostly used for spur gears; and the section shown in (*b*), for bevel gears. The section shown in (*c*) is good for heavy gears, while the oval form (*d*) is a neat form for light gears.

In all of the forms given, the dimension k lies in the plane of the wheel, and the dimension f at right angles to this plane, or parallel with the axis of the wheel.

The stresses in the arms of gears are similar to those in pulleys. Therefore, the explanations given on this point in *Machine Design*, Part 5, will also apply to gears, with this exception: In pulleys, the rim is so thin that it cannot always be depended on to distribute the driving force equally

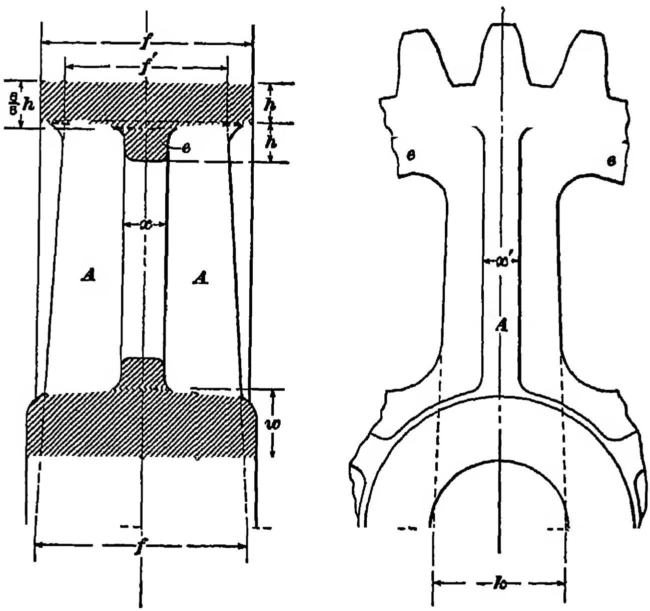


FIG 12

among all the arms. In gears, the rim is too thick to allow any difference between the stresses in the arms, and it is therefore assumed that each arm of a gear transmits to the hub an equal portion of the total load.

In calculating the strength of the arm, the ribs r, Fig. 11, are not taken into account, as they are intended to give the arm lateral stiffness and not to add to its strength in the direction of rotation.

Let z = number of arms in wheel;
k = width of arm, measured at center of wheel (see Fig. 12);
x = thickness of arm;
S_4 = allowable safe bending stress in arm;
R = pitch radius of wheel, in inches;
W = pressure between gears at pitch line.

Each arm may be considered as fixed at the center of the wheel and free at the end, which, though not strictly true, is on the safe side. Then, the total bending moment is WR inch-pounds, and the bending moment on each arm is $\frac{WR}{z}$. The moment of resistance for a rectangular section is

$$\frac{S_4 I}{c} = S_4 \frac{\frac{1}{12} k^3 x}{\frac{1}{2} k} = \frac{1}{6} S_4 k^2 x$$

Hence,
$$\frac{WR}{z} = \frac{1}{6} S_4 k^2 x$$

In some cases, it is preferable to solve this equation for the product $k^2 x$, so as to be able to adjust this product according to circumstances. Then,

$$k^2 x = \frac{6 WR}{z S_4}$$

Inserting in this formula the value of W in formula **2**, Art. **7**,

$$k^2 x = \frac{6 R S_4 f p_1 y}{z S_4} = \frac{6 R f p_1 y}{z} \qquad (1)$$

To avoid strains and breakage in casting, all parts of the wheel should be of nearly the same thickness. Therefore, the thickness of the arm may be made equal to the thickness of the tooth or of the rim.

If $x = \frac{p_1}{2}$, then

$$\frac{k^2 p_1}{2} = \frac{6 R f p_1 y}{z}, \text{ and } k^2 = \frac{12 R f y}{z}$$

Hence,
$$k = 3.5\sqrt{\frac{R f y}{z}} \qquad (2)$$

EXAMPLE.—A $2\frac{1}{2}$-foot gear-wheel with 6 arms has cycloidal teeth with a circular pitch of 2 inches, the width of face is 5 inches. Find the dimensions of the arms.

SOLUTION.—The pitch being 2 in., there will be $\frac{\pi \times 30}{2} = 47$ teeth. From Table I, this gives y a value of .112. By formula **1**,

$$k^2 x = \frac{6 \times 15 \times 5 \times 2 \times .112}{6} = 16.8.$$

If x is made equal to $\frac{p_1}{2} = 1$ in., then $k^2 x = k^2 \times 1 = k^2 = 16.8$ in.,

and $$k = \sqrt{16.8} = 4.1, \text{ or 4 in., nearly. Ans.}$$

If this width k does not appear proportionate to the other dimensions of the wheel, it can be changed by modifying the value of x in the product $k^2 x$.

For instance, if $x = 1.25$ in., then

$$k^2 x = k^2 \times 1.25 = 16.8 \text{ in. and } k^2 = \frac{16.8}{1.25}.$$

$$k = \sqrt{\frac{16.8}{1.25}} = 3.67, \text{ or } 3\tfrac{11}{16} \text{ in. Ans.}$$

21. For arms of elliptical cross-section, the moment of resistance is $\frac{S_4 I}{c}$, which equals $\dfrac{S_4 \dfrac{\pi k^3 (\frac{1}{2} k)}{64}}{\frac{1}{2} k}$ (see Table of Moments of Inertia, *Strength of Materials*, Part 2).

Hence, if the bending moment is to equal the resisting moment,

$$\frac{WR}{z} = \frac{S_4 \pi k^3}{64}, \text{ and } k^3 = \frac{64\, WR}{\pi z S_4}$$

Inserting the value of W from formula **2**, Art. **7**,

$$k^3 = \frac{64 R S_4 p_1 f y}{\pi z S_4} = \frac{64 R f p_1 y}{\pi z}$$

and $$k = 2.73 \sqrt[3]{\frac{R f p_1 y}{z}} \qquad (1)$$

If the diametral pitch p is to be used in the formula,

$$k = 4 \sqrt[3]{\frac{R f y}{z p}} \qquad (2)$$

EXAMPLE.—A gear-wheel 12 inches in diameter has 48 involute teeth of 15° obliquity and 4 arms. The rim is $2\frac{1}{2}$ inches wide. Assuming that the arms are of elliptical cross-section and that the teeth have a diametral pitch of 4, find their dimensions, the thickness being half the breadth.

SOLUTION.—Applying formula 2 and inserting the value of .112 for y, as given in Table I,

$$k = 4\sqrt[3]{\frac{6 \times 2.5 \times .112}{4 \times 4}} = 1.887, \text{ or } 1\tfrac{7}{8} \text{ in., nearly. Ans.}$$

$$\frac{k}{2} = \tfrac{15}{16} \text{ in., nearly. Ans.}$$

22. Formula **1**, Art. **20**, and formula **2**, Art. **21**, give the width of arm measured at the center of the wheel. The arm is tapered from center to circumference. For small gears the taper may be $\frac{3}{8}$ inch per foot on each side; for larger gears, $\frac{1}{4}$ inch per foot on each side. The thickness x remains constant, but in the elliptical form, the arm is tapered in both width and thickness, so that the latter is constantly equal to half the former.

The average thickness x' of the stiffening ribs r, Fig. 11, may be .4 p_1. The ribs in Fig. 11 (a) and (b) are tapered slightly toward the outside, to allow the pattern to be easily drawn from the mold. At the center of the hub, the width of the rib may be f, the same as the width of the tooth face. The rib is tapered so that the width at the rim is $f' = \frac{3}{4} f$ to $\frac{7}{8} f$.

The number of arms to be used in a given gear-wheel is largely a matter of judgment. Reuleaux gives the following formula:

$$z = .55\sqrt[4]{n^2 p_1},$$

in which z = number of arms;
n = number of teeth;
p_1 = circular pitch.

EXAMPLE.—How many arms should be given a gear 4 feet in diameter, the diametral pitch being $1\frac{1}{2}$?

SOLUTION.— $n = 4 \times 12 \times 1\frac{1}{2} = 72$ and $p_1 = \frac{\pi}{p} = \frac{\pi}{1.5} = 2.09$ in. Hence, from the formula,

$$z = .55\sqrt[4]{72^2 \times 2.09} = 5.61$$

Therefore, 6 arms should be used. Ans.

If the formula gives an odd number of arms, the nearest even number may be used, if desired.

23. Hubs, or Naves, of Gear-Wheels.—The thickness of the hub is often made equal to the radius of the shaft

on which the wheel is placed. If the shaft is enlarged for the wheel seat, the thickness of the hub is made equal to the radius of the main portion of the shaft. Then, if d represents the diameter of the shaft, and d_1 the diameter of the enlarged portion, the thickness of the hub w is $\frac{1}{2} d$.

The foregoing rule is very generally used, and gives good proportions when the wheel transmits the full power of the shaft. If, however, a gear transmits only a fraction of the power of the shaft, the hub thickness may be calculated from

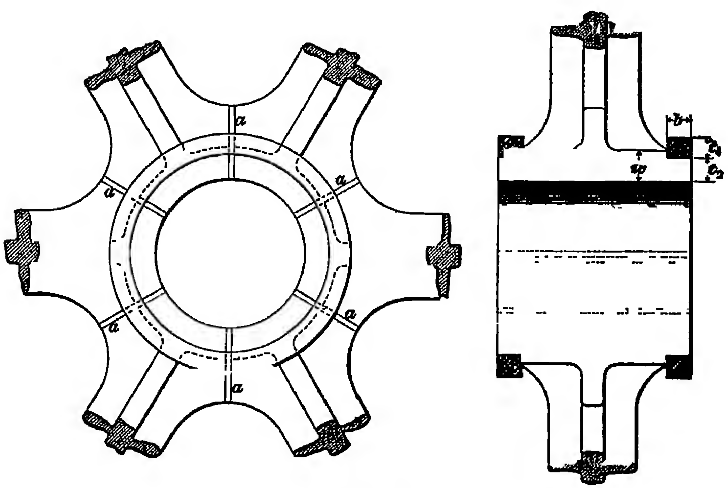


FIG. 13

the following formula, which may also be used for finding the thickness of the hub of any gear-wheel:

$$w = \frac{1}{3}\sqrt[3]{f p_1 R},$$

in which w is the thickness of the hub, and f, p_1, and R the same as in the previous formulas.

The length of the hub varies from f to $1.4 f$. To facilitate the molding, the hub should taper from the center of the arms toward both ends at the rate of about $\frac{1}{2}$ inch per foot.

The hubs of large and heavy wheels may be split as shown in Fig. 13. This relieves the hub of the initial stresses due to unequal contraction in cooling.

Strips of metal a are placed in the slots, and the segments are held together by iron or steel bands shrunk on. The proportions are:

$$t_1 = \tfrac{2}{3}w;\quad t_2 = \tfrac{3}{4}w;\quad b = \tfrac{7}{12}w$$

24. Built-Up Wheels.—For convenience in casting,

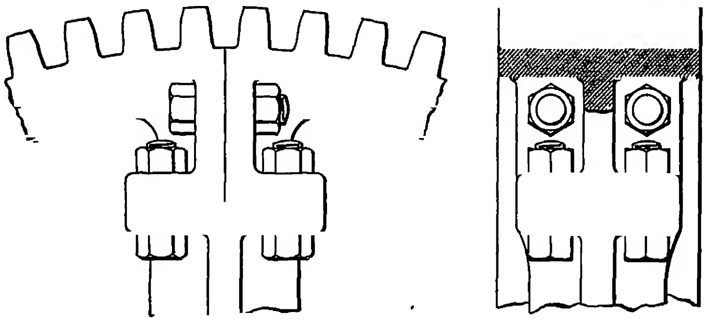


Fig 14

and also for convenience in transportation, large, heavy wheels are made in sections, which are assembled and bolted together. The wheel may be divided in various ways. The hub and arms may be cast separately, and the rim in segments; the hub and arms may be cast together, and the rim in segments; or, finally, each division may include a portion of the hub, an arm, and a segment of the rim. Fig. 14

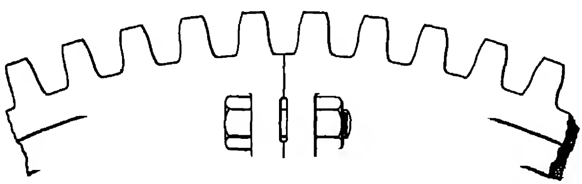


Fig 15

shows one method of bolting the rim, segments, and arm. Fig. 15 shows a method in which the rim and hub of the gear are parted and held together by bolts.

25. Application of Formulas.—In order to show the application of the preceding formulas, the calculations

necessary in designing a spur wheel, Fig. 12, are given in the example that follows.

EXAMPLE.—Compute the leading dimensions for a cast-iron spur wheel with 15° involute teeth The diameter of the wheel is 5 feet 6 inches at the pitch circle, and its arms have a cross-section as shown in Fig 11 (*a*). The power transmitted is 240 horsepower at a speed of 45 revolutions per minute.

SOLUTION —The velocity V of a point in the pitch circle is $\pi D n = 3\ 1416 \times 5\ 5 \times 45 = 778$ ft per min. Hence, from Fig. 5, S_4 is about 3,400 lb. per sq. in. for cast iron From formula **1**, Art. **10**,

$$H = \frac{WV}{33{,}000}, \text{ and } W = \frac{33{,}000\,H}{V} = \frac{33{,}000 \times 240}{778} = 10{,}180 \text{ lb.}$$

Now, from formula **2**, Art. **7**, $W = S_4 f p_1 y$, and $f p_1 = \frac{W}{S_4 y}$. Assuming as a trial value the number of teeth $n = 50$, then, from Table I, $y = .112$. Hence,

$$f p_1 = \frac{10{,}180}{3{,}400 \times .112} = 26\ 7$$

If f is given a value of $3p_1$, then $f p_1 = 3 p_1^2 = 26\ 7$ and $p_1^2 = \frac{26.7}{3} = 8.9$, or 9 in., nearly. $p_1 = 3$.

$$\text{Number of teeth} = \frac{\text{circumference of wheel}}{3} = \frac{3\ 1416 \times 66}{3} = 69.1.$$

Taking 69 as the number of teeth,

$$p_1 = \frac{3.1416 \times 66}{69} = 3\ 005 \text{ in.}$$

From formula **3**, Art. **11**, the breadth of face is practically

$$f = 3p_1 = 3 \times 3 = 9 \text{ in. Ans.}$$

The number of teeth was assumed as 50 while the calculation gives 69, which would make $y = .115$ instead of .112; but as the error is on the safe side, the previous value of y may be retained.

From the formulas of Art **16**, the thickness of tooth is

$$t = \tfrac{1}{2} p_1 = \frac{3.005}{2} = 1\ 503 \text{ in. Ans.}$$

$$\text{Addendum } a = \frac{p_1}{\pi} = .957 \text{ in. Ans.}$$

$$\text{Root } r = .368 p_1 = 1.106 \text{ in. Ans.}$$

The thickness of rim is found from formula **1**, Art. **17**; thus, $h = .4 p_1 + \frac{1}{8}$ in. $= 4 \times 3\ 0 + \frac{1}{8}$ in. $= 1\ 33$, or $1\frac{3}{8}$, in , nearly Ans.

For the size of the shaft, the following formula from *Machine Design*, Part 4, may be applied:

$$d = 68\ 45 \sqrt[3]{\frac{H}{n S_s}} = 68.45 \sqrt[3]{\frac{240}{45 \times 5{,}000}} = 7 \text{ in. Ans.}$$

The bending moment being unknown, it is advisable to use a low value for S_2, say 5,000 lb.

The enlargement for the wheel seat will then have a diameter depending on the depth of the key, which is calculated by means of the formula in *Machine Design*, Part 2

For the thickness of the hub, apply the formula in Art. **23**; thus,

$$w = \tfrac{1}{3}\sqrt[3]{f p_1 R} = \tfrac{1}{3}\sqrt[3]{9 \times 3 \times 33} = 3\tfrac{1}{4} \text{ in., nearly. Ans.}$$

The length of hub may, from Art **23**, be

$$1.4 f = 1.4 \times 9 = 12.6 = 13 \text{ in., nearly. Ans.}$$

For the number of arms, the formula of Art. **22** gives

$$z = .55\sqrt[4]{n^2 p_1} = .55\sqrt[4]{69^2 \times 3} = 6$$

Therefore, use 6 arms. Ans

For the width of arm at the center of the wheel, use formula **2**, Art. **20**, thus,

$$K = 3.5\sqrt{\frac{R f y}{z}} = 3.5\sqrt{\frac{33 \times 9 \times .112}{6}} = 8.24, \text{ or } 8\tfrac{1}{4}, \text{ in., nearly. Ans.}$$

Supposing the arms to taper $\frac{1}{4}$ inch per foot on each side, the width of arm at the pitch line is

$$8\tfrac{1}{4} - (2 \times 2\tfrac{3}{4} \text{ ft.} \times \tfrac{1}{4}) = 6\tfrac{7}{8} \text{ in. Ans.}$$

The thickness of the arm is

$$x = \tfrac{1}{2} p_1 = \tfrac{1}{2} \times 3 = 1.5. \text{ Ans.}$$

The thickness of the stiffening rib is

$$x' = .4 p_1 = .4 \times 3 = 1.2, \text{ or } 1\tfrac{1}{4}, \text{ in., nearly. Ans.}$$

The width of the rib at the hub is, from Art. **22**, equal to $f = 9$ in. At the rim the width is

$$f' = \tfrac{3}{4} f = 6\tfrac{3}{4} \text{ in. Ans.}$$

BEVEL GEARS

26. Strength of Teeth.—It was shown in *Gearing* that the profiles of the teeth of bevel gears are laid off on two cones, sometimes called *back cones*, the sides of which, at the point of contact, are normal to the line of contact of the two pitch cones. A back cone $v_1 o v_2$ is shown in Fig. 16, the side of which, or the length R, is equal to the radius of the pitch circle on which the profiles of the teeth at the larger end are laid off. The circle may be termed the *formative pitch circle*, and its radius the *formative pitch radius*.

The Lewis formulas for spur gears may, in modified form, be applied also to bevel gears, if it is assumed that a bevel gear is equal to a spur gear with a radius R and a

corresponding number of teeth N. The values y for this number of teeth may then be found from Table I. Referring to Fig. 16, let

D = pitch diameter at large end of bevel gear-tooth;
d = pitch diameter at small end of bevel gear-tooth;
p_1 = circular pitch at large diameter D;
p = diametral pitch at large diameter D;
N = the formative number of teeth, that is, the number of teeth that would be contained in the formative pitch circle with radius R;

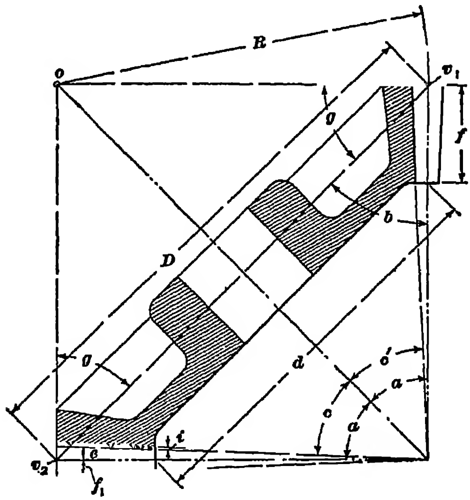


FIG. 16

n_1 = actual number of teeth on gear;
g = angle of edge of bevel gear;
f = face of bevel tooth;
y = factor from Table I, depending on shape of tooth and the number N;
W = working load on teeth;
S_4 = safe working stress for flexure.

The value W may be found by means of the following formula, which is developed from formula **2**, Art. **7**:

$$W = S_4 p_1 f y \frac{d}{D} \qquad (1)$$

In this formula, d should not be less than $\frac{2}{3} D$. If the diameter d is not known, it may be found from the gear drawing.

The number of teeth on the formative and actual pitch circles is proportional to the radii R and $\frac{D}{2}$, respectively; hence, $N : n_1 = R : \frac{D}{2}$; but, from Fig. 16, $R = \frac{D}{2 \cos g}$.

Substituting N for R and n_1 for $\frac{D}{2}$,

$$N = \frac{n_1}{\cos g} = n_1 \sec g \qquad (2)$$

EXAMPLE —Each of a pair of cast-iron miter gears revolving at 150 revolutions per minute has 60 teeth with radial flanks and a circular pitch of $2\frac{1}{4}$ inches at the larger end. If the face is 5 inches wide, what is the working strength?

SOLUTION —The diameter $D = \frac{\text{circumference}}{\pi}$, or

$$D = \frac{n_1 \times p_1}{\pi} = \frac{60 \times 2\ 25}{3\ 1416} = 42\ 97 \text{ in.}$$

From the gear drawing, it is found that with a face of 5 in., $d = 35\ 9$ in. From formula 2,

$$N = \frac{60}{\cos 45} = \frac{60}{707} = 85\text{, nearly}$$

From Table I, $y = \ 071$ The velocity is

$$V = \frac{n_1 \times p_1 \times 150}{12} = \frac{60 \times 2\ 25 \times 150}{12} = 1{,}688 \text{ ft per min.}$$

The value of S_4 for this speed, from Fig 5, is found to be about 2,100 lb. Inserting these values in formula 1,

$$W = 2{,}100 \times 2\ 25 \times 5 \times .071 \times \frac{35\ 9}{42.97} = 1{,}404 \text{ lb.} \quad \text{Ans.}$$

27. Angles of Bevel Gears.—Before proceeding to the construction of bevel gears, it is necessary to consider the important angles and their calculations. In Fig. 17 is shown a pair of bevel gears, the axes of which are at right angles, that is, the angle $C = 90°$. In this figure, the following symbols are used to represent the various dimensions and angles:

C = angle between axes of gears;
D_1 = pitch diameter of pinion, in inches;
D_2 = pitch diameter of gear, in inches;
$B D_1$ = blank diameter of pinion;
$B D_2$ = blank diameter of gear;

n_1 = number of teeth on pinion;
n_2 = number of teeth on gear;
a_1 = center angle of pinion;
a_2 = center angle of gear;
b_1 = face angle of pinion;
b_2 = face angle of gear;

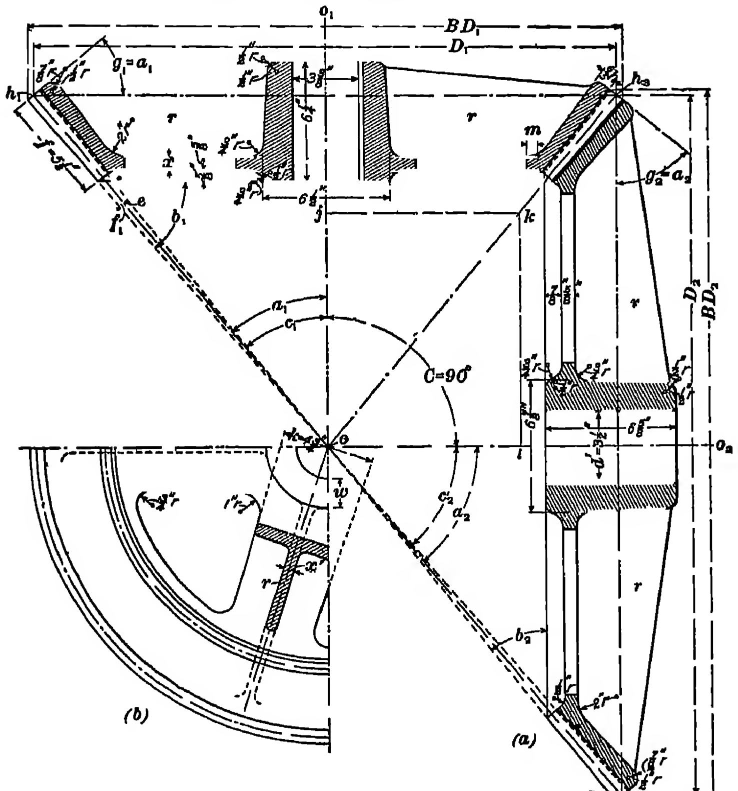


FIG 17

c_1 = cutting angle of pinion;
c_2 = cutting angle of gear;
e = angle of bottom;
f_1 = angle of top = e;
g_1 = angle of edge of pinion = a_1,
g_2 = angle of edge of gear = a_2.

The angles a_1 and a_2 are complementary; therefore, if a_1 is known, $a_2 = 90° - a_1$.

Most of the following formulas are given in *Gearing* or are based on well-known trigonometrical formulas:

$$\tan a_1 = \frac{n_1}{n_2} = \frac{D_1}{D_2} \qquad (1)$$

$$\tan a_2 = \frac{n_2}{n_1} = \frac{D_2}{D_1} \qquad (2)$$

$$\tan f_1 = \frac{2}{n_1} \sin a_1 = \frac{2}{n_2} \sin a_2 \qquad (3)$$

$$D_1 = \frac{n_1}{p}, \text{ and } D_2 = \frac{n_2}{p} \qquad (4)$$

$$B D_1 = 2\left(\frac{D_1}{n_1} \cos a_1 + \frac{D_1}{2}\right) \qquad (5)$$

$$B D_2 = 2\left(\frac{D_2}{n_2} \cos a_2 + \frac{D_2}{2}\right) \qquad (6)$$

$$c_1 = a_1 - e \qquad (7)$$

$$c_2 = a_2 - e \qquad (8)$$

$$b_1 = 90° - (a_1 + e) \qquad (9)$$

$$b_2 = 90° - (a_2 + e) \qquad (10)$$

When bevel gear-teeth are cut in a planer, the angle c', Fig. 16, is taken to be the cutting angle. However, as most teeth are formed by a milling cutter, which cannot adjust itself to the varying height and curvature of the bevel gear-tooth, it is necessary to let the cutter follow a path that gives a close approximation to the true tooth outline. Consequently, the angle c, Fig. 16, is made the cutting angle, and this angle will be the one adopted in the succeeding pages of this Section. The clearance i is produced by a cut parallel with the side of the cutting angle c.

28. Calculation of Bevel Gears.—In order to show the application of the preceding formulas, the calculation and layout of a pair of bevel gears will be illustrated by means of the following example:

EXAMPLE.—A pair of cast-iron bevel gears, the axes of which are at right angles to each other, are intended to transmit 65 horsepower. The ratio of their diameters is as 5 is to 6. The smaller gear, which

is the driver, is to revolve at 190 revolutions per minute; the width of its face is limited to $5\frac{1}{2}$ inches, and its pitch diameter to 30 inches. Calculate the pitch and the number of teeth on each gear, if the teeth are to be involute with an obliquity of 15°.

SOLUTION.—If the pitch diameter of the smaller gear is 30 in., then that of the larger gear is $30 \times \frac{6}{5} = 36$ in. In the drawing, Fig. 17 (*a*), the pitch diameters of the smaller and the larger gear are indicated by the letters D_1 and D_2, respectively.

The velocity V of the smaller gear is

$$\frac{\pi D_1 \times 190}{12} = 1{,}492 \text{ ft. per min.}$$

As $H = 65 = \frac{W V}{33{,}000}$,

$$W = \frac{33{,}000 \times 65}{1{,}492} = 1{,}438 \text{ lb.}$$

The values of p_1, f, d, and y, cannot be inserted in formula **1**, Art. **26**, $W = S_4 p_1 f y \frac{d}{D}$, before the number of teeth in the smaller gear is known. Selecting a diametral pitch of $p = 2$, then $n_1 = D_1 p = 30 \times 2 = 60$. The formative number of teeth N has now to be found from formula **2**, Art **26**, $N = \frac{n_1}{\cos g_1}$; first finding the value of $g_1 = a_1$ from formula **1**, Art **27**, $\tan a_1 = \frac{30}{36} = .833$. Therefore, $a_1 = g_1 = 39° 48'$. Inserting the value of cos 39° 48′ in the formula $N = \frac{n_1}{\cos g_1}$,

$$N = \frac{60}{7683} = 78\ 09, \text{ or } 78, \text{ teeth, nearly}$$

From Table I, the value of y for 78 teeth is 116, and from Fig. 5, the value of S_4 for a speed $V = 1{,}492$ is about 2,300 lb.

As the width f of the face is known, it is possible to find the diameter d from the gear drawing, being 23 in., nearly.

Solving formula **1**, Art. **26**, for p_1, then

$$p_1 = \frac{W D}{S_4 f y d} = \frac{1{,}438 \times 30}{2{,}300 \times 5.5 \times\ 116 \times 23} = 1.278 \text{ in.}$$

The number of teeth on the smaller gear corresponding to this p_1 is

$$n_1 = \frac{\pi D_1}{p_1} = \frac{\pi\, 30}{1\ 278} = 73.7$$

The ratio of the number of teeth on the smaller gear to those on the larger gear being 5 6, the nearest numbers that will fill this requirement are 75 and 90. With 75 teeth on the smaller gear, the pitch $p_1 = \frac{\pi\, 30}{75}$ $= 1\ 257$ in , which corresponds nearly to a diametral pitch $p = 2\frac{1}{2}$ in.

As the tentative value of the number of teeth is 60, the value of y used is larger than required and therefore on the safe side.

29. Proportions of Bevel Gears.—The rules and formulas used in designing spur gears are equally applicable to bevel gears. In referring to the tooth dimensions, those of the formative teeth constructed on the formative pitch circle are meant. The letter R in the formulas of Art. 20 is replaced by $\frac{D_1}{2}$ and $\frac{D_2}{2}$ in the following formulas. As the formulas refer equally to both the smaller and the larger gear, only one set will be given.

Let h = thickness of rim;
m = depth of rib, usually taken equal to h;
k = width of arm, measured at center of gear;
x = thickness of arm;
x' = thickness of web on arm;
w = thickness of hub;
z = number of arms;
d' = diameter of shaft;
n_1 = number of teeth;
n = number of revolutions;
H = horsepower.

Then,

$$h = .4\,p_1 + \tfrac{1}{8}\,m = \frac{5}{4\,p} + \tfrac{1}{8} \text{ inch} \qquad (1)$$

$$k = 3.5\sqrt{\frac{D_1 f y'}{2\,z}} \qquad (2)$$

$$x = \frac{p_1}{2} \qquad (3)$$

$$z = .55\sqrt[4]{n_1{}^2\,p_1} \qquad (4)$$

$$w = \tfrac{1}{3}\sqrt[3]{f\,p_1\frac{D_1}{2}} \qquad (5)$$

$$d' = 68.45\sqrt[3]{\frac{H}{n\,S_s}} \text{ (see \textit{Machine Design}, Part 4)} \qquad (6)$$

$$x' = .4\,p_1 \qquad (7)$$

30. The application of the preceding formulas may be shown by means of the following example:

EXAMPLE.—Retaining the dimensions of the gears in example of Art. 28, calculate the dimensions referring to the rims, arms, and hubs. The shafts of the gears are to be made of wrought iron, and

the safe shearing stress S_s is taken as 5,000 pounds; if of steel, S_s should be taken as 7,000 pounds.

SOLUTION —From the formulas of Art. **29**,

$$h = 4 \times 1\,257 + \tfrac{1}{8} = \tfrac{5}{8} \text{ in.}$$

For the gear with $n = 190 \times \frac{5}{6} = 158$, nearly, $n_2 = 90$, and $D_2 = 36$ in.

$$z = .55\sqrt[3]{90^2} \times 1.257 = 5.52, \text{ say } 6$$

$$k = 3.5\sqrt{\frac{36 \times 5.5 \times .116}{2 \times 6}} = 4\,84, \text{ or } 4\tfrac{7}{8}, \text{ in., nearly}$$

$$x = \frac{1\,257}{2} = .628, \text{ or } \tfrac{5}{8}, \text{ in., nearly}$$

$$w = \tfrac{1}{3}\sqrt[3]{5.5 \times 1.257 \times \tfrac{36}{2}} = 1.664, \text{ or } 1\tfrac{11}{16}, \text{ in., nearly}$$

$$d' = 68\,45\sqrt[3]{\frac{65}{158 \times 5{,}000}} = 2.98, \text{ or } 3 \text{ in., nearly}$$

If provision is to be made for a key, the shafts should be enlarged, according to *Machine Design*, Part 2, to at least $3 + \frac{d'}{6} = 3 + \frac{3}{6} = 3\frac{1}{2}$ in.

$$x' = .4 \times 1\,257 = 503, \text{ or } \tfrac{1}{2}, \text{ in., nearly}$$

From Art. **23**, the length of hub varies from f to $1.4\,f$ Selecting the value 1.2, the length is $1.2 \times 5\,5 = 6\,6$, or $6\frac{5}{8}$, in., nearly.

From formula **6**, Art. **27**, $B\,D_2 = 2\left(\frac{D_2}{n_2}\cos a_2 + \frac{D_2}{2}\right)$. From formula **2**, Art. **27**, $\tan a_2 = \frac{D_2}{D_1} = \frac{36}{30} = 1.2$ and $a_2 = 50° 12'$. Hence,

$$B\,D_2 = 2\left(\tfrac{36}{90} \times .64 + \tfrac{36}{2}\right) = 36\,512 \text{ in.}$$

For the pinion with $n_1 = 75$ and $D_1 = 30$ in., using the formulas just given,

$$z = .55\sqrt[3]{75^2} \times 1\,257 = 5.04, \text{ or } 5, \text{ in., nearly}$$

$$k = 3.5\sqrt{\frac{30 \times 5\,5 \times 116}{2 \times 5}} = 4.84, \text{ or } 4\tfrac{7}{8}, \text{ in., nearly}$$

$$x = \frac{1\,257}{2} = .628, \text{ or } \tfrac{5}{8}, \text{ in , nearly}$$

$$w = \tfrac{1}{3}\sqrt[3]{5\,5 \times 1\,257 \times \tfrac{30}{2}} = 1.566, \text{ or } 1\tfrac{9}{16}, \text{ in., nearly}$$

$$d' = 68\,45\sqrt[3]{\frac{65}{190 \times 5{,}000}} = 2.8, \text{ or } 2\tfrac{7}{8}, \text{ in , nearly}$$

To provide for a keyway, the diameter should be at least $2\frac{7}{8} + \frac{d'}{6} = 2\frac{7}{8} + \frac{23}{48} = 3.35$, or $3\frac{3}{8}$, in , nearly

$$x' = \tfrac{1}{2} \text{ inch}$$

The length of the hub may be $1\,13 \times 5\,5 = 6\frac{1}{4}$ in , nearly.

From formula **5**, Art **27**, $B\,D_1 = 2\left(\frac{D_1}{n_1}\cos a_1 + \frac{D_1}{2}\right)$. $a_1 = 90 - a_2 = 90 - 50° 12' = 39° 48'$ Hence,

$$B\,D_1 = 2\left(\tfrac{30}{75} \times 768 + \tfrac{30}{2}\right) = 30.614 \text{ in.}$$

From the formulas of Art. **27**,

$$\tan f = \tfrac{2}{75} \sin 39^\circ 48' = \tfrac{2}{75} \times .64 = .0171,$$

corresponding to an angle $f = 0^\circ 59'$. Angle $e = f$.

$$c_1 = 39^\circ 48' - 0^\circ 59' = 38^\circ 49'$$
$$c_2 = 50^\circ 12' - 0^\circ 59' = 49^\circ 13'$$
$$b_1 = 90^\circ - (39^\circ 48' + 0^\circ 59') = 90^\circ - 40^\circ 47' = 49^\circ 13'$$
$$b_2 = 90^\circ - (50^\circ 12' + 0^\circ 59') = 90^\circ - 51^\circ 11' = 38^\circ 49'$$

From formula **10**, Art. **16**, the clearance is

$$c = .05 \times 1.257 = .0628 \text{ in.}$$

31. The Gear Drawing.—Having ascertained all the necessary data in the example of Art. **30**, it is now possible to begin with the drawing of the gears. First, draw the center lines $o\,o_1$ and $o\,o_2$, Fig. 17 (*a*), of the shafts at the required angle C, in this case 90°, thereby locating the point of intersection o. As the velocity ratio is 5:6, it is possible to lay off the line of tangency $h_3\,o$ of the pitch cones. If one of the diameters is limited in length by the nearness of some part, therefore requiring it to be found tentatively, select any two lengths, such as $o\,i$ and $o\,j$, the ratio of whose lengths is as 5 is to 6. Draw $i\,k$ parallel to $o\,o_1$, and $k\,j$ parallel to $o\,o_2$. The point of intersection k of these lines is a point on the line of tangency; therefore, a line drawn from o through k will give the direction of the line $o\,h_3$. If, as in this case, the pitch diameters are given, the pitch radii $\frac{D_1}{2}$ and $\frac{D_2}{2}$ are used in place of the lengths $o\,i$ and $o\,j$, respectively. Next, draw the other sides $o\,h_1$ and $o\,h_2$ of the two pitch cones. Draw short lines perpendicular to $o\,h_1$, $o\,h_2$, and $o\,h_3$ through the points h_1, h_2, and h_3 on which to lay off the teeth and rims. The foundation of the drawing being laid, the details may be filled in in the usual manner, using for this purpose the dimensions previously calculated.

Some variation as to the location of the arms is possible, as their positions may be either at the center of the hub or near one of its ends, as in this example. Their position will to some extent influence the position of the hub relative to the point o, Fig. 17 (*a*), which position is often previously determined by the position of the shaft bearing.

It should be noted that bevel gears, unlike spur gears, are subjected to a side thrust—hence the importance of a sufficiently long hub and a good fit between hub and shaft. To insure true alinement, which is necessary for the correct intermeshing of the gears, the connection between the two shaft bearings should be rigid and, preferably, close to the hubs.

The web, indicated by the dimension x, is stiffened by a number of ribs r. In large gears, this web is provided with holes to reduce its weight, as in the example shown. The parts between the holes constitute arms, as shown in Fig. 17 (b), which are given the usual taper on each side of $\frac{1}{4}$ inch per foot. Smaller gears have a solid web, while the

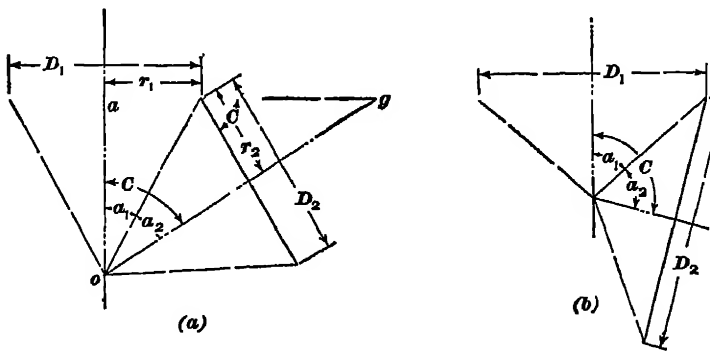


FIG. 18

smallest are generally made with a thicker web, omitting the ribs. All hubs should be tapered according to the rule given in Art. **23**.

The radii of the fillets are more or less arbitrary, but the ones given in the drawing serve the purpose of a general guide.

32. Bevel Gears With Axes Not at Right Angles. When the angle C between shafts, Fig. 18 (a), is smaller than 90°, formulas **1** and **2**, Art. **27**, assume a different form. In the illustration, $\tan a_1 = \frac{r_1}{a\,o}$; $a\,o = \frac{a\,g}{\tan C}$; $a\,g = r_1 + e\,g$ $= r_1 + \frac{r_2}{\cos C}$; hence,

and
$$ao = \frac{r_1 + \dfrac{r_2}{\cos C}}{\tan C}$$

$$\tan a_1 = \frac{r_1 \tan C}{r_1 + \dfrac{r_2}{\cos C}} = \frac{r_1 \sin C}{\left(r_1 + \dfrac{r_2}{\cos C}\right)\cos C} = \frac{r_1 \sin C}{r_1 \cos C + r_2}$$

Then,
$$\tan a_1 = \frac{\sin C}{\dfrac{r_2}{r_1} + \cos C} \qquad (1)$$

Since the number of teeth in each gear is proportionate to the pitch radius or pitch diameter, $\frac{r_2}{r_1} = \frac{n_2}{n_1} = \frac{D_2}{D_1}$, and any of these ratios may be used in the formula.

In the same manner, it may be shown that
$$\tan a_2 = \frac{\sin C}{\dfrac{r_1}{r_2} + \cos C} \qquad (2)$$

When C is greater than 90°, as in Fig. 18 (*b*),
$$\tan a_1 = \frac{\sin(180 - C)}{\dfrac{r_2}{r_1} - \cos(180 - C)} \qquad (3)$$

and
$$\tan a_2 = \frac{\sin(180 - C)}{\dfrac{r_1}{r_2} - \cos(180 - C)} \qquad (4)$$

WORM-GEARING

33. Definitions and Formulas.—In regard to the dimensions of the teeth, the formulas given in Art. 16 for spur gears apply largely to **worm-gears.** When the terms *2 pitch*, *3 pitch*, etc. are used in connection with worm-gearing, they do not refer to diametral pitch, but to the number of threads per inch of axial length of the worm. For a single-threaded worm, these terms will also indicate the number of revolutions of the worm necessary to advance the thread 1 inch. When used in connection with multiple-threaded worms, it is evident, that the terms 2 pitch, 3 pitch, etc. no longer indicate the number of revolutions for 1 inch of advance.

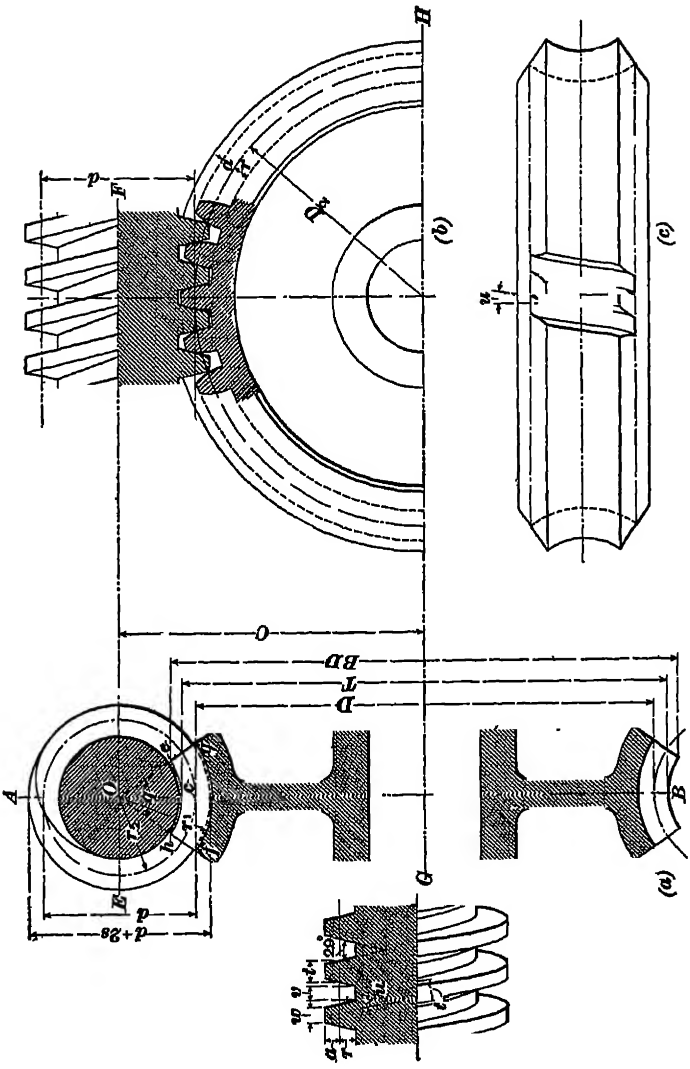


Fig 20

Considerable confusion exists in the application of the term *pitch* to worm-gearing. It has therefore been found necessary to use the term **lead** to indicate the advance of any one thread in 1 revolution of the worm.

The term **pitch** refers to the distance between corresponding points on *adjacent threads*, without reference to the number of threads. When measured in a direction parallel with the axis of the worm, the pitch of the worm-threads is termed **axial pitch.** If measured in a direction at right angles to the threads, it is called the **normal pitch.**

The axial pitch of the worm is equal to the circular pitch at the worm-wheel. Referring to Figs. 19 and 20, let

L = the lead, in inches;
N_1 = number of independent threads on worm;
N = number of teeth on wheel;
$p_1 = \begin{cases}\text{circular pitch of gear;}\\ \text{axial pitch of worm-thread;}\end{cases}$
p_n = normal pitch;
r_1 = radius of the curved base line of a wheel tooth;
r_2 = pitch radius of worm;
C = distance between centers of worm and wheel;
D = pitch diameter of worm-wheel;
d = pitch diameter of worm;
l_x = length of worm-helix in one turn;
T = throat diameter of worm-wheel;
BD = diameter of blank of wheel, to sharp corners;
q = angle of contact with worm;
b = pitch circumference of worm;
u = angle of a wheel tooth with the wheel axis, or helix angle, or angle between the worm-thread and a perpendicular to its axis;
m = number of threads per inch;
a = addendum;
r = root;
t = axial thickness of tooth of worm on pitch line;
v = width of space at bottom between worm-thread;
w = width of worm-thread at top;

v_1 = velocity at pitch circle of worm-wheel, in feet per minute;
v_2 = velocity at pitch circle of worm, in feet per minute;
t_n = normal thickness of tooth;
W = pressure between a worm-thread and a wheel tooth.

Then,
$$p_1 = \frac{1}{m} \qquad (1)$$
$$L = N_1 p_1 \qquad (2)$$

For a 5-pitch single-threaded worm, the pitch $p_1 = \frac{1}{m} = \frac{1}{5} = .2$ inch; the lead $L = N_1 p_1 = 1 \times .2 = .2$ inch. If a worm having the same pitch is double-threaded, then $L = N_1 p_1 = 2 \times .2 = .4$ inch.

As previously explained, the diametral pitch $p = \frac{\pi}{p_1}$. Consequently, if the pitch of the worm is known, the diametral pitch of the worm-wheel is $p = \frac{\pi}{p_1}$. For instance, if the worm-pitch is $p_1 = \frac{1}{5}$ inch, then $p = \frac{\pi}{\frac{1}{5}} = 5\pi = 15.708$. Diametral pitch cannot well be applied to worm-gears, since the pitch of the wheel is fixed by the pitch of the worm, which, in turn, depends on the number of threads per inch that it is possible to cut.

The following formulas apply to worm-gears:

$$p_1 = \frac{\pi T}{N+2} = \frac{\pi D}{N} \qquad (3)$$
$$D = \frac{N p_1}{\pi} = \frac{N}{p} \qquad (4)$$
$$T = D + 2a = \frac{N}{p} + 2a \qquad (5)$$
$$b = \pi d \qquad (6)$$
$$\tan u = \frac{L}{b} = \frac{L}{\pi d} \qquad (7)$$
$$t_n = t \cos u \qquad (8)$$
$$C = \frac{D+d}{2} \qquad (9)$$
$$B\,D = T + 2(r_1 - a - r)\left(1 - \cos\frac{g}{2}\right) \qquad (10)$$

When the angle between the sides of the thread is 29°, which is the angle generally adopted, then

$$v = t - 2r \tan 14.5°$$

and

$$v = .31 p_1 \qquad (11)$$

Also,

$$w = t - 2a \tan 14.5°$$

and

$$w = .335 p_1 \qquad (12)$$

$$p_n = p_1 \cos u \qquad (13)$$

$$l_x = \frac{\pi d}{\cos u} = \frac{b}{\cos u} \qquad (14)$$

The ratio of the regular velocity of the worm to that of the worm-wheel is $\frac{N}{N_1}$.

34. Efficiency of Worm-Gearing.—In the design of worm-gearing, many failures have been caused by disregarding the effects of the angle u of the helix and the pitch diameter of the worm on the efficiency of the gear. The angle has mostly been made too small and the diameter too great.

Extensive experiments carried on in recent years show that a maximum efficiency is obtained when the helix angle is 45°, but that this efficiency suffers little loss if the angle is decreased to 30° or increased to 60°. This rule applies to worms that run under heavy pressure and with ordinary lubrication; but if the worm runs in an oil bath, the angle may be as low as 20°. With light pressures and ordinary lubrication, the angle may be decreased to 12°, but it is not safe to go below this value.

Experiments by Wilfred Lewis show that the amount of work lost in friction between worm and wheel does not increase at the same rate as the speed, and that, therefore, the efficiency increases up to a certain point. If the pressure W is equal to the working strength of the gear, then the speed of the worm at the pitch line should not exceed 300 feet, the most favorable speed being about 200 feet per minute.

In Fig. 21 is shown the relation between the allowable tooth pressure W and the velocity of two hardened-steel worms, one having a helix angle $u = 20°$, and the other an angle of 10°. Each worm runs in an oil bath and drives a

wheel of phosphor-bronze. For instance, if a worm of a 20° helix angle exerts a tangential pressure on the wheel of 2,000 pounds, the diagram indicates that its velocity at the pitch circle should not exceed 380 feet per minute; while a worm of 10° helix angle running at the same velocity should not exert a pressure greater than 1,000 pounds. A slight increase in pressure may be allowed for angles greater than 20°. These data are based on experiments made by Professor Stribeck, and may assist in finding a suitable ratio between pressure and speed.

There is a loss of energy from the friction between worm and wheel, and in the worm step bearing. Taking this loss

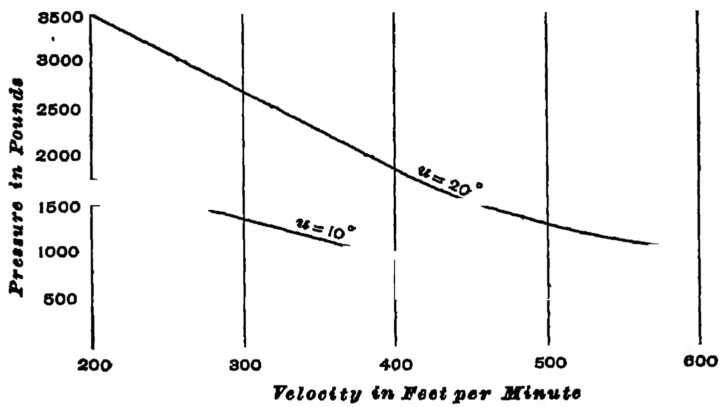


FIG. 21

into consideration, the efficiency e of a worm-gear is found approximately by the following formula by Barr:

$$e = \frac{\tan u\,(1 - f \tan u)}{\tan u + 2f}$$

An average value for f is .05.

35. Application of Formulas.—The following example will serve to show the application of the formulas given in Art. 33.

EXAMPLE.—A double-threaded hardened-steel worm drives a phosphor-bronze worm-wheel at a ratio of 25 : 1. The shafts are at right angles, and the distance between centers is 13.5 inches. If the worm

is to make 350 revolutions per minute, find the circular pitch of the worm-wheel, the pitch diameters, and the helix angle, it being one of the conditions that the latter shall be at least 15°. Also, determine the maximum tangential pressure permissible on the worm-wheel.

SOLUTION.—If the distance between centers is not definitely determined, there is a greater choice in diameters and angles, and the solution is therefore simpler. In the present instance, it is assumed that no variation is allowed, and the solution may therefore not give the desired result at first trial.

The worm being double-threaded and the velocity ratio 25 : 1, it follows that the worm-wheel must have $N_1 \times 25 = 2 \times 25 = 50$ teeth.

The first dimension to select is the circular pitch p_1. It is important to choose a pitch that can be cut with the available cutters in the milling machine. Selecting $p_1 = 1.5$ in., as a trial value, the circumference of the worm-wheel pitch circle is $p_1 N = 1.5 \times 50 = 75$ in. and the diameter $D = \frac{75}{\pi} = 23.873$ in. From formula **9**, Art. **33**, $C = \frac{D+d}{2}$; therefore, $d = 2C - D = 2 \times 13.5 - 23.873 = 3.127$ in., and

$$r_2 = \frac{d}{2} = \frac{3.127}{2} = 1.564 \text{ in. Ans.}$$

From formula **2**, Art **33**,

$$L = N_1 p_1 = 2 \times 1.5 = 3 \text{ in.}$$

From formula **7**, Art. **33**,

$$\tan u = \frac{L}{\pi d} = \frac{3}{\pi \times 3.127} = .3054$$

This corresponds to an angle $u = 16° 59'$. Ans.

From formula **13**, Art. **33**, the normal pitch is $p_n = p_1 \cos u = 1.5 \times .9564 = 1.435$ in.

From formula **14**, Art. **33**,

$$l_x = \frac{\pi d}{\cos u} = \frac{\pi \times 3.127}{.9564} = 10.27 \text{ in.}$$

With 350 revolutions per minute, the velocity of the worm at the pitch circle is

$$v_2 = \frac{350 \times l_x}{12} = \frac{350 \times 10.27}{12} = 299.5, \text{ or } 300, \text{ ft. per min., nearly}$$

From the diagram, Fig. 21, the allowable pressure $W = 2{,}700$ lb. With this pressure, the total energy transmitted to the worm-wheel is equal to the velocity at its pitch circle multiplied by W. The wheel makes $\frac{350}{25} = 14$ rev. per min.; and, as the pitch circumference of the wheel is 75 in., it follows that

$$v_1 = \frac{75 \times 14}{12} = 87.5 \text{ ft. per min.}$$

The energy transmitted is $W v_1 = 2{,}700 \times 87.5 = 236{,}250$ ft.-lb. per min. = 7.16 H. P.

From the formula of Art. **34**, the efficiency of the gear is

$$e = \frac{.3054\,(1 - .05 \times .3054)}{.3054 + 2 \times .05} = .74$$

Hence, the energy delivered to the worm must be 236,250 ÷ .74 = 319,257 ft.-lb. per min.

If P is the tangential force at the pitch circle of worm, then $P r_2$ is the turning moment. The velocity $v_2 = \frac{350\,\pi\,d}{12} = 286.5$ ft. per min. As the energy delivered to the worm is $P v_2 = 319{,}257$ ft.-lb., it follows that

$$P = \frac{319{,}257}{286.5} = 1{,}114 \text{ lb.}$$

The turning moment $P r_2 = 1{,}114 \times 1.564 = 1{,}742$ in.-lb.

Before the torsional strength of the worm can be ascertained, it is necessary to calculate the diameter of the worm at the bottom of the threads, that is, the core diameter.

From formula **11**, Art. **16**, $r = .368\,p_1 = .368 \times 1.5 = .552$ in. Hence, the core diameter of the worm is $d - 2r = 3.127 - 1.104 = 2.023$ in.

From the formula relating to shafts in *Machine Design*, Part 4, the twisting moment, $P r_2 = \frac{S_2 d^3}{5.1} = 1{,}742$. Transposing and solving for the safe working stress,

$$S_2 = \frac{5.1 \times 1{,}742}{2.023^3} = 1{,}073 \text{ lb.}$$

This value is very low for steel and the torsional moment can therefore easily be sustained by the worm if it is cut on the shaft. If the worm is keyed on the shaft, the strength of the latter must be calculated.

The next step is to determine whether the worm-wheel teeth are able to sustain a pressure of 2,700 lb. If the angle q, Fig. 20, is 90°, the length of the base of any tooth is found in the following manner:

The radius of curvature of the tooth base line fg, Fig. 20, is $r_1 = r_2 + r = 1.564 + .368 = 1.932$ in.

The circumference of a circle with this radius is $2\pi \times 1.932 = 12.14$. Hence, the length of base line is $\frac{12.14 \times 90°}{360°} = 3$ in., nearly.

To ascertain the working strength of this tooth, apply formula **2**, Art. **7**, $W = S_4 f p_1 y$. Inserting the value of 9,000 for S_4 for bronze, and .112 for y,

$$W = 9{,}000 \times 1.5 \times 3 \times .112 = 4{,}536 \text{ lb.} \quad \text{Ans.}$$

36. Construction of Worm-Gearing.—Worm-wheels may be constructed in various ways. The wheel may be similar to a spur wheel, except that the teeth make an angle with the axis of the wheel equal to that of the threads of the worm measured at the pitch diameter, as in Fig. 20 (*c*).

For careful and accurate work, the wheel is cut by a special milling tool called a *hob*. This hob is a steel worm that has been cut in a lathe, notched to form a milling cutter, and then hardened and tempered. The teeth cut by this hob must evidently have the correct form to gear with a worm having the same pitch as the hob. When the wheel teeth are constructed by this method, there is much closer contact between the worm and the wheel. Therefore, the durability and efficiency of the mechanism are greatly increased.

The rim and arms of the worm-wheel may be given the same proportions as the rim and arms of a spur wheel of

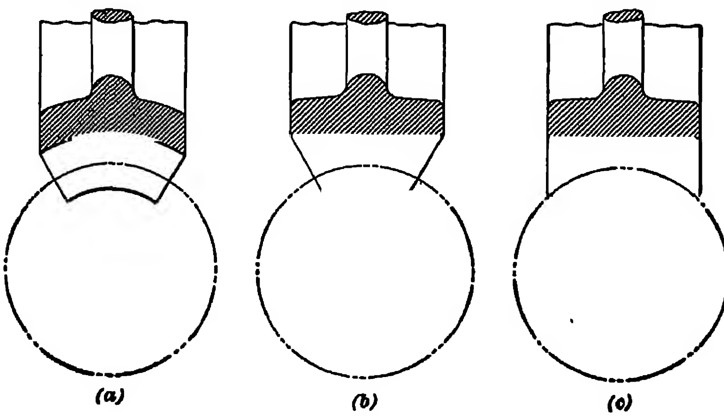


FIG. 22

the same diameter and pitch, except that the width of the face of the wheel is generally 1.5 to 2.5 p_1, the angle q, Fig. 20 (a), varying from 60° to 90°.

The rim may have the forms shown in Fig. 22. The form shown at (a) is best, as there is line contact between the thread and the tooth.

The ratio $\frac{r_2}{p_1}$ varies generally from 1.25 to 3; it may be more, but should be as small as possible.

When the worm is cut on the shaft, the lower values may be used. When, however, the worm is cast and keyed on

the shaft, the ratio must be made greater. The number of threads in the worm varies from 1 to 4, depending on the velocity ratio to be transmitted.

Usually, it is undesirable to give the worm-wheel less than thirty teeth; hence, if the desired velocity ratio is less than 30, the worm must have more than one thread. For example, if the worm-shaft is to make 20 revolutions while the wheel shaft makes 1 revolution, it is possible to have either $N = 40$ and $N_1 = 2$, or $N = 60$ and $N_1 = 3$.

The length of the worm is from $3p_1$ to $6p_1$, usually $4p_1$; that is, there are usually four turns, as shown in Fig. 20 (*b*).

37. The Worm-Gear Drawing.—After calculating the principal data relating to the diameters and the pitch of a worm-gear, the drawing may be made in the following manner:

Draw the main center line AB of the gear, Fig. 20 (*a*). Locate the center O of the worm, and draw the individual center lines EF and GH a distance apart equal to C. Determine the point of tangency c by laying off $\frac{D}{2}$ or r, along AB. On each side of point c, lay off the addenda a and roots r, in this manner determining the working depths and full depths of the teeth. Draw the angle q of the desired size and, inside this angle, the two arcs he and fg, indicating the height of the wheel teeth. With the radius r_2 describe the pitch circle of the worm and the addendum and root circles, thus determining the depth of its thread. Make the rim of the thickness specified for a spur gear, and draw the arms or web, as the case may be, connecting the rim with the hub.

In the other view, Fig. 20 (*b*), show the tooth outlines of the wheel and the circles limiting their heights. A working drawing does not require complete projections of the worm-threads and wheel teeth, and the helixes may, as in Fig. 20 (*b*), be indicated by straight instead of curved lines. Similarly, the cross-section of the worm and gear may simply show the threads and teeth as rack and spur teeth, respectively. The

necessary dimensions and other information required are inserted in the usual manner. The drawing shows the worm without a shaft, although the worm may be made either in one piece with the shaft or keyed to it.

SPIRAL GEARS

SHAFTS AT RIGHT ANGLES

38. Definitions.—When the number of threads on a worm, and, consequently, the helix angle, is increased to such an extent that any one thread on the worm does not make a complete turn, then worm-gearing comes under the general class of **spiral gears.** Distinction is sometimes made between spiral gears in which the axes are parallel or at an angle, the former being then called **twisted gears.**

In the twisted gears, as shown in Fig. 23, the action is similar to that of spur gears, except that in the former there is a greater number of teeth in contact, resulting in greater strength of the gears and smoother action. There is also an end thrust caused by the obliquity of the teeth. When it is desirable to neutralize this end thrust, two gears in which the teeth incline in opposite directions are united into one, as shown in Fig. 24. Such gears are called **double-helical,** or **herring-bone, gears,** and are used where great strength is required, as in rolling mills.

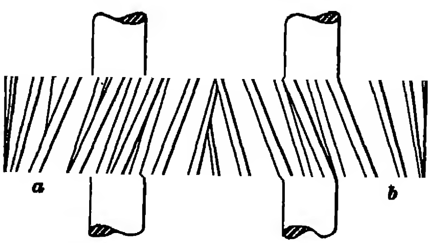


FIG 23

In spiral gears, the interaction between the teeth is changed to one consisting of a combined rolling and sliding. This may be seen from Fig. 25, in which the arrows indicate the directions of rotation. It will be noticed that in Fig. 23

the teeth incline in opposite directions relative to the wheel axes, while in Fig. 25 they incline in the same direction, though not necessarily at the same angle. The distinction between the two directions of inclination is the same as that between left- and right-handed screw threads. In Fig. 23, the gear a is a *right-hand* and b a *left-hand spiral gear;* in Fig. 25, both a and b are left-hand gears. If both were made right-hand gears, the relative directions of rotation would be changed. In spiral gears with axes at *right angles*,

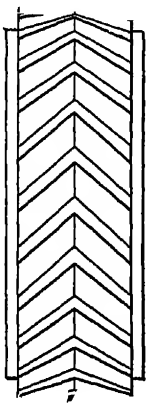


FIG 24

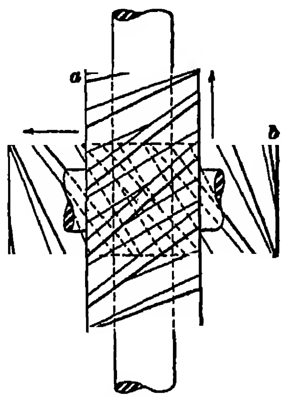


FIG. 25

the teeth on both gears are of the *same* hand, that is, either left hand or right hand.

39. Normal Helix.—The terms *circular*, *axial*, and *normal pitch* have already been defined, but in view of the importance of the normal pitch in the calculations of spiral gears, it is necessary to explain more fully its relation to the tooth helix.

Fig. 26 (a) shows a cylinder with a helix $a\,b\,b_1$, the length $a\,b\,b_1$ representing a complete turn of the helix. Normal to this helix is another one $c\,c_1\,d_1$, which intersects the other helix at the points $c_1\,d_1$. The helix $a\,b\,b_1$ is the **tooth helix** to which the teeth in spiral gears conform, and the helix $c\,c_1\,d_1$ is the **normal helix**.

40. Normal Pitch.—The helix $a\,b\,b_1$ would constitute one tooth in a spiral gear. To insure continuous action of such a gear, it is necessary to increase the number of helixes. This is done in the same manner in which the number of threads is increased on a screw; that is, by dividing the lead of the helix into a suitable number of parts and inserting a helix at each point of division. In Fig. 26 (a), fifteen additional helixes *1–1′*, *2–2′* . . . *15–15′* have been added, making it a sixteen-threaded screw, or a spiral gear with sixteen teeth. It is evident that the normal helix $c_1\,d_1$ is also divided into a number of parts equal to the number of teeth, and that the line $c_1\,d$ is the *normal pitch* of the teeth. Therefore,

normal pitch × number of teeth = length of normal helix

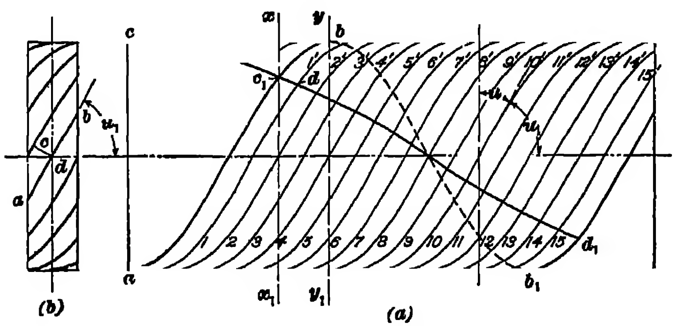


FIG. 26

The length of the normal helix considered is that intercepted by *one* tooth helix.

Only a portion of the spiral gear shown is required, as the length included between the lines $x\,x_1$ and $y\,y_1$. This part is shown separately in Fig. 26 (b), in which $a\,b$ is one of the sixteen teeth and $c\,d$ the normal pitch.

Let H_1 = length of the normal helix;

N_1 = number of teeth on gear;

p_n = normal pitch.

Then,
$$H_1 = N_1 p_n$$

41. Relation Between Helix Angle and Number of Teeth.—Two systems of designating the helix angle are in

use. In one system, the helix angle corresponds to the angle u, Fig. 26 (a), while in the other, the angle u_1 between the cylinder axis and a plane tangent to the helix is considered as the helix angle. As the latter angle is the one used by manufacturers of spiral gears, it will be taken as the helix angle in this Section. The normal helix angle is a complement of the helix angle.

The value of H_1 is a maximum when the helix angle u_1 is of minimum value, and vice versa.

In Fig. 27 is shown a spiral gear on which u_1 is nearly zero. The normal pitch is $a\,d$, and the normal helix $a\,c\,e$, intersecting the tooth helix $a\,b$ at points a and e. It will be noticed that only a small fraction of the tooth helix $a\,b$ is included between the points of intersection, but that the normal helix makes nearly one complete turn. In a single-threaded screw, on the other hand, the intercepted portion of the tooth helix is a maximum and the normal helix a minimum length, being equal to the normal pitch. This may be clearly seen if it is assumed that $a\,c\,e$, Fig. 27, is the tooth helix and $a\,e$ the normal pitch.

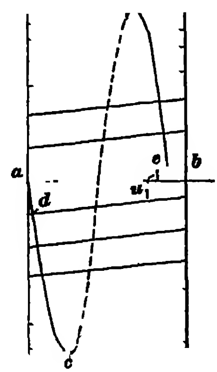


FIG. 27

From Figs. 26 (b) and 27, the effect of the angle u_1 on the number of teeth may be seen. If, in both cases, the diameters of the pitch cylinders and the normal pitches are the same, but the angle u_1 of the gear in Fig. 27 is decreased, then this gear will contain a larger number of teeth than that in Fig. 26.

A comparison of Figs. 26 and 27 shows that with a given normal pitch and pitch diameter it is possible to increase the number of teeth simply by decreasing the helix angle. On the other hand, with a given normal pitch and helix angle, it is possible to increase the number of teeth only by increasing the diameter. Consequently, in two gears of the same normal pitch, the diameters need not necessarily be proportional to the number of teeth in each. The diameters of two gears will be proportional to the number of teeth only when the

helix angles in both gears are equal. With shafts at right angles, this proportion will hold true for two gears in mesh when each of the helix angles is 45°.

42. Speed Ratio.—In a pair of gears with helix angles of 45°, it is unimportant which of the two gears is the driver; but in case of different angles, the *greater* one is given to the *driver*. Fig. 28 shows parts of the developed pitch surfaces

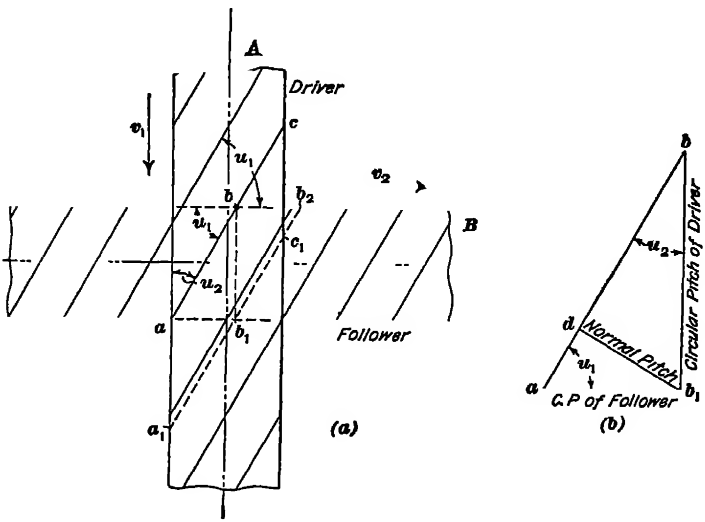


FIG 28

of two gears, the arrows indicating the directions of motion. The line of contact of tooth $a\,b\,c$ of the driver is shown coincident with the line of contact of the tooth $a\,b$ of the follower. When tooth $a\,b\,c$ moves downwards until the point b occupies the position b_1, the tooth $a\,b$ is pushed to the right and the point a has moved into the position b_1. The two teeth now occupy the positions $a_1\,b_1\,c_1$ and $b_1\,b_2$, respectively

The relative linear motion of the two gears is shown separately in Fig. 28 (b), where the ratio $\frac{b\,b_1}{a\,b_1}$ represents the relative pitch-line speeds of the gears. The helix angle of the driver is u_1 and that of the follower $u_2 = (90 - u_1)$.

Then, $$\frac{a\,b_1}{b\,b_1} = \cot u_1$$

From this formula it is evident that in spiral gearing the velocities of the pitch lines of two gears in mesh may be different.

Fig. 28 (*b*) also indicates the ratio between the circular pitches of driver and follower, the length $b\,b_1$ representing that of the driver and $a\,b_1$ that of the follower. The normal pitch is represented by $d\,b_1$. The circular pitch p_1 is found in the usual manner; thus,

$$p_1 = \frac{\text{pitch circumference}}{\text{number of teeth}}.$$

43. Adjustment of Speed Ratio.—As there are two methods of adjusting the velocity ratio of a pair of spiral gears—either by varying the diameters or by varying the angles—it would seem that by applying them singly or in combination it would be a simple matter to obtain any desired velocity ratio. But, if the distance between centers of shafts, the number of teeth in each gear, and the lengths of the normal helixes are fixed, then changes are possible only in the ratio of the diameters and the helix angles. There are also limitations in the choice of the latter, depending on whether they can be cut with the appliances at hand. To readjust all these values by analytical methods until the desired results are obtained, may often require a great deal of time and patience. Fortunately, there are graphical methods by which the problem may be easily solved, as will be shown further on.

Let N_1 = number of teeth in driver;
N_2 = number of teeth in follower;
C = center distance between shafts;
p = diametral pitch;
p_1 = circular pitch;
D_1 = pitch diameter of driver;
D_2 = pitch diameter of follower;
u_1 = tooth helix angle of driver;
u_2 = tooth helix angle of follower;

e = angle between shafts;
H_1 = length of normal helix of driver between points of intersection with tooth helix;
H_2 = length of normal helix of follower;
R_1 = revolutions per minute of driver;
R_2 = revolutions per minute of follower;
v_1 = pitch-line velocity of driver;
v_2 = pitch-line velocity of follower;
L_1 = lead of tooth helix of driver;
L_2 = lead of tooth helix of follower.

As the circular pitch in Fig. 28 (*b*) is proportional to the pitch-line velocity of the gear, it follows that $b\,b_1 = v_1$ and $a\,b_1 = v_2$. Hence,

$$\frac{v_2}{v_1} = \cot u_1$$

As the pitch-line velocity of a gear is proportional to the number of revolutions per minute, for gears of the same diameter, the preceding formula may be written

$$\frac{R_2}{R_1} = \cot u_1$$

If the diameters are not the same, the formula assumes the following form:

$$\frac{R_2}{R_1} = \frac{D_1}{D_2} \cot u_1 \qquad (1)$$

Other formulas derived from general principles in gearing and from trigonometrical ratios are:

$$D_1 = \frac{p_1 N_1}{\pi} \qquad (2)$$

$$p_1 = \frac{\pi D_1}{N_1} \qquad (3)$$

$$H_1 = p_1 N_1 \cos u_1 = \pi D_1 \cos u_1 \qquad (4)$$

$$H_2 = p_1 N_2 \sin u_1 = \pi D_2 \sin u_1 \qquad (5)$$

$$2\,C = D_1 + D_2 \qquad (6)$$

$$L_1 = \pi D_1 \cot u_1 \qquad (7)$$

$$L_2 = \pi D_2 \tan u_1 \qquad (8)$$

44. Problems in Spiral Gearing.—In spiral-gear problems, the conditions are somewhat similar to those in spur gearing, that is, the distance between centers of shafts

and the speed ratio are given. The pitch diameters, number of teeth, and helix angle are to be found.

Before beginning the solution, certain assumptions have to be made, and these depend on several conditions. For instance, an angle u_1 may be selected and the corresponding diameters found, or vice versa. Since it is advantageous to have an angle somewhere near the more efficient values, as given in Art. **34**, it is desirable to select the angle, find the diameters, and then readjust the angle again, if the conditions require it.

The angle referred to in Art. **34** as the helix angle, and shown in Fig. 19, is the complement to the angle considered as the helix angle in spiral gearing. The rule there given when applied to spiral gears will be as follows: Maximum efficiency is obtained when the helix angle of the driver is 45°, but this angle may be increased to 60° without showing any serious decrease in efficiency. This refers to gears under heavy pressure and ordinary lubrication, but if the gears run in an oil bath, the helix angle of the driver may be as great as 70°. With light pressures and ordinary lubrication, the angle may be increased to about 78°.

With a given angle u_1, the diameters may be found from formula **1**, Art. **43**, by transposing and solving for $\frac{D_2}{D_1}$; or,

$$\frac{D_2}{D_1} = \frac{R_1}{R_2} \cot u_1 \qquad (1)$$

From formula **6**, Art. **43**, $2C = D_1 + D_2$ and

$$D_2 = 2C - D_1$$

Solving formula **1** for D_2,

$$D_2 = \frac{D_1 R_1}{R_2} \cot u_1$$

Inserting the value of D_2 given above,

$$2C - D_1 = \frac{D_1 R_1}{R_2} \cot u_1,$$

$$D_1 + \frac{D_1 R_1}{R_2} \cot u_1 = 2C$$

and

$$D_1 \left(1 + \frac{R_1}{R_2} \cot u_1\right) = 2C$$

Finally, $$D_1 = \frac{2C}{1 + \frac{R_1}{R_2} \cot u_1} \qquad (2)$$

45. Length of Normal Helix.—A more difficult part of spiral-gear problems is to find suitable values of the normal helixes H_1 and H_2 that will give a whole number of teeth with a given normal pitch p_n.

The relation between the normal helix and the diameter of a gear may be seen from Fig. 29, which shows an exten-

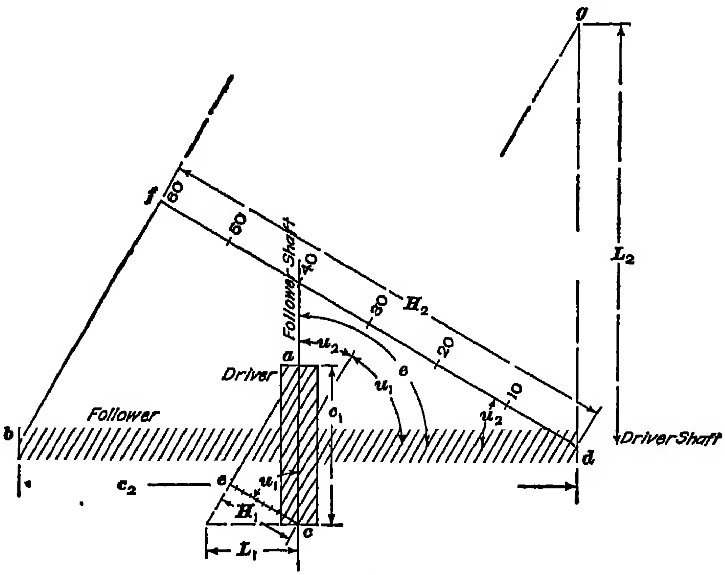


FIG. 29

sion of the diagram given in Fig. 28 (*a*), the whole length of the pitch circumference of each gear being shown. The circumference of the driver is c_1 and that of the follower c_2. The angle of the driver tooth helix is u_1 and that of the follower $u_2 = 90° - u_1$. The several angles in these gears are all complements of one another. On each gear, one of the teeth is extended to indicate the angle of the tooth helix with the shaft. The normals ec and df represent the lengths of the intercepted parts of the normal helixes, and are indicated by H_1 and H_2. The axis of the driver shaft is bd and

that of the follower ac. The point g is the intersection of the perpendicular at d and the line bf produced, the length L_2 being equal to dg.

From the rules for right triangles,

$$H_1 = c_1 \cos u_1 = \pi D_1 \cos u_1 \qquad (1)$$

and

$$H_2 = c_2 \sin u_1 = \pi D_2 \sin u_1 \qquad (2)$$

For convenience in the graphical solutions, these formulas are used in the following forms:

$$\frac{H_1}{\pi} = D_1 \cos u_1 \qquad (3)$$

$$\frac{H_2}{\pi} = D_2 \sin u_1 \qquad (4)$$

46. Number and Formation of Teeth.—By choosing a suitable value for the normal pitch, it is now possible to find the number of teeth on each gear and to ascertain whether they will be whole numbers or not.

As shown in Fig. 28 (*b*), the normal pitch is the same for both gears; hence, from the formula of Art. **40**, $H_1 = N_1 p_n$, the following formulas may be deduced:

$$\frac{H_1}{p_n} = N_1 \qquad (1)$$

$$\frac{H_2}{p_n} = N_2 \qquad (2)$$

The teeth of spiral gears are involute, and the same standard set of milling cutters is used as for ordinary spur gears. However, in selecting a suitable cutter for a given spiral gear, the determining factor is *not* the circular but the normal pitch of the gear; that is, the *circular pitch* of the teeth formed on a spur gear by the given cutter, sometimes called the *pitch* of the cutter, must equal the *normal pitch* of the spiral gear The *diametral pitch* of the gear, as used in the subsequent calculations, is based on the circular pitch of the *cutter*, and not on the circular pitch of the gear.

As, for practical reasons, only the circular or the diametral pitch of the cutter, and not the normal pitch p_n, is needed in the following calculations, the value p_n will be replaced by the corresponding circular pitch p_1 of the cutter.

From formula **2**, Art. **16**, $p_1 = \frac{\pi}{p}$; hence, $p_n = p_1 = \frac{\pi}{p}$. Inserting this value of p_n in formulas **1** and **2**,

$$N_1 = p\frac{H_1}{\pi} \qquad (3)$$

$$N_2 = p\frac{H_2}{\pi} \qquad (4)$$

In the graphical solutions given subsequently, the values $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$ are found directly. Formulas **3** and **4** show that by multiplying these values by the diametral pitch p, the number of teeth in each gear may be found at once. If these numbers are not whole numbers, they must be changed into the nearest whole numbers that will constitute the required speed ratio, after which the values $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$ must be readjusted.

NOTE —Formulas **3** and **4** prove that the values $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$ represent the diameters of ordinary spur gears with the same number of teeth as the spiral gears, that is, a spiral gear with a helix angle u_1 and diameter D_1 will contain the same number of teeth as a spur gear of diameter $\frac{H_1}{\pi}$, both being cut with the same cutter.

47. Application of the Formulas.—The application of the preceding formulas may be shown by the following example:

EXAMPLE.—The distance between the shaft centers of two spiral gears is to be 12 inches, nearly. The shaft angle e, Fig 29, is 90°, and the ratio of the revolutions of the driver to those of the follower is 6 to 1. It is desirable to have the tooth helix angle u_1 not more than 60° Calculate the diameters of the gears and the number of teeth on each gear.

SOLUTION.—First, make a trial calculation with the angle $u_1 = 60°$ and find D_1 and D_2. From formula **2**, Art **44**,

$$D_1 = \frac{2C}{1 + \frac{R_1}{R_2}\cot u_1} = \frac{2 \times 12}{1 + 6 \times \cot 60} = 5.376 \text{ in.}$$

$$D_2 = 2C - D_1 = 24 - 5\ 376 = 18.624 \text{ in.}$$

From formula **3**, Art. **45**,

$$\frac{H_1}{\pi} = D_1 \cos u_1 = 5.376 \times 5 = 2\ 688 \text{ in.}$$

From formula **4**, Art. **45**,

$$\frac{H_2}{\pi} = D_2 \sin u_1 = 18\,624 \times .866 = 16.128 \text{ in.}$$

If a diametral pitch of $p = 4$ is chosen, then, from formulas **3** and **4**, Art. **46**, $N_1 = p\frac{H_1}{\pi}$, and $N_2 = p\frac{H_2}{\pi}$; hence,

$$N_1 = 4 \times 2\,688 = 10\,752$$

and

$$N_2 = 4 \times 16.128 = 64.512$$

The fractional numbers represent the ratio 1 to 6. The nearest whole numbers of the same ratio are $N_1 = 10$ and $N_2 = 60$. If the angle of $u_1 = 60°$ is to be retained, a decrease in the number of teeth will result in a decrease in the lengths H_1 and H_2, followed by shorter diameters

From formula **3**, Art. **46**, it follows that $\frac{H_1}{\pi} = \frac{N_1}{p}$. The reduced value of $\frac{H_1}{\pi} = \frac{N_1}{p} = \frac{10}{4} = 2.5$ in. Likewise, from formula **4**, Art. **46**, the new value of $\frac{H_2}{\pi} = \frac{N_2}{p} = \frac{60}{4} = 15$ in.

The revised value of D_1, from formula **3**, Art. **45**, is

$$D_1 = \frac{H_1}{\pi \cos u_1} = \frac{2\,5}{.5} = 5 \text{ in.}$$

From formula **4**, Art. **45**,

$$D_2 = \frac{H_2}{\pi \sin u_1} = \frac{15}{.866} = 17.32 \text{ in.}$$

$$2\,C = D_1 + D_2 = 5 + 17.32 = 22.32 \text{ in.}$$

$$C = \frac{22.32}{2} = 11.16 \text{ in.} \quad \text{Ans.}$$

The diagram in Fig. 29 is drawn to scale and shows the dimensions and angles as finally calculated. The lengths H_1 and H_2 are shown subdivided into a number of parts, each equal to $p_n = p_1$ of milling cutter, the total number being equal to the number of teeth.

The leads L_1 and L_2 of the tooth helixes are found by means of formulas **7** and **8**, Art **43**; thus,

$$L_1 = \pi D_1 \cot u_1 = \pi\, 5 \times \cot 60° = 9.069 \text{ in.}$$

$$L_2 = \pi D_2 \tan u_1 = \pi\, 17.32 \times \tan 60° = 94.24 \text{ in.}$$

48. Formative Number of Teeth.—It is not sufficient to know the circular pitch of the cutter, as, according to the system of the Brown & Sharpe Manufacturing Company, an involute gear of a given pitch, whether a spur, bevel, or spiral gear, may be cut with any one of eight different cutters. Which cutter to select depends on the number of teeth in the gear, as the curvature of the pitch circle and,

therefore, the radial divergence of the teeth, will change with the diameter.

In order to select the proper cutter for a spiral gear, it is necessary to ascertain the curvature of the normal helix and the number of teeth contained in a pitch circle of this curvature. Let this number be termed the *formative* number of teeth $= N_f$, and let the radius and diameter of their pitch circle be R_f and D_f, respectively.

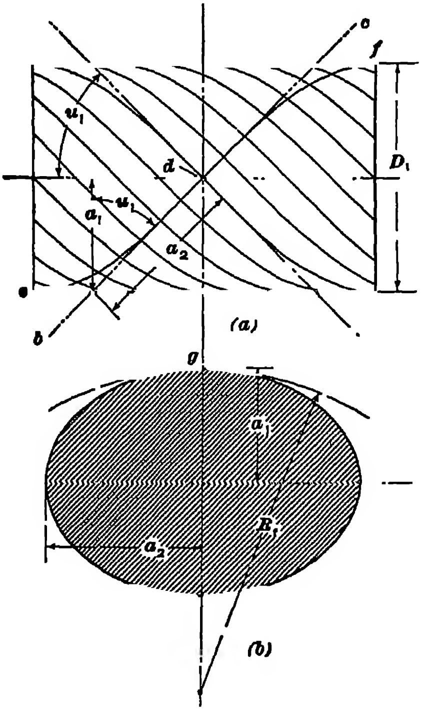


Fig 30

Fig. 30 (a) represents the pitch cylinder of a spiral gear of diameter D_1 and helix angle u_1. If a plane $b\,c$ intersects the cylinder so as to be tangent to the normal helix $e\,f$ at the point d, the outline of the cross-sectional area will be an ellipse, as shown separately in Fig. 30 (b). In this ellipse, $a_1 = \frac{D_1}{2}$, and, as seen from Fig. 30 (a),

$$a_2 = \frac{a_1}{\cos u_1} = \frac{D_1}{2 \cos u_1}.$$

In an ellipse, the radius of curvature at the point g is

$$R_f = \frac{a_2^2}{a_1} = \frac{D_1^2}{4 \cos^2 u_1} \times \frac{2}{D_1} = \frac{D_1}{2 \cos^2 u_1}$$

As $R_f = \frac{D_f}{2}$,

$$\frac{D_f}{2} = \frac{D_1}{2 \cos^2 u_1}$$

and

$$D_f = \frac{D_1}{\cos^2 u_1} \qquad (1)$$

Adapting formula **1**, Art. **16**, $n = p\,d$, and, consequently, $N_t = p\,D_t$. Inserting the value of D_t from formula **1**,

$$N_t = \frac{p\,D_1}{\cos^2 u_1}$$

By combining formula **3**, Art **45**, with formula **3**, Art. **46**,

$$D_1 = \frac{N_1}{p \cos u_1}$$

Substituting the latter value in the formula $N_t = \frac{p\,D_1}{\cos^2 u_1}$,

$$N_t = \frac{p}{\cos^2 u_1} \times \frac{N_1}{p \cos u_1}$$

Finally, for the *driver*,

$$N_t = \frac{N_1}{\cos^3 u_1} \qquad (2)$$

The same formula serves for the follower by inserting N_2 and u_2 for N_1 and u_1, respectively. Hence, for the *follower*,

$$N_t = \frac{N_2}{\cos^3 u_2} \qquad (3)$$

49. Graphical Solutions.—The solving of problems in spiral gearing and worm-gearing may be greatly facilitated by means of graphical methods. The diagrams employed for this purpose show very clearly the conditions that have to be complied with and the changes that may be made without affecting the whole calculation. Some problems in which the distance between centers is fixed will require very extended calculations if solved analytically; they may, however, be solved readily by graphical methods.

In Fig. 31, the triangles *a c e* and *b d f* of Fig. 29 are rearranged so as to let their hypotenuses adjoin and form one continuous line. The triangle *a b O*, Fig. 31, contains all the data referring to the *driver*, and *d e O* all the data for the *follower*. For the purpose of reducing the size of the diagram and for certain conveniences in calculation, the diameters D_1 and D_2 are used as hypotenuses in place of the circumferences c_1 and c_2, as in Fig. 29. This causes a corresponding reduction in the other dimensions, a fact that must be distinctly remembered during the application of these diagrams.

As D_1 is $\frac{1}{\pi}$ of the pitch circumference, it follows that all other dimensions in each triangle are reduced in the same ratio. For instance, the side $b\,O$ becomes $\frac{H_1}{\pi}$ and $d\,O$ becomes $\frac{H_2}{\pi}$, instead of H_1 and H_2, respectively. As mentioned in Art. **46**, the values $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$ represent the equivalent diam-

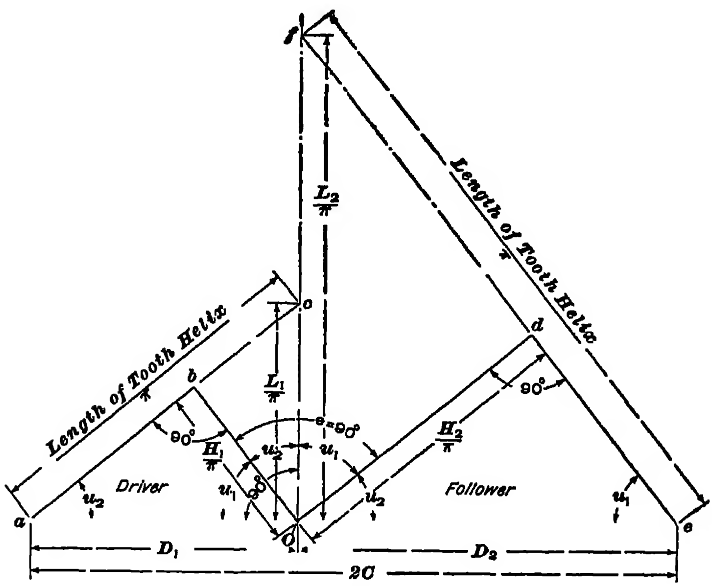


FIG. 31

eters of two spur gears having the same number of teeth as in the corresponding spiral gears, and the same cutter is used in both cases. The values $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$ may therefore be called the *equivalent diameters*.

Referring to the driver, if u_1 is the tooth helix angle, the line $a\,c$ would represent the length of the tooth helix for one turn, provided $O\,a$ were equal to the pitch circumference; but, for the reason given,

$$ac = \frac{\text{length of tooth helix}}{\pi}$$

$$Oc = \frac{L_1}{\pi}$$

$$Ob = \frac{H_1}{\pi}$$

Referring to the follower, the angle $u_2 = 90° - u_1$ = tooth helix angle.

$$ef = \frac{\text{length of tooth helix}}{\pi}$$

$$Of = \frac{L_2}{\pi}$$

$$Od = \frac{H_2}{\pi}$$

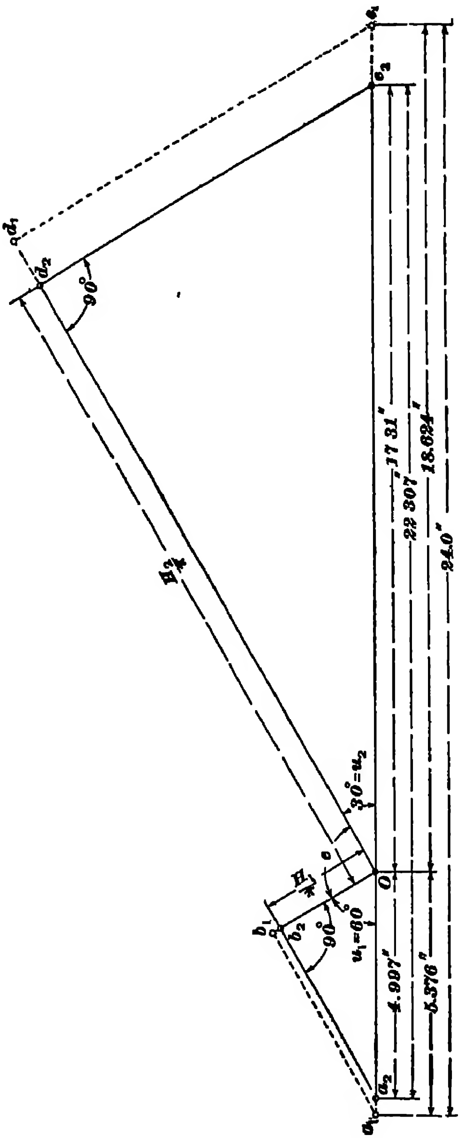
$$e = \text{angle between shafts} = 90°$$

In the subsequent solutions, the following terms are mostly used: D_1, D_2, u_1, u_2, $\frac{H_1}{\pi}$, and $\frac{H_2}{\pi}$.

50. Effect of Changes in the Diagram.—Considering the relation between the various angles and lines in Fig. 31, the following facts will be observed:

1. Retaining the values D_1 and D_2, a change of angle u_1 changes H_1, H_2, L_1, and L_2.
2. Retaining the values H_1 and H_2, a change of angle u_1 results in a change in the diameters D_1 and D_2.
3. With a given angle u_1, a variation of the values H_1 and H_2 is followed by a variation in the diameters D_1 and D_2.
4. The point O may be shifted to the left or the right without changing the value $2C$ or helix angles u_1 and u_2, but will result in changing the values D_1, D_2, H_1, H_2, L_1, and L_2. Or, the values H_1 and H_2 may remain constant while D_1, D_2, u_1, u_2, L_1, and L_2 change.

51. Examples of Graphical Solutions.—The application of the foregoing principles to the solution of spiral-gear problems is shown by means of Fig. 32, which solves graphically the problem solved analytically in Art. 47. The construction may proceed along different lines, depending on



e_1
e_2
d_1
d_2
90°
17.31"
22.307"
18.624"
24.0"
H_2
$30° = u_2$
O
H_1
90°
$u_1 = 60°$
b_1
b_2
4.997"
5.376"
a_2
a_1

Fig. 32

the limitations imposed. Assuming that the angle $u_1 = 60°$, $e = 90°$, C about 12 inches, and the speed ratio is 6 to 1, calculate the value D_1 from formula **2**, Art. **44.** Thus,

$$D_1 = \frac{2\,C}{1 + \frac{R_1}{R_2}\cot u_1} = 5.376 \text{ inches}$$

Draw the horizontal line $a_1 e_1 = 2\,C = 24$ inches, to as large a scale as the circumstances will allow, and from a_1 lay off the value of D_1, thereby locating the point O.

NOTE.—The length D_1 could be determined graphically, but the method indicated seems preferable as it is more accurate; also the diagram requires less space and is less encumbered by construction lines.

From O draw line $O\,b_1$ at an angle equal to the helix angle $u_1 = 60°$ with line $a_1 e_1$. From a_1 draw a line $a_1 b_1$ perpendicular to $O\,b_1$ and intersecting it at b_1, thus obtaining the length $\frac{H_1}{\pi}$.

Next, draw from O the line $O\,d_1$ at an angle $u_2 = 90° - u_1 = 30°$ with $a_1 e_1$, and from e_1 a perpendicular to $O\,d_1$ intersecting it at d_1, thus obtaining the length $\frac{H_2}{\pi}$. This completes the diagram.

From formulas **3** and **4**, Art. **46**, $N_1 = p\frac{H_1}{\pi}$ and $N_2 = p\frac{H_2}{\pi}$. By scaling the drawing, it is found that $\frac{H_1}{\pi} = 2.675$ inches and $\frac{H_2}{\pi} = 16.15$ inches. Selecting 4 as the diametral pitch, $N_1 = 4 \times 2.675 = 10.8$ and $N_2 = 4 \times 16.15 = 64\,6$.

As the speed ratio is 1 to 6, the numbers of teeth nearest the numbers giving this ratio are 10 and 60, respectively. The new values of $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$ may be found from the formulas for N_1 and N_2. Thus,

$$\frac{H_1}{\pi} = \frac{N_1}{p} = \frac{10}{4} = 2.5 \text{ inches}$$

and

$$\frac{H_2}{\pi} = \frac{N_2}{p} = \frac{60}{4} = 15 \text{ inches}$$

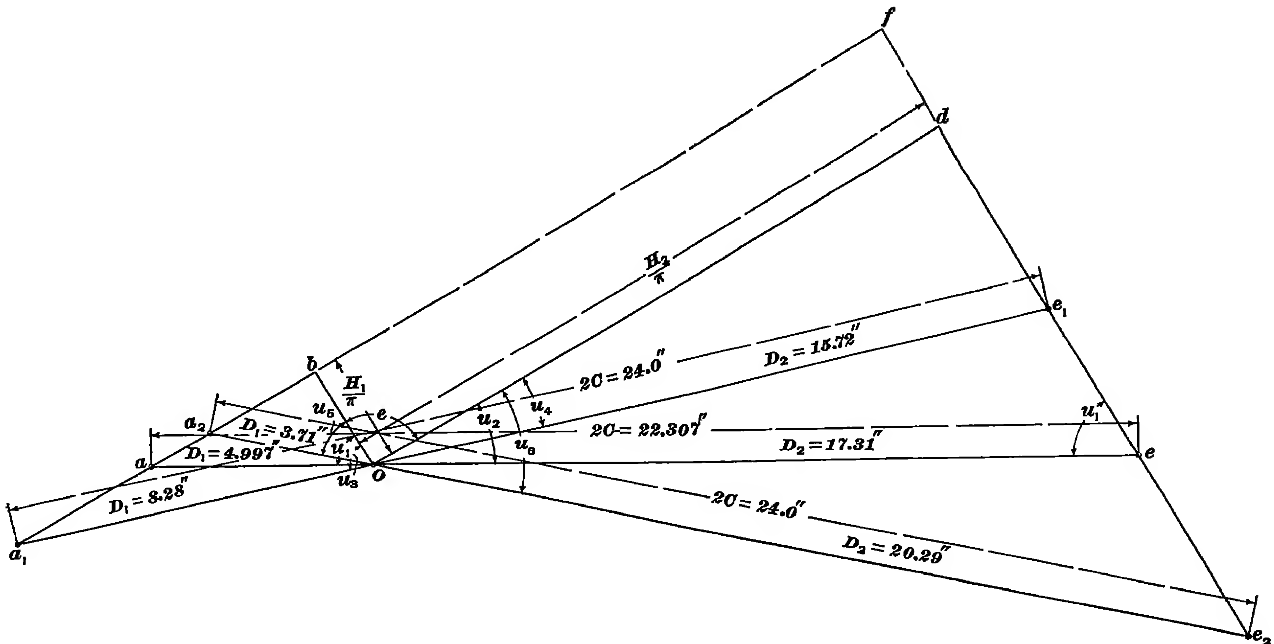

f
d
b
e
e
e1
e2
a
a1
a2
O
H2/π
H1/π
2C = 24.0″
2C = 22.307″
2C = 24.0″
D2 = 15.72″
D2 = 17.31″
D2 = 20.29″
D1 = 3.71″
D1 = 4.997″
D1 = 8.28″
u1
u2
u3
u4
u5
u6

Fig 33

The lines Ob_1 and Od_1 must therefore be correspondingly shortened, whereby point b_1 is transferred to the position b_2 and point d_1 to d_2.

Draw line b_2a_2 parallel with a_1b_1, intersecting ae_1 at a_2. The length Oa_2 is the required diameter D_1. Through the point d_2 draw a line d_2e_2 parallel with d_1e_1, intersecting a_1e_1 at e_2. The length Oe_2 is the new diameter D_2, and the sum $D_1 + D_2 = 2C = 4.997 + 17.31 = 22.307$ inches.

$$C = \frac{22.307}{2} = 11.154 \text{ inches}$$

In the diagram, the preliminary construction lines are shown dotted. In any of these diagrams, the leads $\frac{L_1}{\pi}$ and $\frac{L_2}{\pi}$ are found graphically in the manner indicated in Fig. 31.

52. **Fixed Center Distance.**—In some cases, the conditions are such that no variation in the center distance C is allowed. Under such conditions, a graphical solution is especially advantageous, as the drawing of *one* line will accomplish the same results that would otherwise be obtained only after extended and tedious calculation.

Let it be assumed that in the preceding example the center distance C must be exactly 12 inches. The solution could be made at once in Fig. 32, but for the sake of greater clearness, it is shown separately in Fig. 33.

The length $2C = 22.307$ inches, as found in the preceding solution, is shown as line aOe, Fig. 33. This line has now to be replaced by one 24 inches long. As the number of teeth is not to be changed, the values $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$ and, consequently, the position of the point O must be retained. Any variation must therefore be made in the lengths of the diameters, with corresponding changes in the angles u_1 and u_2.

Observing that the line ab is perpendicular to the line de, the former may be produced so as to intersect line de at f; these lines then form two sides of a right triangle afe. It is now necessary to adjust the hypotenuse $2C$ (24 inches long)

to these sides in such a position that its ends will be located on the lines $a\,b$ and $d\,e$, respectively, produced, if necessary, and that it simultaneously will pass through the point O. The easiest way to find this position is to lay a draftsman's scale along the line $a\,e$ with its zero point at a. While the rule is touching point O, its zero point is moved along line $a\,b$ until the point of intersection between the rule and the line $d\,e$ indicates the required length of $2\,C$. Two solutions are possible: one with the hypotenuse at $a_1 e_1$, giving a value of $D_1 = 8.28$ inches; $D_2 = 15.72$ inches; $u_2 = 72°\ 26'$, and $u_4 = 17°\ 34'$; the other with the hypotenuse at $a_2 e_2$, when $D_1 = 3.71$ inches, $D_2 = 20.29$ inches, $u_2 = 47°\ 38'$, and $u_4 = 42°\ 22'$.

Which one of these solutions to choose depends on conditions. In general, it is preferable to have D_1 small and u_1 approach 45°. If neither combination is satisfactory, then new values of p and $\frac{H_1}{\pi}$ must be selected and the process repeated.

NOTE —The accuracy of the values obtained may be tested by transforming formulas 3 and 4, Art. 45, into the forms $\cos u_1 = \frac{H_1}{\pi\,D_1}$ and $\sin u_1 = \frac{H_2}{\pi\,D_2}$. If all values are correct, then the two values of u_1 should correspond within the limits of accuracy required.

It should also be noted that in order to solve this problem graphically, the whole numbers selected for N_1 and N_2, Art 52, must be *smaller* than the fractional values found at first. These values were 10 8 and 64 6, respectively. If 11 and 66 had been chosen instead of 10 and 60, the diameters D_1 and D_2 would have been increased and the hypotenuse of 24 inches would be too short to intersect the sides $a\,f$ and $e\,f$ of the triangle $a\,f\,e$

53. Fixed Helix Angles.—A helix angle of 45° for the driver is frequently used on spiral gears. In this case, both gears will have the same angle, and under these circumstances, the diameter of each gear will be directly proportional to the number of teeth on each. In case lengths $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$ do not give the required number of teeth, adjustments can be made only in the diameters, as the angles are to remain 45°.

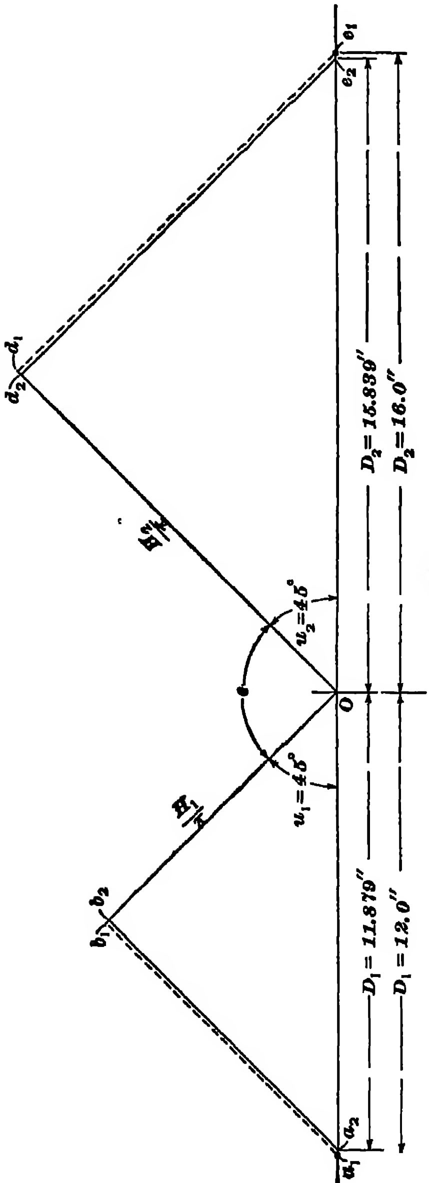


FIG. 34

EXAMPLE.—In the diagram shown in Fig. 34, the distance between the shaft centers of a pair of gears is to be 14 inches, approximately. The angle $e = 90°$, and the ratio of the revolutions of the driver to those of the follower is $\frac{R_1}{R_2} = \frac{4}{3}$. The helix angles u_1 and u_2 are 45°. Calculate the diameters of the gears and the number of teeth on each.

SOLUTION.—The diameters of the gears in this special case may be found by the proportions $(2\,C - D_2) : D_2 = 3 : 4$, or by the general formula **2**, Art. **44**

$$D_1 = \frac{2\,C}{1 + \frac{R_1}{R_2}\cot u_1}$$

By this formula,

$$D_1 = \frac{28}{1 + \frac{4}{3} \times 1} = 12 \text{ in.}$$

Then,

$$D_2 = 2\,C - D_1 = 28 - 12 = 16 \text{ in.}$$

The position of point O being known, the positions of points b_1 and d_1 may be determined in the manner already described. It is found that $\frac{H_1}{\pi} = 8.484$ in., and that $\frac{H_2}{\pi} = 11.312$ in. If a diametral pitch is chosen as $p = 5$, then,

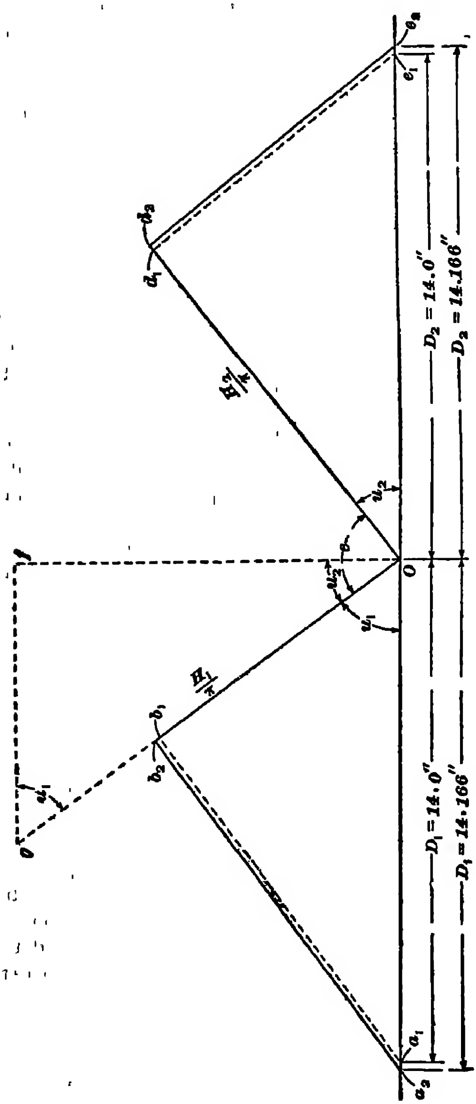


FIG. 35

from formulas 3 and 4, Art. 46,

$$N_1 = p\,\frac{H_1}{\pi} = 5 \times 8.484 = 42\,42$$

and
$$N_2 = p\,\frac{H_2}{\pi} = 5 \times 11\,312 = 56\,56$$

The nearest whole numbers of the required ratio are $N_1 = 42$ and $N_2 = 56$.

The corresponding new values of $\frac{H_1}{\pi}$ and $\frac{N_2}{\pi}$ are now

$$\frac{H_1}{\pi} = \frac{N_1}{p} = \frac{42}{5} = 8.4$$

and
$$\frac{H_2}{\pi} = \frac{N_2}{p} = \frac{56}{5} = 11.2$$

These new values are shown in Fig. 34 at $O\,b_2$ and $O\,d_2$. By drawing the parallel lines $a_2\,b_2$ and $d_2\,e_2$, the corrected diameters D_1 and D_2 are found. The correctness of these values may be checked by the formulas $D_1 = \frac{H_1}{\pi \cos u_1}$ and $D_2 = \frac{H_2}{\pi \sin u_1}$, which are derived from formulas 3 and 4, Art 45. From these formulas it is found that $D_1 = 11\,879$ in. and $D_2 = 15\,839$ in, and $2\,C = 27.718$ in. and $C = 13\,859$ in.

If it is desired to have the value of C nearer 14 in., other values of p may be tried For instance, with $p = 11$, $D_1 = 11.957$ in., $D_2 = 15.943$ in., $2\,C = 27.9$ in., and $C = 13\,95$ in.

54. Fixed Diameters.—A special case of **fixed diameters** is that where both gears have equal diameters. This case is illustrated by means of Fig. 35.

It is evident from the diagram that any necessary adjustment cannot be made in the helix angle, as such change will be followed by inequality of the diameters. Consequently, any needed adjustment can be made only by increasing or decreasing both diameters by an equal amount. From the fact that the diameters are equal, it follows that the number of teeth on each must be inversely proportional to its number of revolutions, or

$$\frac{N_1}{N_2} = \frac{R_2}{R_1}$$

In this case, the helix angle can be found directly, in the manner shown in Fig. 35.

EXAMPLE —The center distance of the shafts of a pair of spiral gears is 14 inches, and the shaft angle $e = 90°$ The speed ratio is 4 · 3 Calculate the helix angle and the number of teeth on each gear, if the diameters are equal.

SOLUTION —Lay off $a_1 O$ and $O e_1$, Fig 35, equal to D_1 and D_2, making each 14 inches Erect a perpendicular Of of any convenient length and divide it into four equal parts. From f draw a line fg perpendicular to Of and make it equal to $\frac{3}{4} Of$ Connect points g and O; then, the angle fgO = angle $gOa_1 = u_1$ From e_1 draw $e_1 d_1$ parallel with Og and from a_1 and o the perpendiculars $a_1 b_1$ and $O d_1$, the points of intersection b_1 and d_1 determining the lengths $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$, respectively, which are found to be 8 4 in. and 11 2 in.

Multiplying the latter values by a suitable diametral pitch will determine the number of teeth in each case If necessary, the diameters may then be readjusted to give a whole number of teeth.

Selecting a diametral pitch of 6,

$$N_1 = p\frac{H_1}{\pi} = 6 \times 8.4 = 50.4$$

and

$$N_2 = p\frac{H_2}{\pi} = 6 \times 11.2 = 67.2$$

The nearest whole numbers of teeth with a ratio of $\frac{3}{4}$ will be 51 and 68. Increasing the values $\frac{H_1}{\pi}$ and $\frac{H_2}{\pi}$ correspondingly,

$$\frac{H_1}{\pi} = \frac{N_1}{p} = \frac{51}{6} = 8\ 5 \text{ in.}$$

and

$$\frac{H_2}{\pi} = \frac{N_2}{p} = \frac{68}{6} = 11.333 \text{ in.}$$

Finally, D_1 = 14.166 in , and D_2 = 14.166 in Both diameters must therefore be lengthened by an amount equal to 166 in

If a value of p = 5 is taken, then no alterations will be necessary, as

$$N_1 = p\frac{H_1}{\pi} = 5 \times 8.4 = 42$$

and

$$N_2 = p\frac{H_2}{\pi} = 5 \times 11\ 2 = 56$$

55. Proportions of Spiral Gears.—The formulas used to find the proportions of blanks and teeth of spiral gears are the same as those given in Art. 16 for spur gears, but it is important to note the new basis for these formulas when applied to spiral gears.

For *spur gears*, the formulas are based on the circular and the corresponding diametral pitch of the gears.

For *spiral gears*, the formulas are based on the normal pitch of the gears, and the diametral pitch is based on the latter. This means that the formulas for spiral gears of diameters D_1, D_2, etc. are based on the circular pitch of spur gears of

diameters $\frac{H_1}{\pi}$, $\frac{H_2}{\pi}$, etc. The relation between these diameters, $\frac{H_1}{\pi}$, $\frac{H_2}{\pi}$, and D_1, D_2 was explained in Art. **46.** The same milling cutter is used for a spiral gear of diameter D_1 as for a spur gear of diameter $\frac{H_1}{\pi}$, and consequently the preceding calculations refer simply to the pitch of the cutter used—either circular or diametral—instead of the normal pitch.

In these formulas, p_1 is the circular pitch and p the diametral pitch of the milling cutter.

For convenience, the more important formulas are here repeated, the various letters indicating the terms being as follows:

a = addendum;
c = clearance at bottom of tooth;
$r = a + c$ = root;
e = working depth of tooth;
$a + r$ = whole depth of tooth;
t = thickness of tooth along normal pitch line;
D_1 and D_2 = diameters of pitch circles;
BD = outside diameter of gear = blank diameter;
N_1 and N_2 = number of teeth in gears.

Then,
$$a = \frac{1}{p} = \frac{p_1}{\pi} \quad (1)$$

$$t = \frac{p_1}{2} = \frac{1.5708}{p} \quad (2)$$

$$c = \frac{t}{10} \quad (3)$$

$$a + r = 2a + \frac{t}{10} = \frac{2.1571}{p} \quad (4)$$

$$\left.\begin{aligned} BD &= D_1 + 2a = D_1 + \frac{2}{p} \\ &= D_2 + 2a = D_2 + \frac{2}{p} \end{aligned}\right\} \quad (5)$$

EXAMPLE —In the example given in Art **51**, the diameter D_1 of the driver is 4 997 inches, the number of teeth $N_1 = 10$, and the angle $u_1 = 60°$ If the diametral pitch p of the cutter is 4, find the dimensions of the blank and of the teeth, and also the formative number of teeth by which to select a cutter.

SOLUTION —From the formulas given in this article, the following values are found:

From formula **1**, $a = \frac{1}{p} = \frac{1}{4}$ in.

From formula **2**,

$$t = \frac{1.5708}{p} = \frac{1.5708}{4} = .3927 \text{ in.}$$

From formula **3**, $c = \frac{t}{10} = .0393$ in.

From formula **4**,

$$a + r = \frac{2.1571}{p} = \frac{2.1571}{4} = .5393 \text{ in.}$$

From formula **5**,

$$BD = D_1 + \frac{2}{p} = 4.997 + \frac{2}{4} = 5.497 \text{ in.}$$

In order to ascertain the formative number of teeth by which to select a cutter, formula **2**, Art **48**, should be used Then,

$$N_f = \frac{N_1}{\cos^3 u_1} = \frac{10}{\cos^3 60^\circ} = \frac{10}{.125} = 80$$